

Testing B -violating signatures from exotic instantons in future colliders^{*}

Andrea Addazi^{1,1)} Xian-Wei Kang^{2,3,2)} Maxim Yu. Khlopov^{4,5,3)}

¹ Dipartimento di Fisica, Università di L'Aquila, 67010 Coppito AQ and LNGS, Laboratori Nazionali del Gran Sasso, 67010 Assergi AQ

² Institute of Physics, Academia Sinica, Taipei 115

³ Institute for Advanced Simulation, Jülich Center for Hadron Physics, and Institut für Kernphysik, Forschungszentrum Jülich, D-52425 Jülich

⁴ Centre for Cosmoparticle Physics Cosmion; National Research Nuclear University MEPhI (Moscow Engineering Physics Institute), Kashirskoe Sh., 31, Moscow 115409

⁵ APC laboratory 10, rue Alice Domon et Léonie Duquet, 75205 Paris Cedex 13

Abstract: We discuss possible implications of exotic stringy instantons for baryon-violating signatures in future colliders. In particular, we discuss high-energy quark collisions and $\Lambda-\bar{\Lambda}$ transitions. In principle, the $\Lambda-\bar{\Lambda}$ process can be probed by high-luminosity electron-positron colliders. However, we find that an extremely high luminosity is needed in order to provide a (somewhat) stringent bound compared to the current data on $NN\rightarrow\pi\pi, KK$. On the other hand, (exotic) instanton-induced six-quark interactions can be tested in near future high-energy colliders beyond LHC, at energies around 20–100 TeV. The Super proton-proton Collider (SppC) would be capable of such measurement given the proposed energy level of 50–90 TeV. Comparison with other channels is made. In particular, we show the compatibility of our model with neutron-antineutron and $NN\rightarrow\pi\pi, KK$ bounds.

Keywords: CEPC, SppC, exotic instanton, baryon number violation

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1 Introduction

In recent companion papers, implications of exotic stringy instantons in $B-L$ violating rare processes and baryogenesis have been explored [1–12].

As known, in open string theories, instantons are (Euclidean) D-branes wrapping n -cycles on the Calabi-Yau (CY) manifold. These solutions have been calculated and classified in the literature (see Ref. [13] for a complete review on these subjects).

In this paper, we will suggest that the exotic instantons can be tested in collider physics. As shown in Ref. [8], six quarks $\Delta B=2$ violating transitions can be generated by one exotic instanton in the context of a low scale string theory scenario with $M_S \simeq 10\div 100$ TeV.

The model suggested in Ref. [8] is theoretically motivated by baryogenesis and the hierarchy problem of the Higgs mass, while embedding a theory of quantum gravity. As regards baryogenesis, the tunneling probability of $B-L$ violating processes can be enhanced in the thermal bath, as it happens for standard model electroweak sphalerons [14, 15]. This implies a $B-L$ first order phase transition which could explain the observed matter-antimatter asymmetry. On the other hand, a low string scale theory can alleviate the strong hierarchy problem of the Higgs mass of 17 orders, reducing the hierarchy to a small number (comparable to the Yukawa coupling of the electron). The possibility of testing exotic instantons in a collider may allow testing of low scale string theory in the fully non-perturbative regime. This may also be important

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1) E-mail: andrea.addazi@inf.nlngs.it

2) E-mail: kxw198710@126.com

3) E-mail: khlopov@apc.in2p3.fr



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to understand the issues arising from geometric moduli stabilization in string compactification. In fact, string instantons and string fluxes may generate non-perturbative effective potentials stabilizing the geometric moduli and allowing for a string vacuum safety.

In particular, we will show how exotic instantons can generate $\Lambda - \bar{\Lambda}$ transitions, and $q\bar{q} \rightarrow \bar{q}\bar{q}q\bar{q} \rightarrow \bar{q}\bar{q}\bar{q}\bar{q}$ high-energy collisions. Electron-positron colliders with luminosity much higher than Belle and BaBar could be used to (indirectly) test such a scenario. Indeed, we find that an extremely high luminosity is needed, which looks unrealizable in the near future. On the other hand, we argue how comparable measurements in neutron-antineutron physics and in high energy colliders beyond the LHC can provide tests for our model in the near future. In this sense, the high energy frontier is the preferred experimental direction of our model, with respect to the high luminosity one.

