Running vacuum model in a non-flat universe *

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Abstract: We investigate observational constraints on the running vacuum model (RVM) of $\Lambda = 3\nu(H^2 + K/a^2) + c_0$ in a spatially curved universe, where ν is the model parameter, K corresponds to the spatial curvature constant, a represents the scalar factor, and c_0 is a constant defined by the boundary conditions. We study the CMB power spectra with several sets of ν and K in the RVM. By fitting the cosmological data, we find that the best fitted χ^2 value for RVM is slightly smaller than that of ΛCDM in the non-flat universe, along with the constraints of $\nu \leqslant O(10^{-4})$ (68% C.L.) and $|\Omega_K = -K/(aH)^2| \leqslant O(10^{-2})$ (95% C.L.). In particular, our results favor the open universe in both ΛCDM and RVM. In addition, we show that the cosmological constraints of $\Sigma m_\nu = 0.256^{+0.224}_{-0.234}$ (RVM) and $\Sigma m_\nu = 0.257^{+0.219}_{-0.234}$ (ΛCDM) at 95% C.L. for the neutrino mass sum are relaxed in both models in the spatially curved universe.

Keywords: cosmological parameters from CMBR, cosmological simulations, dark energy theory

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1 Introduction

Since the recent discovery from type Ia supernova data [1-3] that our universe has been expanding at an accelerated rate, many dark energy models have been proposed to explain these phenomena [4-9]. The simplest one is the ΛCDM model, in which Λ represents the cosmological constant term. However, ΛCDM encounters some difficulties, primarily the "fine tuning" [10,11] and "coincidence" [12-14] problems.

The running vacuum model (RVM) [15,16] has been introduced to solve the "coincidence problem", where the cosmological constant term is assumed to vary with the Hubble parameter H. This model links the existence of dark energy to the theoretical mechanism of the quantum field, which may trigger the primordial inflation scenario [17], and fits with the observational data better than ΛCDM [18]. In the literature, the spatially flat RVM has been extensively investigated [18-35].

Recently, the Planck Legacy 2018 analysis by Valentino, Melchiorri, and Silk in Ref. [36] has suggested that the universe is closed at 99% C.L. [37]. Similar conclusions have been obtained by Park and Ratra in the context of non-spatially-flat DE models [38-41]. Nevertheless, when the BAO data set is included together with the CMB, the evidence in favor of a non-spatially-flat universe disappears completely [36,42,43]. These interesting results encourage us to study RVM in a non-flat universe [44,45], in addition to a flat one [18]. With the involvement of non-zero spatial curvature, it is inevitable to encounter degeneracies between the curvature and other parameters. One of them is the famous "geometrical degeneracy" [46,47] in CMB power spectra, caused by different sets of parameters that lead to the same value of the angular diameter distance of the last scattering. Conversely, when fitting with observational data, the non-zero spatial curvature also broadens the constraints of the cosmological parameters [48].

In this work, we concentrate on the running cosmological constant in the non-flat universe, $\Lambda = 3\nu(H^2 +$

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$K/a^2)+c_0$, where ν and c_0 are the model parameters, a represents the scalar factor, and K corresponds to the spatial curvature constant. We first study the CMB power spectra in this non-flat RVM and discuss the degeneracy between ν and the density parameter of curvature $\Omega_K=-K/(aH)^2$. We then constrain the cosmological parameters of both non-flat RVM and ΛCDM with the observational data using the Markov chain Monte Carlo (MCMC) method, and we compare the results with those in the flat universe. The effectiveness of RVM versus ΛCDM in the non-flat universe is also tested based on the minimal χ^2 values.

This paper is organized as follows. In Sec. 2, we introduce the non-flat RVM and derive the background evolution equations. We compare the CMB power spectra of RVM in the non-flat universe along with the Planck 2018 data and show the constraints of the cosmological parameters in Sec. 3. Our conclusions are presented in Sec. 4.

