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Search for decay modes of heavy and superheavy nuclei*

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Abstract: Spontaneous fission (SF) with a new formula based on a liquid drop model is proposed and used in the calculation of the SF half-lives of heavy and superheavy nuclei ($Z = 90–120$). The predicted half-lives are in agreement with the experimental SF half-lives. The half-lives of α decay (AD) for the same nuclei are obtained by using the Wentzel-Kramers-Brillouin (WKB) method together with Bohr-Sommerfeld (BS) quantization condition considering the isospin-dependent effects for the cosh potential. The decay modes and branching ratios of superheavy nuclei ($Z =$ 104-118) with experimental decay modes are obtained, and the modes are compared with the experimental ones and with the predictions found in the literature. Although some nuclei have predicted decay modes that are different from their experimental decay modes, decay modes same as the experimental ones are predicted for many nuclei. The SF and AD half-lives, branching ratios, and decay modes are obtained for superheavy nuclei ($Z = 119–120$) with unknown decay modes and compared with the predictions obtained in a previous study. The present results provide useful information for future experimental studies performed on both the AD and SF of superheavy nuclei.

Keywords: alpha decay, spontaneous fission, superheavy, Wentzel-Kramers-Brillouin (WKB) method

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1 Introduction

The α decay (AD) of nuclei is an important tool to study the structure of light, medium, and heavy, as well as superheavy nuclei produced at the accelerator centers around the world [1–8]. When superheavy nuclei are produced in the laboratory, they transition from the excited state to the ground state through the α decay chains. The observation and counting of these α particles provides information about the identification of the new synthesized superheavy nuclei. The spontaneous fission (SF) is another key decay energetically feasible for heavy and superheavy nuclei ($Z \geqslant 90$) [9, 10]. It was first proposed by Bohr and Wheeler [11], and subsequently observed by Flerov and Petrjak [12]. Since the SF of ^{238}U was discovered, many actinide nuclei with this type of radioactive decay have been reported in the experiments [13]. More recently, the SF half-lives of many superheavy nuclei were observed in different laboratories [14–16]. In fact, SF is an important limiting factor that describes the stability of synthesized superheavy nuclei. Theoretically, AD and SF comprise the same physical mechanism, i.e., the quantum mechanical tunneling effect.

In recent years, the studies on the AD of heavy and superheavy nuclei have become interesting and popular [17–26]. Many models and methods have been applied to investigate the AD of nuclei, such as the liquid drop model [27, 28], the cluster model [29], empirical formulas [30–32], and others [33–35]. It is difficult to model the interaction between nucleons in nuclei, as nucleus is a many-body system that contains numerous nucleons. In the cluster model, especially in terms of binary clustering, the many-body system can be reduced to a two-body (the core and surrounding α particle) system, and the problem can be easily solved [36, 37]. In this sort of model, α is assumed as to already exist in the nucleus before the decay, and it can be tunneled through the Coulomb barrier. This phenomena can be described as quantum tunneling. In the Gamow model, a formula between the half-life and Q-value was proposed, and followingly this relation and the formula were also produced by Geiger and Nuttall [38– 40]. More recently, a two-potential approach has been applied to calculate the half-lives of ADs for even-even nuclei [41], odd-A nuclei [42], and doubly odd nuclei [43]. In these studies, authors have used cosh-type nuclear potential including the isospin effects to calculate the AD half-lives of nuclei.

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The SF half-lives were obtained by using macroscopic-microscopic methods over the deformation parameters and nuclear shapes [44–47]. Because the case of SF is more complex than the AD, and many difficulties in the fission arise such as the mass and charge numbers of the two fragment nuclei and the number of emitted neutrons [48], the complete microscopic explanation of such a multidimensional system is extremely hard. The most realistic calculations of the SF half-lives can be performed by investigation of the multidimensional deformation space [45, 46]. Another method applied to calculating the SF half-lives is the phenomenological technique. A systematic study of the relation between the proton number (Z) and the mass number (A), as well as the half-lives, should make it possible to achieve a deep understanding of this phenomenon. There are different models employed to compute the SF half-lives in the literature [49, 50]. A semi-empirical formula was proposed by Swiatecki [51], upon which it was applied to obtain the SF half-lives of even-even, odd-A, and odd-odd nuclei. By using this formula, the author successfully reproduced the experimental data. Recently, a generalized Swiatecki formula [52, 53] with a set of new parameters was used to reproduce the experimental SF half-lives of the heavy and superheavy nuclei. Another possible decay mode in this region is the multicluster-accompanied fission, investigated in Ref. [54].

The study in Ref. [55] has shown that among the formulae used to calculate AD half-lives, the SemFIS2 formula performs the best in this prediction. In addition, the UNIV2 formula with the fewest parameters, as well as the VSS, SP and NRDX formulas with fewer parameters work well in the prediction of the AD half-lives of superheavy nuclei [56–64]. With regard to the cluster decay, there are many different studies on the calculations of cluster decay half-lives of nuclei considering various approaches in the literature [65–69].

Xu et al. [70] systematically investigated the AD and SF half-lives for heavy and superheavy nuclei with a proton number $Z \geqslant 90$. The AD half-lives were obtained by the deformed version of the density-dependent cluster model (DDCM). The SF half-lives of nuclei from ^{232}Th to ^{286}114 were calculated with the parabolic potential approximation by considering the nuclear structure effects. The competition between the AD and SF was analyzed in detail, and the branching ratios of these two decay modes were predicted for the unknown cases.

Bao et al. [71] obtained the AD half-lives of superheavy nuclei within the framework of the unified fission model (UFM) and the analytical formula. A modified formula based on Swiatecki's formula was proposed for explaining of the SF half-lives, which included the shell correction and isospin effect terms inside. The stability of superheavy nuclei against AD and SF, as well as the competition between them, were discussed. For nuclei with Z = 119–120, they interpreted the existing experimental decay modes and predicted decay modes of yet unknown nuclei.