This paper is organized as follows. In Section 2 we discuss our theoretical model, in Section 3 its phenomenology in electron-positron colliders, and in Section 4 its phenomenology and parameter space in comparison with several other different possible channels, before coming to our conclusions.

2 Theoretical model

B and L number conservation can be dynamically violated by non-perturbative quantum gravity effects known as exotic stringy instantons. Exotic stringy instantons are Euclidean D-branes (or E-branes), intersecting physical D-brane stacks. We consider IIA string-theories¹⁾. In this case, “exotic instantons” are E2-branes, wrapping different 3-cycles on a Calabi-Yau compactification with respect to physical D6-branes. A MSSM can be embedded in a quiver theory with three or more nodes. In the low energy limit, these quivers can produce gauge theories $U(3)_a \times U(2)_b \times U(1)_c$ or $U(3)_a \times Sp(2)_b \times U(1)_c \times U'(1)_s$ or $U(3)_a \times U(2)_b \times U(1)_c \times U'(1)_d$ or eventually higher node extensions²⁾. The basic fundamental elements are: i) D6-branes wrapping 3-cycles in the Calabi-Yau CY_3 ; ii) Ω -planes; iii) E2-instantons; iv) open strings³⁾.

Open strings are attached to D6 and E2-branes. Let us also recall that open (un)oriented strings attached to two intersecting D6-brane stacks will reproduce (MS)SM

matter fields as lower energy excitations, in the limit $\alpha'_s \rightarrow 0$. The number of intersections among the stacks will correspond to the number of generations. As an example, the three generations of lepton superfields L come from open strings attached to a $U(2)$ or $Sp(2)$ stack and a $U(1)$ stack, with these stacks intersecting each other three times. The hypercharge $U(1)_Y$ is reconstructed as a massless linear combinations of the $U(1)$ s in the model. For example, for $U(3)_a \times U(2)_b \times U(1)_c$,

$$Y = q_a Q_a + q_b Q_b + q_c Q_c, \tag{1}$$

where $Q_{a,b,c}$ corresponds to $U(1)_a \subset U(3)_a$ and $U(1)_b \subset U(2)_b$ and $U(1)_c$ respectively. On the other hand, two linear independent combinations of $U(1)$ s orthogonal to (1) will be anomalous⁴⁾.

Let us consider the presence of an E2-brane intersecting the $U(3)$ -stack two times, the $U(1)$ -stack two times, and the $\hat{U}(1)$ -stack (image of $U(1)$ with respect to Ω) four times. For $\alpha'_s \rightarrow 0$, this construction generates effective Lagrangian terms among the ordinary superfields U^c, D^c and fermionic moduli (also called modulini):

$$\mathcal{L}_{\text{eff}} \sim k_f^{(1)} U_f^i \tau_i \alpha + k_f^{(2)} D_f^i \tau_i \beta, \tag{2}$$

where τ^i modulini live at $U(3)$ -E2 intersections, α modulini at $U(1)$ -E2 and β at $\hat{U}(1)$ -E2. Here, we consider an E2-instanton with a Chan-Paton factor $O(1)$. As prescribed by instanton calculation, we will integrate out modulini at the D6-E2 intersections, and obtain

$$\begin{aligned} \mathcal{W}_{E2-D6-\hat{D}6} &= \int d^6 \tau d^4 \beta d^2 \alpha e^{-\mathcal{L}_{\text{eff}}} \\ &= \mathcal{Y}^{(1)} \frac{e^{-S_{E2}}}{M_S^3} \epsilon_{ijk} \epsilon_{i'j'k'} U^i D^j D^k U^{i'} D^{j'} D^{k'}, \end{aligned} \tag{3}$$