2 Evolution of RVM in a curved universe

We start with the Einstein field equation of the RVM, given by

$$R_{\alpha\beta}-\frac{1}{2}g_{\alpha\beta}R+g_{\alpha\beta}\Lambda=\kappa^2T_{\alpha\beta}, \tag{1}$$

where $\kappa^2=8\pi G$ is set to 1 for simplicity, $R=g^{\alpha\beta}R_{\alpha\beta}$ represents the Ricci scalar, $T_{\alpha\beta}$ stands for the energy-momentum tensor for matter and radiation, and Λ corresponds to the dynamical cosmological constant.

The spatially isotropic and homogeneous universe can be described by the Robertson-Walker metric:

$$\mathrm{d}s^2=-\mathrm{d}t^2+a^2(t)\left\{\frac{\mathrm{d}r^2}{1-Kr^2}+r^2\mathrm{d}\theta^2+r^2\sin^2\theta\mathrm{d}\phi^2\right\}, \tag{2}$$

where a is the scale factor, while K is a constant describing the spatial curvature, with $K=1,0,-1$ corresponding to closed, flat, and open universes, respectively. Then, the Friedmann equations can be expressed as

$$H^2=\frac{1}{3}(\rho_m+\rho_r+\rho_\Lambda)-\frac{K}{a^2}, \tag{3}$$

$$\dot{H}=-\frac{1}{2}(\rho_m+\rho_r+\rho_\Lambda+P_m+P_r+P_\Lambda)+\frac{K}{a^2}, \tag{4}$$

where $\rho_{m,r,\Lambda}$ ($P_{m,r,\Lambda}$) are the energy densities (pressures) of matter, radiation, and dark energy, respectively, and $H=\mathrm{d}a/(a\mathrm{d}t)$ represents the Hubble parameter. We note that $\rho_\Lambda=\kappa^{-2}\Lambda$, and the density parameters are given by

$$\Omega_{m,r}=\frac{\rho_{m,r}}{3H^2}, \tag{5}$$

$$\Omega_\Lambda=\frac{\Lambda}{3H^2}, \tag{6}$$

$$\Omega_K=-\frac{K}{a^2H^2}. \tag{7}$$

In the non-flat universe, the running cosmological constant term is set to

$$\Lambda=3\nu H^2+3\nu\frac{K}{a^2}+c_0, \tag{8}$$

where ν is a non-negative model parameter to ensure that the energy density of dark energy is positive in the early universe; c_0 is given by $c_0=-3\nu(H_0^2+K)+\Lambda_0$, where H_0 and Λ_0 are the values of the Hubble parameter and cosmological constant, respectively. The model becomes ΛCDM when $\nu=0$. The corresponding equations of state in this model can be defined as

$$w_{m,r,\Lambda}=\frac{P_{m,r,\Lambda}}{\rho_{m,r,\Lambda}}=0,\frac{1}{3},-1. \tag{9}$$

For the energy transformations from dark energy to matter and radiation, the modified continuity equations are given by

$$\dot{\rho}_{m,r}+3H(1+w_{m,r})\rho_{m,r}=Q_{m,r}, \tag{10}$$

$$\dot{\rho}_\Lambda+3H(1+w_\Lambda)\rho_\Lambda=-Q, \tag{11}$$

with $Q_m+Q_r=Q$, $Q_{m,r}$ can be written as

$$Q_{m,r}=-\frac{\dot{\rho}_\Lambda(\rho_{m,r}+P_{m,r})}{\rho_m+\rho_r+P_m+P_r}=3\nu H(1+w_{m,r})\rho_{m,r}. \tag{12}$$

By combining Eqs. (3)-(12), we derive the energy densities as functions of the scale factor:

$$\rho_m(a)=\rho_m^{(0)}a^{-3\xi}, \tag{13}$$

$$\rho_r(a)=\rho_r^{(0)}a^{-4\xi}, \tag{14}$$

$$\rho_\Lambda(a)=\rho_\Lambda^{(0)}+\left(\xi^{-1}-1\right)\left[\rho_m^{(0)}\left(a^{-3\xi}-1\right)+\rho_r^{(0)}\left(a^{-4\xi}-1\right)\right], \tag{15}$$

where $\rho_{m,r.\Lambda}^{(0)}$ are current values and $\xi=(1-\nu)$. Consequently, the Friedmann equation defined in Eq. (3) can be rewritten as

$$H^2=H_0^2\left\{\frac{1}{\xi}\left[\Omega_m^0\left(a^{-3\xi}-1\right)+\Omega_r^0\left(a^{-4\xi}-1\right)\right]+\Omega_K^0\left(a^{-2}-1\right)+1\right\}, \tag{16}$$

where $\Omega_{m,r,K}^0$ are current values of density parameters.