Santhosh et al. [72] attempted to reproduce the experimental AD half-lives and modes of the decay of superheavy nuclei with the Coulomb and proximity potential model for deformed nuclei (CPPMDN), which is a deformed version of the Coulomb and proximity potential model (CPPM). A modified formula was proposed to obtain the SF half-lives by including the microscopic shell correction in the formula. A complete theoretical analysis is on the half-lives was conducted, and the decay modes of experimentally synthesized superheavy nuclei were obtained for the first time. More recently, Santhosh et al. have predicted the decay modes and half lives of all even Z isotopes of the superheavy elements within the range $104 \leqslant Z \leqslant 136$, and they have compared the results of the AD half-lives with the SF half-lives [73].

The aim of the present study is to perform a comprehensive investigation of both the AD and SF half-lives and to predict decay modes for superheavy nuclei with the known and yet unknown experimental decay modes. The half-lives are obtained for superheavy nuclei (Z = 104–118), for the SF with the new formula and for the AD using the WKB method together with the BS quantization condition for cosh potential, including the isospin effects. The new formula is used to obtain the SF values of nuclei with Z = 108–120 and the logarithmic values of the SF. These are then compared with the results of other models. Branching ratios for the SFs and ADs are calculated, and the modes of decays are predicted for Z = 104–118 nuclei, which have the known experimental decay modes. The predictions are in good agreement with experiment. Branching ratios for the SF and ADs are likewise obtained, and the decay modes are predicted for Z = 119–120 nuclei with experimental decay modes that are still unknown. Different decay modes from the predicted ones in the literature are obtained for some nuclei.

In Section 2, the theoretical background and equations required for the SF and AD half-lives in the WKB method together with BS are presented. The obtained numerical results and discussion can be found in Section 3. Section 4 is devoted to discussion.

2 Theoretical models

2.1 Spontaneous fission

Although the SF process is described as the quantum tunneling effect in physics, it is difficult to solve such a multidimensional penetration problem. This problem can be simplified to a one-dimensional WKB approach. Similarly to the AD, the only unknown term is the potential, and the so-called Hill-Wheeler formula can be obtained in a parabolic potential [70]. By modeling the potential, the following expression of spontaneous fission was given by Xu et al. [70]

$$T_{\rm SF}=\frac{\ln 2}{n.P_{\rm SF}}=e^{2\pi[c_0+c_1A+c_2Z^2+c_3Z^4+c_4(N-Z)^2-Q_{\rm SF}]},\qquad(1)$$

where $Q_{\rm SF}=0.13323\dfrac{Z^2}{A^{1/3}}-11.64$. Eq. (1) has five parameters that were obtained from fitting to the experimental SF half-lives of 45 even-even nuclei from ^{232}Th to ^{286}114. These values are given in Ref. [70] as $c_0=-195.09227$, $c_1=3.10156$, $c_2=-0.04386$, $c_3=1.40301\times10^{-6}$, $c_4=-0.03199$.

In this study, moving from the idea of Xu et al. [70], a new function is proposed and used by establishing similarity with the nuclear liquid drop model. It is given by

$$T_{\rm SF}=e^{2\pi[aA+bA^{2/3}+cZ(Z-1)/A^{1/3}+d(N-Z)^2/A+eZ^4+f]},\qquad(2)$$

where Z, N and A are the proton, neutron and mass numbers of the parent nuclei, and a, b, c, d, e, f are the adjustable parameters that can be obtained by fitting to experimental SF half-lives. This equation is given in terms of years.

Eq. (2) is a new semi-empirical formula proposed for spontaneous fission half-lives. It can be considered as the modified form of the formula of Xu et al. [70]. Hence, this formula was inspired by the binding energy formula of the liquid drop model. Each term in the liquid drop model is assumed to correspond to a change of the SF half-lives with Z, N, and A. The aA term, i.e., the volume effect, is used to model an increase in SF half-lives with A. The $bA^{2/3}$ term, which depicts the surface effect, shows an increase of SF half-lives proportional to $A^{2/3}$. The $cZ(Z-1)/A^{1/3}$ term, depicting the Coulombic effect, is used to model an increase in SF with $Z(Z-1)/A^{1/3}$, and the $d(N-Z)^2/A$ represents the isospin effects. Finally, the eZ^4 term is added to formula to consider a higher-order correction of the Coulomb term, which describes the transition from asymmetric to symmetric charge distributions for various fission nuclei [70]. Furthermore, the f parameter is added to the expression to take into account other contributions to SF half-lives. The obtained fitting parameters are given by $a=-10.0987592959$, $b=119.319858732$, $c=-0.516609881059$, $d=-9.52538327068$, $e=1.92155604207\times10^{-6}$, and $f=-1496.05967574$. In the fitting used in the calculations, the curve fit function was used in Scipy in Python 2.7 program language based on Spyder 2 with Anaconda [74]. As the SF is considered to be dependent on the binding energy of the nucleus and the Q-value, this can be modeled in terms of the liquid drop model, as in Eq. (2). It is not necessary to consider $Q_{\rm SF}$ separately in Eq. (2), as the equation already includes this term.

When comparing this new formula for the SF with the formula of Xu et al. [70], there is one extra term, which depicts the surface term, in the new formula that resultantly comprises six parameters. However, this does not include the $Q_{\rm SF}$ term, which is within the formula of Xu et al. [70]. Since all parameters include the atomic number A in this new form, the values of fitting parameters are changed as well. Moreover, even if most of the parameters of this formula were inspired by the terms of the binding energy formula of the liquid drop model, this model produces the experimental SF values of similar rms to the formula of Xu et al., which was obtained from the basically effective potential, including the nuclear, Coulomb, and isospin potential.