where $\mathcal{Y}_{f_1 f_2 f_3 f_4 f_5 f_6}^{(1)} = k_{f_1}^{(1)} k_{f_2}^{(1)} k_{f_3}^{(2)} k_{f_4}^{(2)} k_{f_5}^{(2)} k_{f_6}^{(2)}$ is the flavor matrix determined by the couplings $k^{(1,2)}$ derived from mixed disk amplitudes. We can assume these as free parameters, parametrizing our ignorance about the particular geometry of the E2-instanton considered. A superpotential (3) can generate diagrams like the one in Fig. 2. In particular, $n - \bar{n}$ and $\Lambda - \bar{\Lambda}$ transitions are generated by the two effective operators

$$\begin{aligned} \mathcal{O}_{n\bar{n}} + \mathcal{O}_{\Lambda\bar{\Lambda}} &= \frac{y_1}{\mathcal{M}_{E2}^3 \mathcal{M}_{SU SY}^2} (u^c d^c d^d)(u^c d^c d^c) \\ &+ \frac{y_2}{\mathcal{M}_{E2}^3 \mathcal{M}_{SU SY}^2} (u^c d^c s^c)(u^c d^c s^c), \end{aligned} \tag{4}$$

1) For classic papers on open string theories see Refs. [19–23]. In open string theories, calculations of scattering amplitudes are much simpler than F-theories or heterotic string theories.

2) See Refs. [16–18, 25–28] for (incomplete) literature on intersecting D-brane models.

3) Before the orientifold projection, in order to restore the correct balance of arrows one has also to introduce flavor branes for some of these models. See Ref. [24] for a discussion of unoriented quivers with flavour branes.

4) In string theory, anomalous $U(1)$ s can be cured through the generalized Green-Schwarz mechanism. The two anomalous vector bosons Z', Z'' get a mass of the order of the string scale, through the Stückelberg mechanism. Peculiarly, they have to interact with Z, γ through generalized Chern-Simon (GCS) terms in order to cancel the anomalies. See Refs. [29, 31] for an extensive discussion of these aspects.

where $\mathcal{M}_{E2}^3 = e^{+S_{E2}} M_S^3$, $y_1 = \mathcal{Y}_{111111}^{(1)}$ and $y_2 = \mathcal{Y}_{112112}^{(1)}$. More details and analysis of explicit quivers were extensively discussed in Ref. [8]. These operators correspond to the generation of an effective Majorana mass for the neutron and for the Λ baryon.

$e^{-S_{E2}}$ is the effective action of an $E2$ -brane wrapping 3-cycles on the CY_3 . It is related to the string coupling as

$$e^{-S_{E2}} = e^{-\mathcal{V}_{E2}/g_s + i\sum_r q_r a_r}, \quad (5)$$

where \mathcal{V}_{E2} is the volume wrapped by the $E2$ -brane and the imaginary part consists of a sum all over Ramond-Ramond axions. For $l > l_S$, exotic instantons have to respect the universal string theory bound on non-perturbative effects:

$$|e^{-S_{E2}}| \leq e^{-\frac{2}{g_s}} \quad (6)$$

interpreted as a bound from the instanton-antinstanton virtual pair diagram, i.e as a bound on their partition function. This bound is very important for the considerations in the following. In fact, $g_s \ll 1$ will suppress $E2$ -transitions. On the other hand, scenarios in which $g_s = 0.25 \div 1$ suggest a coupling strong as $e^{-10} \div e^{-2} \simeq 5 \times 10^{-5} \div 1.1 \times 10^{-1}$, with a small volume \mathcal{V}_{E2} . This result is peculiarly different with respect to non-perturbative classical configurations in field theories, like sphalerons, usually suppressed as e^{-1/g_{YM}^2} . In particular, for electroweak gauge instantons, the suppression factor can be proportional to e^{-10^4} or so. Such a high string scale as the one desired in our case has to be consistently compatible with the Yang-Mills coupling and M_{Pl}/M_S ratio. Let us recall that, by dimensional reduction to 4d,