3 Numerical calculations

To study the degeneracy between the cosmological parameters, we first modify the **CAMB** [49] program to generate theoretical CMB power spectra for both the RVM and ΛCDM. The results are presented in Sec. 3.1. We then use the **CosmoMC** package [50], which is a Markov Chain Monte Carlo (MCMC) engine exploring the cosmological parameter space, to constrain RVM and

ΛCDM from the observational data. For simplification, we use Ω_K afterward to represent the density parameter of curvature at the present time, except where specifically indicated.

3.1 CMB power spectra of the models

There is "geometrical degeneracy" between the curvature and other parameters in the CMB power spectra. To see this effect, we compare the CMB power spectra of RVM and ΛCDM with various values of ν and Ω_K along with the observational data from Planck 2018 [43]. From previous studies [18,33,34] with $0 \leqslant \nu \leqslant O(10^{-3})$ in RVM for the flat universe and the result of $-0.007 \geqslant \Omega_K \geqslant -0.095$ at 99% C.L. in Ref. [36], we choose $0 \leqslant \nu < 0.01$ and $0 \geqslant \Omega_K \geqslant -0.01$ to see the degeneracy between ν and Ω_K in the CMB power spectra. Furthermore, the ΛCDM model is recovered when $\nu = 0$ and $\Omega_K = 0$ in Eq. (8).

In Fig. 1, we present the CMB power spectra for the TT, EE, and TE modes from the **CAMB** package. It can be seen that $0 \leqslant \nu \leqslant O(10^{-3})$ (solid lines) and $0 \geqslant \Omega_K \geqslant -O(10^{-2})$ (dashed lines) fit well with the data from Planck 2018. The residues with respect to ΛCDM are plotted in Fig. 2. We find that the geometrical degeneracy with $(\nu, \Omega_K) = (0.001, 0)$ (green solid line) and $(0.0, -0.01)$ (purple dashed line) has similar results in the CMB power spectra. However, only ν can cause strong suppression of the TT mode spectra when $\nu > 0$. In addition, the effects of ν and K show an additive property in the CMB power spectra (red dash-dotted line).

3.2 Global fitting

To constrain the cosmological parameters of RVM and ΛCDM in the non-flat universe, we use the **CosmoMC** package with an MCMC engine to explore the parameter space with combinations of the observational data sets, which include the CMB temperature fluctuation from *Planck 2018* with TT, TE, EE, low- l polarization from SMICA [43,51-53], BAO data from the 6dF Galaxy Survey [54] and BOSS [55], supernova (SN) data from the JLA compilation [56], the weak lensing (WL) data from CFHTLenS [57] and direct large-scale structure (LSS) formation data, and the data points of $f\sigma_8$ listed in Table 1. The priors of parameters are given in Table 2. Because of the tension between the geometry data (SNIa, BAO, etc.) and growth data (WL, $f\sigma_8$) [74], we choose the two combinations CMB+BAO+SN and CMB+BAO+SN+WL+ $f\sigma_8$ in our fits. To calculate the best fitted values of χ^2, we use