2.2 α decay

The AD half-life can be obtained using the following formula

$$T_{1/2}=\hbar\frac{\ln 2}{\Gamma},\qquad(3)$$

where Γ denotes the decay width for the decay. According to the semi-classical WKB method, the α-decay width Γ is given by,

$$\Gamma=P_\alpha F\frac{\hbar^2}{4\mu}\exp\left[-2\int_{r_2}^{r_3}k(r){\rm d}r\right],\qquad(4)$$

where P_α is the preformation probability of the α particle in a parent nuclei [75, 76]. In the half-life calculations, similar studies [77] and the experimental study, the preformation probabilities are specified as $P_\alpha=1.0$ for even-even nuclei, $P_\alpha=0.6$ for odd-A nuclei, and $P_\alpha=0.35$ for odd-odd nuclei. In Eq. (4), the normalization factor is

$$F=1\Bigg/\int_{r_1}^{r_2}\frac{1}{k(r)}{\rm d}r\cos^2\left(\int_{r_1}^{r}k(r'){\rm d}r'-\frac{\pi}{4}\right),\qquad(5)$$

where the squared cosine term might be replaced by 1/2 without significant loss of accuracy [75, 77]. In Eqs. (4) and (5), the wave number $k(r)$ is given by,

$$k(r)=\sqrt{\frac{2\mu}{\hbar^2}|Q-V_{\rm eff}(r)|},\qquad(6)$$

where Q is Q-value for the AD, and $V_{\rm eff}(r)$ is the effective potential between the α and core nuclei that stems from the binary clustering model that assumes the parent nuclei as the α particle surrounding the daughter (core) nuclei. Hence, the only unknown term in these equations is the effective potential between α and the core, and it is given by,

$$V_{\rm eff}(r)=V_N(r)+V_C(r)+V_L(r),\qquad(7)$$

where r is the separation radius between the center of mass of the α particle and the daughter nucleus.

In this study, the modified form proposed by Brink and Takigawa in Ref. [78] was used instead of the Coulomb potential to solve the discontinuity in the Coulomb potential in WKB semi-classical calculations as follows

$$\widetilde{V}_{\rm C}(r) = \frac{Z_d Z_\alpha e^2}{r}(1 - e^{-\varphi r - \frac{1}{2}(\varphi r)^2 - 0.35(\varphi r)^3}), \quad \varphi R = \frac{3}{2}, \tag{8}$$

where Z_α and Z_d are the charge numbers of the α and daughter nuclei, and R is the Coulomb radius. In Eq. (7), the last term is Langer modified centrifugal barrier potential [79] that is given by

$$V_L(r) = \frac{\hbar^2 (L+1/2)^2}{2\mu r^2}, \tag{9}$$

with the WKB being valid for one-dimensional problems, the above modification from $L(L+1) \to (L+\frac{1}{2})^2$ is essential to ensure the correct behavior of the WKB wave function near the origin as well as the validity of the connection formulas used in Ref. [80]. In this study, $L = 0$ is used in the calculations.

Although the forms of the Coulomb and centrifugal potentials are known very well, the shape of the nuclear potential in Eq. (7) is the only unknown term. As the analytical formula for the nuclear interaction between the α and core nuclei cannot be written, various potential models, phenomenological or microscopic, should be used to determine the nuclear interaction. In this study, the nuclear potential is considered as phenomenological cosh potential similar to Ref. [41]. The cosh potential was proposed by Buck and Pilt [81], and it is a symmetrized form of the Woods-Saxon form,

$$V_N(r) = -V_0 \frac{1+\cosh\left(\dfrac{\lambda}{a}\right)}{\cosh\left(\dfrac{r}{a}\right)+\cosh\left(\dfrac{\lambda}{a}\right)}, \tag{10}$$

where V_0 and a are the depth of the nuclear potentials and diffuseness parameters, respectively. The studies using this form of potential were conducted to obtain both the AD and exotic decay half-lives of heavy nuclei [77, 82, 83]. Furthermore, λ, the renormalization factor, is obtained by the Bohr-Sommerfeld quantization. As the isospin effect plays a important role in nuclear physics, one should take into account isospin effect in AD calculations as well. If protons and neutrons in the nucleus have a different nucleon density, the asymmetry of the isospin might affect the motion of alpha particles on the surface, and the nuclear interaction potential between the α and core nuclei would be isospin-dependent. In Ref. [41], the authors have added a parameter related to the isospin in the depth of the nuclear potential to include it in their considerations. They have used this potential form to investigate the isospin effects on the α-decay half-lives for the even-even nuclei from $Z = 62$ to $Z = 118$ using the two-potential approach. Considering this effect improved the results by 6.8% in Ref. [41]. In this study, to be able to consider the isospin effects on the AD as well as the SF, the isospin-dependent potential parameter $V_0 = 192.42 + 31.059(N-Z)/A$ MeV similar as Ref. [41] and $a = 0.75$ fm were used. V_0 and a were obtained phenomenologically to obtain the best AD half-life values that are close to the experiment.

Moreover, λ is determined separately for each decay by applying the Bohr-Sommerfeld quantization condition. The λ in the Eq. (10) can be calculated for every single decay by using the Bohr-Sommerfeld quantization rule,

$$\int_{r_1}^{r_2} \sqrt{\frac{2\mu}{\hbar^2}(Q - V_{\rm eff}(r))}{\rm d}r = (G-L+1)\frac{\pi}{2}, \tag{11}$$

where G are global quantum numbers coming from the Wildermuth condition [84], and they are used as follows [75, 76]

$$\begin{aligned} &G = 22 \quad (N > 126), \\ &G = 20 \quad (82 < N \leqslant 126), \\ &G = 18 \quad (N \leqslant 82). \end{aligned} \tag{12}$$

In the semiclassical WKB approximation, there are three classical turning points, which are r_1, r_2, and r_3. They are obtained by numerical solutions of the equation of $V_{\rm eff}(r) = Q$, where Q is the α-decay energy for special decays [77, 85].