$$\frac{1}{g_{YM}^2} = \frac{M_S^3 \mathcal{V}_3}{(2\pi)^4 g_s}, \quad (7)$$

$$M_{Pl}^2 = \frac{M_S^8 \mathcal{V}_6}{(2\pi)^7 g_s^2}, \quad (8)$$

where \mathcal{V}_3 is the volume wrapped by D6-branes (stacks) corresponding to YM bosons, and \mathcal{V}_6 is the total internal volume. The hierarchy among YM couplings and a high string coupling is understood as a volume suppression hierarchy. Generically, the volumes of three-cycles are not directly related to the total internal volume, even if in simple compactifications like the isotropic toroidal one they are related as $\mathcal{V}_3 \sim \sqrt{\mathcal{V}_6}$. This allows more variability among string scale and Planck scale hierarchies.

Furthermore, the suppression factor $e^{-S_{E2}}$ can also be compensated by coefficients in $k_f^{(1)}$ of mixed disk amplitudes. These coefficients can be higher than one and their combinations can give rise to an appreciable enhancement of $\mathcal{Y}^{(1)}$ flavor components¹⁾.

2.1 Exotic instantons in high energy collisions

In this section, we will consider two-quark high energy collisions induced by exotic instantons. For $s \ll \Lambda^2$, the scattering amplitude is just reduced to a contact six-quark interaction

$$\mathcal{A}(q_{f_1}^c q_{f_2}^c \rightarrow \bar{q}_{f_3}^c \bar{q}_{f_4}^c \bar{q}_{f_5}^c \bar{q}_{f_6}^c) \simeq \mathcal{Y}_{f_1 f_2 f_3 f_4 f_5 f_6}^{(1)} \frac{\Lambda^{-3}}{M_{SUSY}^2} \quad (9)$$

with $q^c = u^c, d^c, \dots, t^c$ RH quarks and $\Lambda^{-3} \simeq M_S^{-3} e^{-S_{E2}}$. A less suppressed channel is $q^c q^c \rightarrow \bar{q}^c \bar{q}^c \bar{q}^c \bar{q}^c$. In the low energy limit, its amplitude is

$$\mathcal{A}(q_{f_1}^c q_{f_2}^c \rightarrow \bar{q}_{f_3}^c \bar{q}_{f_4}^c \bar{q}_{f_5}^c \bar{q}_{f_6}^c) \simeq \mathcal{Y}_{f_1 f_2 f_3 f_4 f_5 f_6}^{(1)} \Lambda^{-3}. \quad (10)$$

The corresponding quark-squark cross section can be evaluated by integrating the square of the amplitude over the whole 4-dimensional phase space $d\Phi_4$. We define the Mandelstam variables $s_{12} = (p_1 + p_2)^2$, $s_{34} = (p_3 + p_4)^2$, where $p_{1,2,3,4}$ are squark momenta (final states) and $s = E_{CM}^2$. One finds that

$$d\sigma_{q_{f_1}^c q_{f_2}^c \rightarrow \bar{q}_{f_3}^c \bar{q}_{f_4}^c \bar{q}_{f_5}^c \bar{q}_{f_6}^c} = d\hat{\Omega} \frac{\mathcal{C}}{4\Lambda^6} \mathcal{P}_2(s, s_{12}, s_{34}), \quad (11)$$

where

$$\mathcal{C} = \frac{C_4}{(8\pi^2)^3} \text{Tr} \left\{ \mathcal{Y}^{(1)\dagger} \mathcal{Y}^{(1)} \right\}, \quad (12)$$

with C_4 a combinatorial factor depending on the number of equal particles in the final state, and $d\hat{\Omega}$ the integral over all scattering angles

$$d\hat{\Omega} = d\cos\theta \frac{d\phi}{2\pi} d\cos\hat{\theta}_{12} \frac{d\hat{\phi}_{12}}{2\pi} d\cos\hat{\theta}_{34} \frac{d\hat{\phi}_{34}}{2\pi}. \quad (13)$$