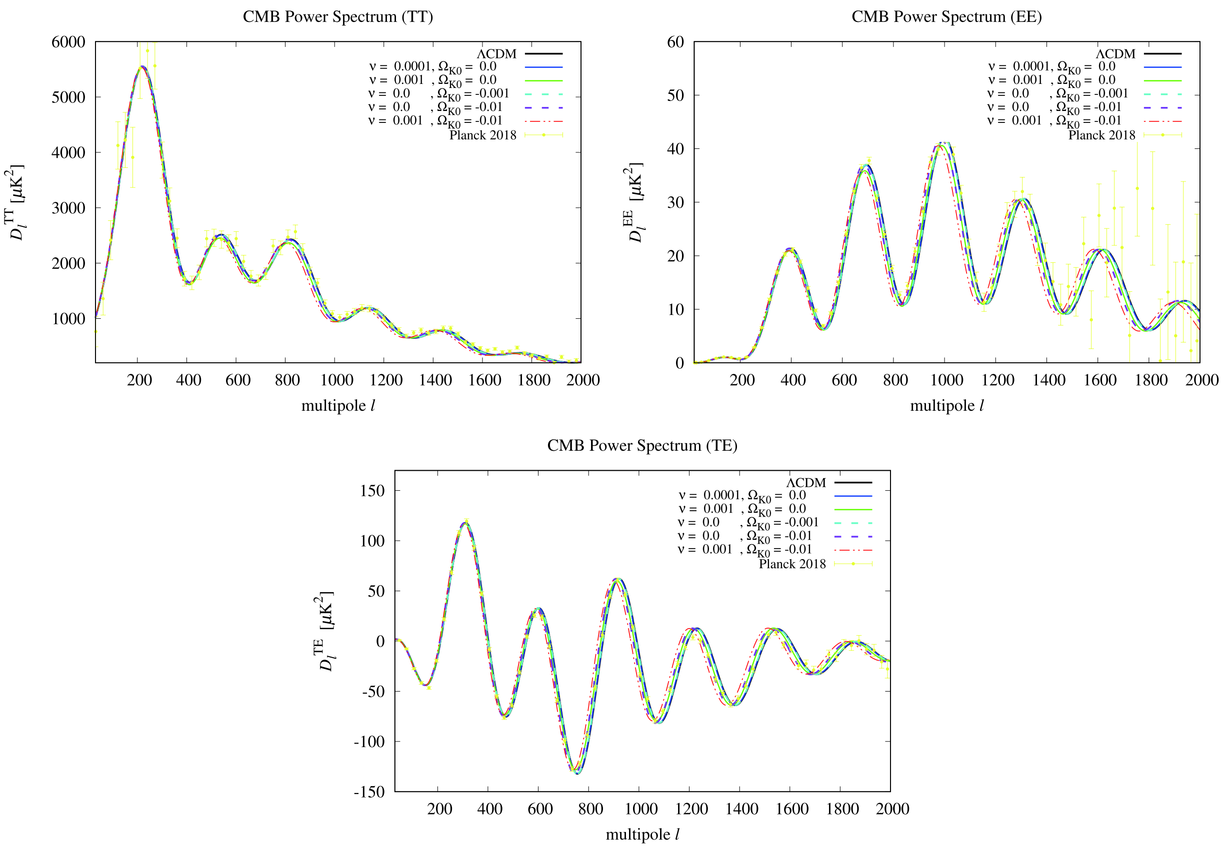

Fig. 1. (color online) Power spectra of CMB TT, EE, and TE for RVM and ΛCDM in flat and non-flat universes along with the Planck 2018 data.