3 Results and discussion

The SF half-lives of even-even nuclei with $Z = 90-114$ were calculated using the proposed formula in Eq. (2) to find how the present formula obtains experimental SF half-lives. The obtained results and results of other models (by Xu et al. [70], by Bao et al. [71], and by Santhosh et al. [72]) are listed in Table 1. In Table 1, the first column depicts the nuclei, the second column depicts the proton number Z and neutron number N for parent nuclei, respectively. The Exp. column shows the experimental $\log_{10}T_{\rm SF}$ values (in years) of spontaneous fission (SF) of nuclei with $Z = 90-114$, which are taken from Refs. [13, 86]. The results obtained by Xu et al. [70] (Xu), Bao et al. [71] (Bao), and Santhosh et al. [72] (KPS) are also presented in Table 1.

To compare the results, the rms deviations of the decimal logarithmic values are calculated using the following equation,

$$\sigma = [\frac{1}{n-1}\sum_{k=1}^{n}[\log_{10}(T_{\rm SF}^{\rm cal}) - \log_{10}(T_{\rm SF}^{\rm exp})]^2]^{1/2}, \tag{13}$$

where n denotes the number of the related nuclei [32]. The rms deviation (σ) was computed for the present model calculations. The obtained value is presented in Table 2. In Table 2, σ values were also presented for Xu [70], Bao [71], and KPS [72]. As depicted in Table 2, $\sigma = 1.22$ was obtained for this present model.

The parameters obtained by fitting and Eq. (2) have

Table 1. Calculated $\log_{10} T_{\mathrm{SF}}$ (in years) and results of other models for SF half-lives of nuclei with $Z = 90$–114.

nuclei	Z	N	Exp.	Xu[70]	Bao[71]	KPS[72]	present
^{232}Th	90	142	21.08	21.88	22.22	21.87	21.13
^{234}U	92	142	16.18	16.03	16.04	16.44	15.87
^{236}U	92	144	16.40	16.56	16.26	16.36	16.42
^{238}U	92	146	15.91	16.38	16.04	15.35	16.17
^{236}Pu	94	142	9.18	9.71	9.65	10.24	9.81
^{238}Pu	94	144	10.68	10.99	10.24	11.18	11.18
^{240}Pu	94	146	11.06	11.55	10.84	11.40	11.74
^{242}Pu	94	148	10.83	11.40	10.92	10.81	11.51
^{244}Pu	94	150	10.82	10.54	11.08	9.57	10.52
^{240}Cm	96	144	6.28	5.02	4.52	5.40	5.28
^{242}Cm	96	146	6.85	6.33	5.34	6.62	6.65
^{244}Cm	96	148	7.12	6.92	6.69	7.00	7.23
^{246}Cm	96	150	7.26	6.80	7.35	6.74	7.03
^{248}Cm	96	152	6.62	5.96	7.41	5.67	6.06
^{250}Cm	96	154	4.05	4.41	4.61	3.37	4.35
^{242}Cf	98	144	−1.33	−1.27	−1.17	−0.71	−1.13
^{246}Cf	98	148	3.26	2.12	2.09	2.75	2.43
^{248}Cf	98	150	4.51	2.74	3.27	3.42	3.02
^{250}Cf	98	152	4.23	2.65	4.31	3.25	2.84
^{252}Cf	98	154	1.93	1.84	2.11	1.76	1.90
^{254}Cf	98	156	−0.78	0.32	−0.82	−0.33	0.23
^{246}Fm	100	146	−6.60	−5.01	−4.15	−4.14	−4.94
^{248}Fm	100	148	−2.94	−2.93	−2.43	−1.92	−2.76
^{250}Fm	100	150	−0.10	−1.57	−0.67	−0.35	−1.37
^{252}Fm	100	152	2.10	−0.92	0.89	0.36	−0.77
^{254}Fm	100	154	−0.20	−0.98	−1.04	−0.26	−0.92
^{256}Fm	100	156	−3.48	−1.76	−3.71	−1.61	−1.83
^{252}No	102	150	−6.54	−6.04	−5.38	−4.70	−6.00
^{254}No	102	152	−3.04	−4.65	−3.28	−3.12	−4.61
^{256}No	102	154	−4.77	−3.97	−4.72	−2.90	−3.99
^{254}Rf	104	150	−12.14	−10.62	−9.35	−9.14	−10.74
^{256}Rf	104	152	−9.71	−8.48	−6.98	−6.73	−8.57
^{258}Rf	104	154	−9.35	−7.06	−7.74	−5.63	−7.17
^{260}Rf	104	156	−9.2	−6.36	−8.87	−5.24	−6.54
^{262}Rf	104	158	−7.18	−6.36	−8.32	−5.20	−6.65
^{258}Sg	106	152	−10.04	−12.34	−9.63	−10.19	−12.48
^{260}Sg	106	154	−9.65	−10.17	−9.80	−8.31	−10.31
^{262}Sg	106	156	−9.32	−8.72	−10.41	−7.13	−8.91
^{264}Sg	106	158	−8.93	−7.98	−9.42	−6.30	−8.26
^{266}Sg	106	160	−7.86	−7.96	−7.48	−5.80	−8.35
^{264}Hs	108	156	−10.2	−11.02	−12.10	−9.14	−11.10
^{270}Ds	110	160	−8.6	−9.46	−10.22	−7.23	−9.39
^{282}112	112	170	−10.58	−9.39	−11.28	−7.21	−9.40
^{284}112	112	172	−8.5	−11.43	−9.65	−8.14	−11.52
^{286}114	114	172	−8.08	−7.12	−5.95	−4.45	−6.44

Table 2. Rms values for all models.