The notation $\hat{\phi}_{ij}$ indicates variables evaluated with respect to the rest frame of q_{ij} . \mathcal{P}_2 is a complicated polynomial of s, s_{12}, s_{34} and subleading logarithmic functions as follows:

$$\begin{aligned} \mathcal{P}_2(x, y, z) = & -\zeta_1(y)\zeta_1(z)[-4y\log\zeta_2^+(x, y, z) \\ & + x\log\zeta_2^-(x, y, z) + y^2 + \zeta_3(x, y, z)] \end{aligned}$$

where

$$\begin{aligned} \zeta_1(x) = & \sqrt{1 - \frac{2(m_1 + m_2)}{x} + \frac{(m_1 - m_2)^2}{x^2}} \\ \zeta_2^\pm(x, y, z) = & -\frac{x^2 - 2x(x+y) + (y-z)^2 - x \pm y \mp z}{\Lambda^2} \end{aligned}$$

$$\zeta_3(x, y, z) = \frac{1}{3} \left[x^2 + 5x(y+z) - \frac{2(y-z)^2}{x} \sqrt{\zeta_2^+ + x - y + z} \right]$$

with $m_{1,2,3,4}$ squark masses in the final state.

For $s \rightarrow \Lambda^2$, the cross section grows rapidly. $\bar{s} = \Lambda^2$ corresponds to the bound of the unitarity break-down.

1) However, recent trends in non-perturbative string theory and string phenomenology suggest that for distances of $l < l_S$, the 3-cycle volume factor can be collapsed on the CY_3 singularity [32, 33]. In this regime, the saddle point approximation can no longer be trusted, so our previous estimates are no longer valid in this case. In this case an enhancement of exotic instanton processes is expected.

In other words, our effective cross-section would badly break the Froissart bound.

For $s \simeq \Lambda^2$, the contact interaction approximation loses validity; scatterings are probing the fully non-perturbative regime, and an exotic instanton is a fully non-perturbative configuration. Resonant production at $s \simeq \Lambda^2$ corresponds to an infinite series of stringy amplitudes with six open-string insertions $\sum_{g=0}^{\infty} \mathcal{A}^g(z_1, z_2, z_3, z_4, z_5, z_6)$, where g is the genus (loops). This is a technical problem in the high energy limit common to all non-perturbative (Euclidean) configurations, like QCD instantons, sphalerons and so on. However, let us note that in our model $\Lambda = e^{+S E_2} M_S > M_S$. As a consequence, an infinite tower of stringy higher spin states are excited at Λ and they will unitarize the S -matrix. This is a generic conclusion coming from unitarization arguments of string theory amplitudes, also connected to CPT symmetry in string theory. We consider the problem in the fixed angle kinematical regime, which is also particularly suitable with the experimental set-up of colliders. We argue that the fixed angle scattering amplitudes of (six) open strings are expected to fall exponentially with energy as

$$\mathcal{A}_{s > M_S^2}^{\text{tree-level}} = \mathcal{A}^{QFT} e^{-\alpha' s \log \alpha' s + \dots} \quad (14)$$

while N-loop amplitudes behave as¹⁾ [34]

$$\mathcal{A}_{s > M_S^2}^{N\text{-loops}} = \mathcal{A}^{QFT} e^{-\frac{1}{N} \alpha' s \log \alpha' s + \dots} \quad (15)$$

In Fig. 1, we show the qualitative universal part (flavor and combinatory independent) of the two-quark four-squark cross section, assuming squark masses smaller than Λ -scale. For $E_{CM} \simeq \Lambda$ the process lies in the fully non-perturbative stringy regime. From the physical point of view, we expect that, with the growing of CM energy, the $E2$ -brane starts to oscillate and its dynamics is described by oscillations of the moduli fields. The number of moduli is a topological invariant of the exotic instanton solution, associated with the invariance of intersections with physical D-branes. At the non-perturbative scale, open strings associated to moduli have to reggeize. Loop-corrections to tree-level mixed-disk amplitudes at fixed angle are expected to add exponentially suppressed contributions in the form of Eq. (15). We conjecture that this is the main contribution to the unitarization of the scattering amplitude for the fixed angle kinematical regime and $s \gg \Lambda^2$.