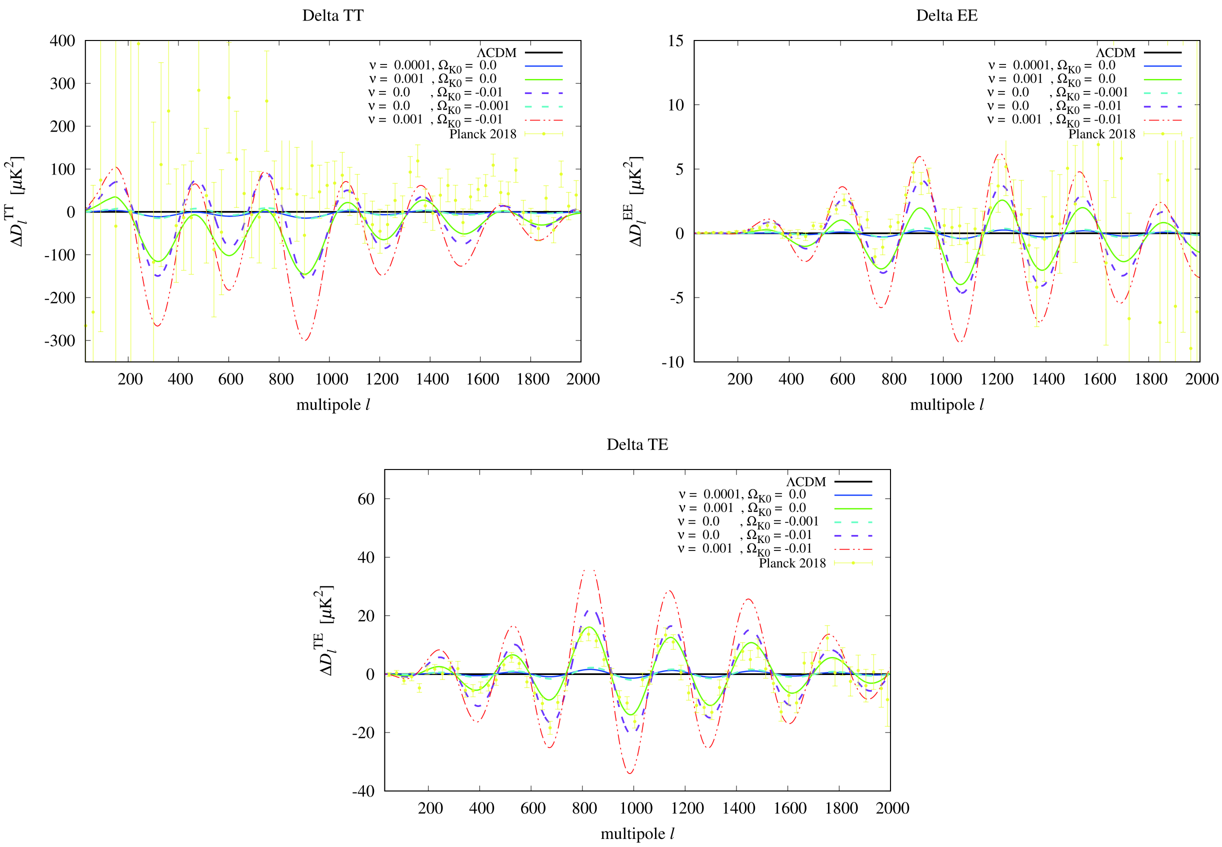

Fig. 2. (color online) Residuals of ΔD_ℓ^{TT}, ΔD_ℓ^{EE}, and ΔD_ℓ^{TE} in RVM with respect to ΛCDM for CMB power spectra, respectively, along with the observational data from Planck 2018.

Table 1. Data points of $f\sigma_8$.

	z	$f\sigma_8$	Ref.		z	$f\sigma_8$	Ref.		z	$f\sigma_8$	Ref.
1	1.36	0.482 ± 0.116	[58]	10	0.59	0.488 ± 0.06	[66]	19	0.35	0.440 ± 0.05	[61,69]
2	0.8	0.470 ± 0.08	[59]	11	0.57	0.444 ± 0.038	[67]	20	0.32	0.394 ± 0.062	[67]
3	0.78	0.38 ± 0.04	[60]	12	0.51	0.452 ± 0.057	[64]	21	0.3	0.407 ± 0.055	[65]
4	0.77	0.490 ± 0.18	[61,62]	13	0.5	0.427 ± 0.043	[65]	22	0.25	0.351 ± 0.058	[68]
5	0.73	0.437 ± 0.072	[63]	14	0.44	0.413 ± 0.080	[63]	23	0.22	0.42 ± 0.07	[60]
6	0.61	0.457 ± 0.052	[64]	15	0.41	0.45 ± 0.04	[60]	24	0.17	0.51 ± 0.06	[61,70]
7	0.60	0.390 ± 0.063	[63]	16	0.4	0.419 ± 0.041	[65]	25	0.15	0.49 ± 0.15	[71]
8	0.6	0.433 ± 0.067	[65]	17	0.38	0.430 ± 0.054	[64]	26	0.067	0.423 ± 0.055	[72]
9	0.60	0.43 ± 0.04	[60]	18	0.37	0.460 ± 0.038	[68]	27	0.02	0.36 ± 0.04	[73]

$$\chi_c^2=\sum_{i=1}^{n}\frac{(T_c(z_i)-O_c(z_i))^2}{E_c^i}, \qquad (17)$$

where c denotes the type of the data, n is the number of the data in each data set, T_c represents the theoretical value derived from **CAMB** at redshift z_i, and O_c (E_c) corresponds to the observational value (covariance).