σ (rms values)			
Xu et al.[70]	Bao et al.[71]	KPS [72]	present
1.27	1.12	1.69	1.22

been used to calculate the SF half-lives and compare them with the results of three different models for the even-even superheavy nuclei with Z = 108, 110, 112, 114, 116, 118, and 120 as listed in Table 3, 4, and 5, respectively. In these Tables, the Z, N, A depict the proton, neutron and mass number of nuclei, respectively. The Xu column lists the results of Xu et al. [70], the Bao lists the results of Bao et al. [71], the KPS shows the results of Santhosh et al. [72], and the "present" column depicts the obtained $\log_{10} T_{\mathrm{SF}}$ values in terms of second in this study.

As shown in the tables, even if the logarithmic values of Xu, Bao, KPS, and the present study exhibit similar behaviors of change according to the mass number of the parent nuclei, their size is different. However, the results of Bao et al. show slightly different behavior in comparison to the others.

A successful model should produce both experimental SF half-lives and predict the decay modes of nuclei. Superheavy nuclei decay through the AD, followed by the SF. If the half-lives of AD are shorter than the SF, then nuclei survive the fission and therefore decay through the AD. The α decay half-lives for even-even nuclei from Z = 104 to Z = 118 were calculated within the framework of the WKB method and BS quantization rule by considering the isospin-dependent effects and the SF half-lives using the proposed formula (Eq. (2)). The obtained results are shown in Table 6. To make predictions about which decay is dominant for each nuclei, the branching ratios for SF (%) $((T_\alpha/(T_{\mathrm{SF}}+T_\alpha))\times 100)$ and α decay (%) $((T_{\mathrm{SF}}/(T_{\mathrm{SF}}+T_\alpha))\times 100)$ were calculated, and subsequently the modes of decays were predicted and compared with the decay modes in Ref. [72] as well as the experimental ones, as seen in Table 6. In Table 6, the nuclei column shows the related superheavy nuclei, $Q_\alpha^{\mathrm{exp.}}$ shows the experimental Q-value taken from Ref. [87]. T_{SF} and T_α are the calculated values for SF and AD, respectively. $\mathrm{BR}_{\mathrm{SF}}(\%)$ and $\mathrm{BR}_\alpha(\%)$ show the calculated branching ratio values for SFs and ADs, respectively. The present column shows the dominant decay modes in

Table 3. Comparison of calculated $\log_{10}T_{\rm SF}$ (s) with other models for $Z = 108$ and $Z = 110$.

Z	N	A	Xu[70]	Bao [71]	KPS [72]	present
108	150	258	−12.26	−7.71	−10.18	−12.46
108	152	260	−8.63	−5.32	−6.31	−8.72
108	154	262	−5.72	−4.77	−3.61	−5.77
108	156	264	−3.53	−4.47	−1.64	−3.60
108	158	266	−2.05	−3.18	−0.04	−2.19
108	160	268	−1.29	−0.89	1.23	−1.53
108	162	270	−1.24	0.69	1.74	−1.61
108	164	272	−1.90	−0.04	1.09	−2.40
108	166	274	−3.28	−3.43	−0.79	−3.90
108	168	276	−5.38	−6.43	−3.07	−6.10
108	170	278	−8.18	−6.58	−5.08	−8.98
108	172	280	−11.70	−5.40	−7.19	−12.52
110	154	264	−8.63	−7.5	−6.68	−8.41
110	156	266	−5.69	−5.25	−3.94	−5.47
110	158	268	−3.47	−4.76	−1.63	−3.30
110	160	270	−1.96	−3.1	0.27	−1.89
110	162	272	−1.17	−1.04	1.61	−1.22
110	164	274	−1.1	−1.03	1.62	−1.27
110	166	276	−1.73	−2.41	0.38	−2.04
110	168	278	−3.09	−4.79	−1.26	−3.51
110	170	280	−5.15	−4.53	−2.4	−5.67
110	172	282	−7.93	−10.27	−3.99	−8.50
110	174	284	−11.42	−7.65	−5.97	−12.00

Table 4. Comparison of calculated $\log_{10}T_{\rm SF}$ (s) with other models for $Z = 112$, $Z = 114$ and $Z = 116$.

Z	N	A	Xu[70]	Bao [71]	KPS [72]	present
112	158	270	−4.67	−8.33	−3.34	−3.95
112	160	272	−2.42	−5.9	−0.87	−1.79
112	162	274	−0.88	−3.51	0.98	−0.37
112	164	276	−0.06	−3.53	1.64	0.31
112	166	278	0.04	−5.7	1.17	0.28
112	168	280	−0.57	−6.02	0.67	−0.46
112	170	282	−1.89	−4.02	0.29	−1.90
112	172	284	−3.93	−2.29	−0.65	−4.02
112	174	286	−6.68	−0.84	−2.11	−6.81
114	160	274	−2.56	−8.4	−1.8	−1.08
114	162	276	−0.29	−6.24	0.53	1.08
114	164	278	1.28	−2.49	2.7	2.51
114	166	280	2.12	−1.03	3.71	3.20
114	168	282	2.25	−0.4	3.97	3.19
114	170	284	1.67	−0.14	3.62	2.47
114	172	286	0.38	1.13	3.05	1.06
114	174	288	−1.64	2.95	2.16	−1.02
116	168	284	5.47	0.21	6.04	7.58
116	170	286	5.63	1.77	6.52	7.59
116	172	288	5.07	2.73	6.33	6.89
116	174	290	3.81	3.58	5.64	5.52
116	176	292	1.82	5.34	4.74	3.48
116	178	294	−0.87	5.84	3.06	0.78
116	180	296	−4.28	5.39	0.72	−2.57
116	182	298	−8.41	4.55	−2.13	−6.56
116	184	300	−13.24	2.64	−5.65	−11.16
116	186	302	−18.79	−2.84	−10.48	−16.39
116	188	304	−25.06	−9.05	−15.86	−22.21