The global fitting results of RVM and ΛCDM in the non-flat universe are plotted in Figs. 3 and 4, while those listed in Table 3 correspond to the cosmological parameters and ν, given at 95% and 68% C.L., respectively. Our results show that $\nu \lesssim 1.39\times10^{-4}$ at 68% C.L. in the non-flat universe of RVM for the data set CMB+BAO+SN+WL+$f\sigma_8$, which is similar to the previous result of 1.54×10^{-4} at 68% C.L. in RVM for a flat universe [33]. Explicitly, we obtain that $\chi^2_{\rm RVM}=3472.32\,(3523.74)$ and $\chi^2_{\Lambda\rm CDM}=3474.92\,(3524.51)$ when fitting with the data set CMB+BAO+SN (CMB+BAO+SN+WL+ $f\sigma_8$), indicating that our results in RVM are consistent with those in ΛCDM for a non-flat universe. For the density parameter Ω_K of the spatial curvature at the present time, our res-

Table 2. Priors for cosmological parameters with the non-flat RVM of $\Lambda = 3\nu(H^2 + K/a^2) + c_0$.

Parameter	Prior
RVM parameter ν	$0.0 \leqslant \nu \leqslant 3.0\times10^{-4}$
Curvature parameter Ω_K	$-0.25 \leqslant \Omega_K \leqslant 0.2$
Baryon density	$0.5 \leqslant 100\Omega_b h^2 \leqslant 10$
CDM density	$0.1 \leqslant 100\Omega_c h^2 \leqslant 99$
Optical depth	$0.01 \leqslant \tau \leqslant 0.8$
Neutrino mass sum	$0 \leqslant \Sigma m_\nu \leqslant 2$ eV
Sound horizon / Angular diameter distance	$0.5 \leqslant 100\theta_{MC} \leqslant 10$
Scalar power spectrum amplitude	$2 \leqslant \ln(10^{10}A_s) \leqslant 4$
Spectral index	$0.8 \leqslant n_s \leqslant 1.2$

ults show that an open universe is preferred, instead of the closed one in Ref. [36]. In particular, when the data of WL and $f\sigma_8$ are included, both RVM and ΛCDM favor an open universe with $|\Omega_K| \leqslant O(10^{-2})$. Our result for the open universe is consistent with that in Ref. [75].

The best-fit values of the neutrino mass sum, Σm_ν, in a non-flat universe are similar to those in a flat universe when fitting with CMB+BAO+SN [18,76-78]. However, it is interesting to see that the constraints on Σm_ν are relaxed for the data set CMB+BAO+SN+WL+ $f\sigma_8$, in which the $f\sigma_8$ data points play the main role. Note that similar results are also obtained in a flat universe, as shown in Ref. [18]. This is because the structure-growth rate of $f\sigma_8$ is a unique indicator of massive neutrinos [79,80]. Specifically, we have $\Sigma m_\nu = 0.256^{+0.224}_{-0.234}$ $(0.257^{+0.219}_{-0.234})$ eV at 95% C.L., resulting in non-zero lower bounds of $\Sigma m_\nu \geqslant 0.022$ (0.023) eV at 95% C.L. for RVM (ΛCDM) in a spatially curved universe. Conversely, becuase of the possible degeneracy among τ, Ω_K, and Σm_ν, the constraints on τ and Ω_K are not improved with the data points of WL+$f\sigma_8$, as seen from Table 3. To obtain better constraints, some additional data could be used, such as 21 cm emission measurements [81,82], which could fix the parameter τ to break the degeneracy of τ, Ω_K, and Σm_ν [79,80]. We note that in our data fitting, we do not specify the neutrino mass hierarchy in Σm_ν. For the cosmological effects of the neutrino mass hierarchy, refer to the discussion in the literature [83-85].

4 Conclusions

We have studied the running vacuum model with a cosmological constant of $\Lambda = 3\nu(H^2 + K/a^2) + c_0$ in a spatially curved universe. We have compared our results for several sets of ν and Ω_K with the Planck 2018 data in the CMB power spectra. We have found that ν and Ω_K have similar effects on the CMB power spectra, but only non-zero values of ν lead to large suppressions in the CMB

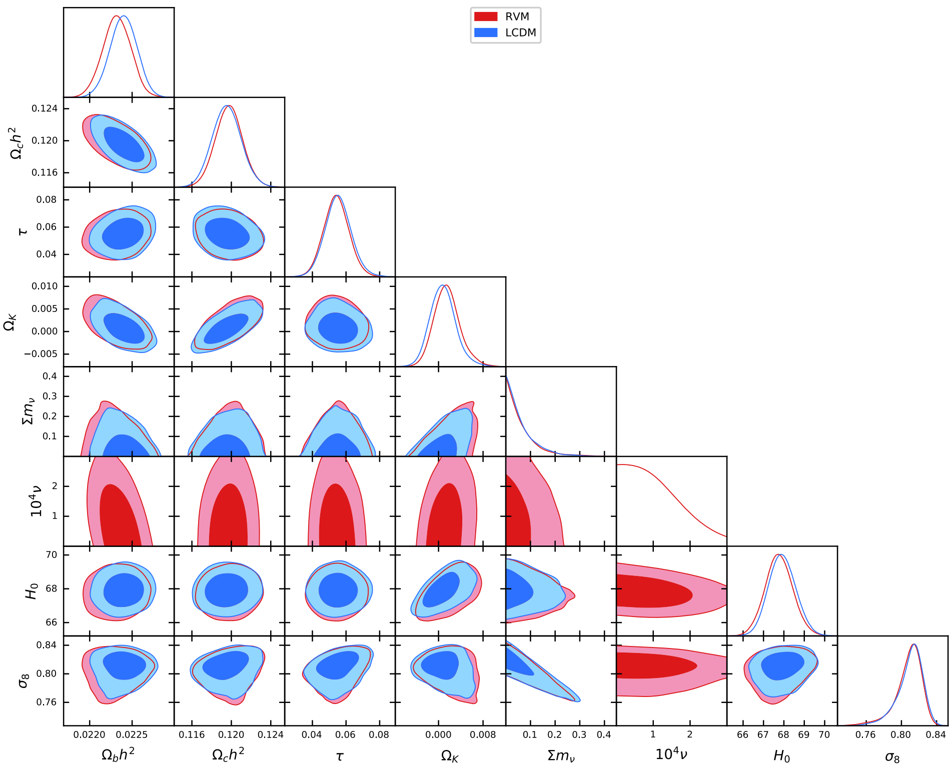

Fig. 3. (color online) One- and two-dimensional distributions of $\Omega_b h^2$, $\Omega_c h^2$, τ, Ω_K, $\sum m_\nu$, $10^4\nu$, H_0, and σ_8 for RVM and ΛCDM in a non-flat universe with the combined data CMB+BAO+SN, where the contour lines represent 68% and 95% C.L., respectively.