present calculations; Ref. [72] shows the predicted decay modes for nuclei in Ref. [72], and the Exp. column shows the dominant decay modes in the experiment taken from Ref. [87]. In the "present" column, the parenthesis is used to depict the dominant decay mode. As can be seen in Table 6, the decay modes predicted in present calculations are in very good agreement with the predicted decay modes in Ref. [72] and the experimental ones, with the exception of some nuclei. When the present results are compared to Ref. [72], the predictions are different for some nuclei even if all other predictions obtained in this study are agreement with the ones in Ref. [72]. The SF values of half-lives are observed to increase with the Z number of parent nuclei, whereas the AD half-lives are tend to decrease.

To be able to make the predictions for the unknown decay modes of superheavy nuclei, the AD half-lives in the WKB method considering the isospin-dependent potential and BS quantization condition, as well as the SF half-lives using the new formula proposed in this study, have been calculated for possible AD chains from isotopes superheavy nuclei with $Z = 119–120$. The calculations of the SF and AD for $Z = 119{,}120$ are obtained and presented in Table 7. In Table 7, the nuclei column depicts the superheavy nuclei, $Q_\alpha^{\rm exp.}$ shows the experimental Q-value taken from Ref. [87], and $T_{\rm SF}$ and T_α are calculated values for SF and AD, respectively. With regard to $BR_{\rm SF}$ and BR_α, they show the calculated branching ratio values for SF and AD, respectively. The "present" column depicts the dominant decay modes in calculations, and the Bao [71] column shows the obtained decay modes by Bao et al. [71]. In the "present" column, the parenthesis is used to depict the dominant decay mode. As seen in the table, similar behaviors have been shown for nuclei with Z numbers ranging from 104 to 118.

It should be also underlined that the reason why the

Table 5. Comparison of calculated $\log_{10} T_{\mathrm{SF}}$ (s) with other models for $Z = 118$ and $Z = 120$.

Z	N	A	Xu[70]	Bao [71]	KPS [72]	present
118	170	288	10.07	1.97	9.12	13.60
118	172	290	10.25	3.24	9.47	13.62
118	174	292	9.73	4.3	9.27	12.95
118	176	294	8.48	4.29	8.33	11.61
118	178	296	6.53	5.49	7.27	9.60
118	180	298	3.86	4.38	5.16	6.94
118	182	300	0.48	2.98	2.57	3.64
118	184	302	−3.62	0.92	−0.59	−0.29
118	186	304	−8.43	−4.5	−5.01	−4.84
120	172	292	16	3.25	12.69	21.38
120	174	294	16.21	4.63	13	21.42
120	176	296	15.71	4.45	12.42	20.78
120	178	298	14.5	4.78	11.53	19.46
120	180	300	12.57	3.58	9.8	17.49
120	182	302	9.93	2.07	7.57	14.87
120	184	304	6.57	−0.23	4.73	11.62
120	186	306	2.5	−5.65	0.69	7.74

predictions for some nuclei are different from each other in the presented SF values in Tables 6 and 7 would come from the fact that the present model does not consider shell effects, magic numbers, and also whether the mass, proton and neutron numbers of the related nuclei are odd or even.

4 Summary

The half-lives of the spontaneous fission (SF) were obtained for heavy and superheavy nuclei with the new formula and α decay (AD) by using the Wentzel-Kramers-Brillouin (WKB) method together with Bohr-Sommerfeld (BS) quantization condition for cosh type potential including the isospin effects. By comparing the SF results with experimental values, rms values were calculated. When the new SF function is used in the calculations to obtain the experimental SF half-lives, the rms values become significantly better. The new formula is applied to obtain the SF half-lives of $Z = 108$–120 nuclei, and the logarithmic values of the SF are obtained and subsequently compared with the results of other models. Even if the logarithmic half-lives obtained by the formulas of Xu et al. [70], Bao et al. [71], KPS, Santhosh et al.

Table 6. Calculated T_{SF}(s), T_α(s), $\mathrm{BR}_{\mathrm{SF}}$(%), BR_α(%), and the predicted decay modes of superheavy nuclei and their experimental modes for nuclei with $Z = 104$ to $Z = 118$.

nuclei	$Q_\alpha^{\mathrm{exp.}}$ [87]	T_{SF}(s)	T_α(s)	$\mathrm{BR}_{\mathrm{SF}}$(%)	BR_α(%)	present	Ref. [72]	Exp. [87]
^{294}118	11.82	4.044e+11	1.976e-03	0.000	100.000	α	α	α
^{294}117	11.18	2.118e+06	8.768e-02	0.000	100.000	α	α	α
^{293}117	11.32	2.638e+07	2.340e-02	0.000	100.000	α	α	α
^{293}116	10.71	1.613e+02	3.704e-01	0.229	99.771	α	α	α
^{292}116	10.78	2.998e+03	1.458e-01	0.005	99.995	α	α	α
^{291}116	10.89	3.816e+04	1.271e-01	0.000	100.000	α	α	α
^{290}116	11.00	3.314e+05	4.029e-02	0.000	100.000	α	α	α
^{290}115	10.41	1.084e+01	1.814e+00	14.336	85.664	α/SF(α)	α	α
^{289}115	10.49	1.410e+02	6.436e-01	0.454	99.546	α	α	α
^{288}115	10.63	1.250e+03	4.719e-01	0.038	99.962	α	α	α
^{287}115	10.76	7.526e+03	1.270e-01	0.002	99.998	α	α	α
^{289}114	9.98	4.935e-03	7.492e+00	99.934	0.066	SF	α	α
^{288}114	10.07	9.620e-02	2.490e+00	96.280	3.720	SF	α	α
^{287}114	10.17	1.279e+00	2.175e+00	62.956	37.044	α/SF(SF)	α	α
^{286}114	10.35	1.158e+01	4.215e-01	3.511	96.489	α	α	$\alpha = 0.6, \mathrm{SF} = 0.4$
^{285}114	9.492	7.104e+01	1.977e+02	73.561	26.439	α/SF(SF)	α	α
^{286}113	9.79	2.013e-03	2.018e+01	99.990	0.010	α/SF(SF)	α	α
^{285}113	10.01	2.739e-02	2.771e+00	99.021	0.979	SF	α	α
^{284}113	10.12	2.531e-01	2.338e+00	90.230	9.770	SF	α	α
^{283}113	10.38	1.582e+00	2.711e-01	14.626	85.374	α	α	α