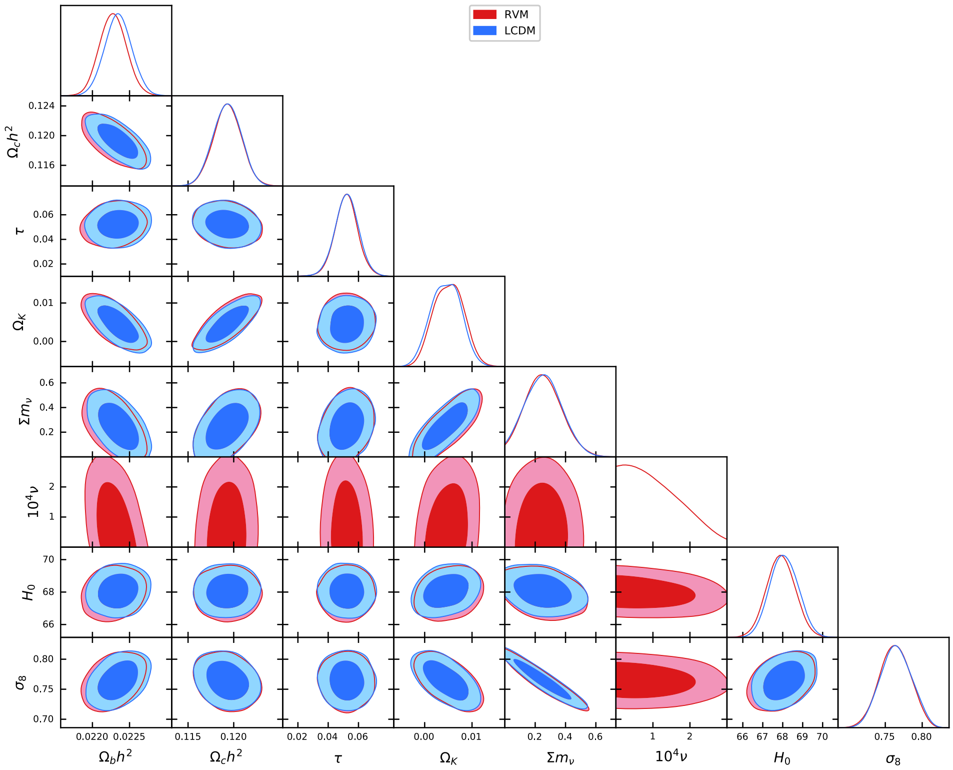

Fig. 4. (color online) One- and two-dimensional distributions of $\Omega_b h^2$, $\Omega_c h^2$, τ, Ω_K, $\sum m_\nu$, $10^4\nu$, H_0, and σ_8 for RVM and ΛCDM in a non-flat universe with the combined data CMB+BAO+SN+WL+$f\sigma_8$, where the contour lines represent 68% and 95% C.L., respectively.

Table 3. Fitting results for RVM and ΛCDM in a non-flat universe, where the cosmological parameters and ν are given at 95% and 68% C.L., respectively.

Parameter	CMB+BAO+SN		CMB+BAO+SN+WL+$f\sigma_8$	
Model	RVM	ΛCDM	RVM	ΛCDM
$\mathbf{100\Omega_b h^2}$	2.23 ± 0.03	2.24 ± 0.03	$2.23^{+0.04}_{-0.03}$	2.23 ± 0.04
$\mathbf{100\Omega_c h^2}$	12.0 ± 0.3	12.0 ± 0.3	11.9 ± 0.3	11.9 ± 0.3
$\mathbf{100}\tau$	$5.43^{+1.52}_{-1.45}$	$5.53^{+1.62}_{-1.52}$	$5.22^{+1.56}_{-1.50}$	$5.23^{+1.54}_{-1.58}$
$\mathbf{10^3\Omega_K}$	$1.55^{+4.91}_{-4.54}$	$0.80^{+4.72}_{-4.53}$	$5.10^{+5.97}_{-5.94}$	$4.53^{+5.92}_{-5.99}$
$\mathbf{\Sigma m}_\nu$ [eV]	< 0.199	< 0.188	$0.256^{+0.224}_{-0.234}$	$0.257^{+0.219}_{-0.234}$
$\mathbf{10^4}\nu$	< 1.36	–	< 1.39	–
H_0 [km/s/Mpc]	$67.8^{+1.4}_{-1.3}$	$67.9^{+1.3}_{-1.2}$	$67.9^{\pm}1.4$	$68.1^{+1.4}_{-1.3}$
σ_8	$0.808^{+0.027}_{-0.034}$	$0.810^{+0.026}_{-0.031}$	$0.764^{+0.040}_{-0.042}$	0.765 ± 0.040
$\chi^2_{\text{best-fit}}$	3472.32	3474.92	3523.74	3524.51

TT mode spectra. In the two combinations of the observational data, we have constrained $\nu \leqslant O(10^{-4})$, together with $|\Omega_K| \leqslant O(10^{-2})$. Notably, the constraints on ν in a non-flat universe are similar to those in a flat universe. From the best fitted values of χ^2, we have shown that RVM is inconsistent with ΛCDM. When fitting with the data set CMB+BAO+SN+WL+ $f\sigma_8$, we have obtained non-zero lower bounds of $\Sigma m_\nu \geqslant 0.022$ and 0.023 eV at 95% C.L. in the non-flat RVM and ΛCDM, respectively, indicating that the involvement of a non-zero Ω_K can provide viable constraints on the absolute neutrino masses in cosmological models.

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