Table 6-continued from previous page

nuclei	$Q_\alpha^{\text{exp.}}$ [87]	T_{SF}(s)	T_α(s)	BR$_{\text{SF}}$(%)	BR$_\alpha$(%)	present	Ref. [72]	Exp. [87]
^{282}113	10.78	6.671e+00	4.401e-02	0.655	99.345	α	α	α
^{285}112	9.32	4.670e-06	1.344e+02	100.000	0.000	SF	SF	α
^{284}112	9.011	9.555e-05	7.680e+02	100.000	0.000	SF	SF	SF
^{283}112	9.66	1.329e-03	1.253e+01	99.989	0.011	SF	α	$\alpha = 1, \text{SF} \leq 0.01$
^{282}112	9.481	1.254e-02	2.530e+01	99.950	0.050	SF	SF	SF
^{281}112	10.46	7.997e-02	7.891e-02	49.665	50.335	$\alpha/\text{SF}(\alpha)$	α	α
^{282}111	9.16	9.884e-06	3.146e+02	100.000	0.000	SF	SF	α
^{281}111	9.41	1.407e-04	3.080e+01	100.000	0.000	SF	SF	$\alpha = 0.1, \text{SF} = 0.9$
^{280}111	9.91	1.356e-03	1.867e+00	99.927	0.073	SF	α	α
^{279}111	10.53	8.813e-03	2.475e-02	73.740	26.260	$\alpha/\text{SF(SF)}$	α	α
^{278}111	10.85	3.848e-02	6.879e-03	15.164	84.836	$\alpha/\text{SF}(\alpha)$	α	α
^{281}110	8.85	9.971e-08	7.932e+02	100.000	0.000	SF	SF	$\alpha = 0.07, \text{SF} = 0.93$
^{279}110	9.85	3.122e-05	7.350e-01	99.996	0.004	SF	SF	$\alpha = 0.1, \text{SF} = 0.9$
^{277}110	10.72	2.037e-03	4.009e-03	66.306	33.694	$\alpha/\text{SF(SF)}$	α	α
^{278}109	9.58	9.365e-07	3.326e+00	100.000	0.000	SF	SF	α
^{277}109	9.808	1.396e-05	4.374e-01	99.997	0.003	SF	SF	SF
^{276}109	10.03	1.404e-04	1.852e-01	99.924	0.076	SF	α	α
^{275}109	10.48	9.495e-04	7.420e-03	88.655	11.345	SF	α	α
^{274}109	10.2	4.296e-03	6.489e-02	93.790	6.210	SF	α	α
^{277}108	8.808	3.512e-08	1.963e+02	100.000	0.000	SF	SF	SF
^{275}108	9.45	1.211e-05	2.040e+00	99.999	0.001	SF	α	α
^{273}108	9.73	8.586e-04	3.209e-01	99.733	0.267	SF	α	α
^{274}107	8.94	1.254e-06	5.383e+01	100.000	0.000	SF	SF	SF
^{272}107	9.18	2.059e-04	9.634e+00	99.998	0.002	SF	α	α
^{271}107	9.42	1.448e-03	1.089e+00	99.867	0.133	SF	α	α
^{270}107	9.06	6.799e-03	2.195e+01	99.969	0.031	SF	α	α
^{271}106	8.67	5.694e-05	9.727e+01	100.000	0.000	SF	SF	$\alpha = 0.6, \text{SF} = 0.4$
^{269}106	8.7	4.390e-03	7.621e+01	99.994	0.006	SF	α	SF
^{270}105	7.965	1.742e-05	1.958e+04	100.000	0.000	SF	SF	SF
^{268}105	7.365	3.137e-03	4.435e+06	100.000	0.000	SF	SF	SF
^{267}105	7.325	2.299e-02	3.774e+06	100.000	0.000	SF	SF	SF
^{266}105	7.495	1.119e-01	1.262e+06	100.000	0.000	SF	SF	SF
^{267}104	6.995	2.382e-03	3.900e+07	100.000	0.000	SF	SF	SF
^{265}104	7.115	2.001e-01	1.122e+07	100.000	0.000	SF	SF	SF

Table 7. Calculated T_{SF}(s), T_α(s), BR$_{\text{SF}}$(%), BR$_\alpha$(%), and the predicted decay modes of superheavy nuclei without known experimental decay modes and predictions of Bao et al. [71].

nuclei	$Q_\alpha^{\text{exp.}}$	T_{SF}(s)	T_α(s)	BR$_{\text{SF}}$(%)	BR$_\alpha$(%)	present	Bao [71]
^{300}120	13.31	3.096e+17	6.485e-06	0.000	100.000	α	α
^{296}118	11.75	3.973e+09	2.872e-03	0.000	100.000	α	α
^{299}120	13.25	3.613e+18	1.384e-05	0.000	100.000	α	α
^{295}118	11.90	4.841e+10	2.195e-03	0.000	100.000	α	α

Table 7-continued from previous page

nuclei	$Q_\alpha^{\text{exp.}}$	T_{SF}(s)	T_α(s)	BR_{SF}(%)	BR_α(%)	present	Bao [71]
$^{298}120$	13.00	2.895e+19	2.502e-05	0.000	100.000	α	α
$^{297}120$	13.14	1.588e+20	2.261e-05	0.000	100.000	α	α
$^{293}118$	12.24	2.297e+12	3.981e-04	0.000	100.000	α	α
$^{289}116$	11.17	1.954e+06	2.511e-02	0.000	100.000	α	α
$^{285}114$	10.27	7.092e+01	1.079e+00	1.499	98.501	α	α
$^{281}112$	10.48	7.984e-02	6.848e-02	46.167	53.833	α/SF(α)	α/SF
$^{296}120$	13.34	5.947e+20	5.638e-06	0.000	100.000	α	α
$^{292}118$	12.23	8.893e+12	2.392e-04	0.000	100.000	α	α
$^{288}116$	11.28	7.825e+06	7.948e-03	0.000	100.000	α	α
$^{284}114$	10.57	2.938e+02	1.090e-01	0.037	99.963	α	α
$^{280}112$	10.86	3.425e-01	4.694e-03	1.352	98.648	α	SF
$^{295}120$	13.27	1.514e+21	1.266e-05	0.000	100.000	α	α
$^{291}118$	12.41	2.332e+13	1.666e-04	0.000	100.000	α	α
$^{287}116$	11.28	2.115e+07	1.364e-02	0.000	100.000	α	α
$^{283}114$	10.87	8.193e+02	3.054e-02	0.004	99.996	α	α
$^{279}112$	11.41	9.854e-01	3.949e-04	0.040	99.960	α	α/SF
$^{294}120$	13.24	2.612e+21	8.579e-06	0.000	100.000	α	α
$^{290}118$	12.59	4.132e+13	4.262e-05	0.000	100.000	α	α
$^{286}116$	11.31	3.849e+07	6.930e-03	0.000	100.000	α	α
$^{282}114$	11.37	1.532e+03	1.189e-03	0.000	100.000	α	α
$^{278}112$	11.77	1.894e+00	3.879e-05	0.002	99.998	α	SF
$^{300}119$	12.57	4.747e+10	2.849e-04	0.000	100.000	α	α
$^{296}117$	11.50	4.415e+03	1.549e-02	0.000	100.000	α	α
$^{292}115$	9.93	2.049e-02	3.954e+01	99.948	0.052	SF	α
$^{288}113$	9.34	3.446e-06	4.439e+02	100.000	0.000	SF	α
$^{284}111$	8.68	1.529e-08	1.177e+04	100.000	0.000	SF	α/SF
$^{280}109$	8.69	1.308e-09	1.842e+03	100.000	0.000	SF	SF
$^{299}119$	12.76	1.192e+12	6.767e-05	0.000	100.000	α	α
$^{295}117$	11.29	1.165e+05	2.730e-02	0.000	100.000	α	α
$^{291}115$	10.19	5.687e-01	4.198e+00	88.069	11.931	α/SF(SF)	α
$^{287}113$	9.34	1.006e-04	2.585e+02	100.000	0.000	SF	α
$^{283}111$	9.02	4.706e-07	5.065e+02	100.000	0.000	SF	SF
$^{298}119$	12.71	2.066e+13	1.454e-04	0.000	100.000	α	α
$^{297}119$	12.42	2.463e+14	3.271e-04	0.000	100.000	α	α
$^{296}119$	12.47	2.013e+15	4.359e-04	0.000	100.000	α	α
$^{292}117$	11.75	2.241e+08	4.043e-03	0.000	100.000	α	α
$^{295}119$	12.75	1.125e+16	6.731e-05	0.000	100.000	α	α
$^{291}117$	11.71	1.299e+09	2.772e-03	0.000	100.000	α	α
$^{287}115$	10.50	7.514e+03	5.951e-01	0.008	99.992	α	α
$^{283}113$	10.40	1.580e+00	2.303e-01	12.722	87.278	α/SF(α)	α
$^{279}111$	10.59	8.799e-03	1.682e-02	65.648	34.352	α/SF(SF)	α/SF

[72], and the present study exhibit a similar behavior of change according the mass number of the parent nuclei, their size is different. However, the results of Bao et al. [71] show a slightly different behavior in comparison to others. The proposed formula, comprising six parameters and excluding the Q_{SF} value, produces the experimental value of SF of nuclei in reasonable rms values.

The branching ratios for SFs and ADs were obtained, and the modes of decays were predicted for $Z = 104$–118 nuclei with known experimental decay modes. The decay modes extracted in calculations are in very good agreement with the experimental ones. Although theoretical predictions of the decay modes of many nuclei are the same as experimental ones, the predictions of decay modes for some nuclei are nevertheless different from experimental results. Furthermore, the branching ratios for SFs and ADs were obtained, and the modes of decay for $Z = 119$ and $Z = 120$ nuclei, which have the unknown experimental decay modes, were predicted. The decay modes are predicted for 45 nuclei that do not have experimental decay modes, and they compared with the predictions of Bao et al. [71]. Even if the predictions of two different models are same for many nuclei, the predictions of decay modes obtained in present study for some nuclei are different from the results of Bao et al. [71]. These are the decay modes of ^{280}112, ^{278}112, ^{292}115, ^{288}113, ^{291}115, and ^{287}113 nuclei.

In this study, different decay modes are obtained for some nuclei in comparison to the predictions provided in the literature for some superheavy nuclei. The present results provide useful information and knowledge for improving theoretical models and possible future experimental studies on superheavy nuclei in terms of both half-lives of the α decays and spontaneous fission.

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