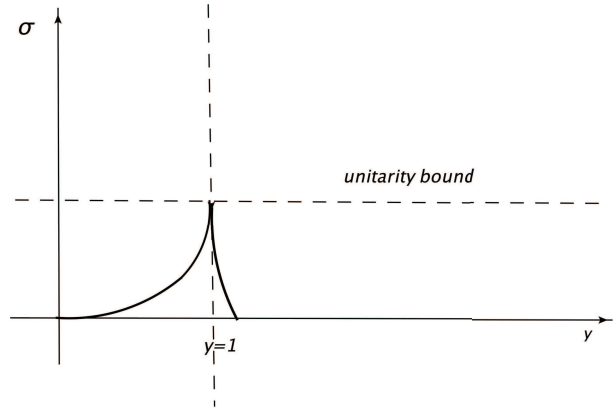


Fig. 1. The two-quark four-squark cross section $\sigma(y)$ (flavor independent part) with respect to the normalized s -variable $y = s/\Lambda^2$ and neglecting squark masses. For $y < 1$, $\sigma \sim y^2$; for $y \simeq 1$ the amplitude will saturate the unitarity bound of string theory. For $y > 1$ and fixing the scattering angle, the amplitude falls off exponentially with the production of the non-perturbative configuration.

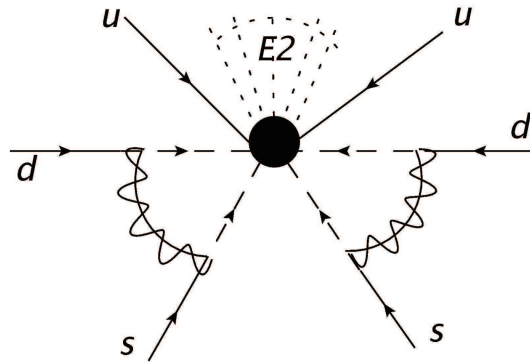


Fig. 2. Diagram for a $\Lambda - \bar{\Lambda}$ transition. u, d, s are right-handed up, down and strange quarks. Four quarks in the diagram are converted to the corresponding squarks through the exchange of two gaugini. The black vertex is directly generated by an exotic instanton.

3 Phenomenology of $\Lambda - \bar{\Lambda}$ transitions

Now let us discuss the implications of $\Lambda - \bar{\Lambda}$ oscillations on the phenomenological side²⁾. So far, no baryon number violating process has been observed in the Standard Model (SM), thus any possible signal would be definitely exciting and efforts along this direction are deserved. New physics effects can be also probed at hadron-hadron colliders. As discussed in Ref. [40], the BES detector

1) The result found by Gross-Mende in Ref. [34] is valid for four open string amplitudes. Their result is expected to be qualitatively the same as for six point amplitudes in 2→4 fixed angle scatterings.

2) Of course, in the Standard Model analysis, these baryon-violating effects need not to be considered, e.g., see the CP violation of $\Lambda_c \rightarrow \Lambda \pi$ decay [35] and also the conventional $N\bar{N}$ scattering [36–39].

with its huge J/ψ data sample has been proposed to measure possible $\Lambda-\bar{\Lambda}$ oscillation by studying the quantum coherent $\Lambda\bar{\Lambda}$ states, following the treatment of $D^0-\bar{D}^0$ mixing [41]. To avoid complications due to mixing, the charged B was used as an example in Ref. [42]. The eigenstates emerging from the oscillations can be written as

$$\begin{aligned} |\Lambda_H\rangle &= p|\Lambda\rangle + q|\bar{\Lambda}\rangle, \\ |\Lambda_L\rangle &= p|\Lambda\rangle - q|\bar{\Lambda}\rangle, \end{aligned} \quad (16)$$

with the normalization condition $|p|^2 + |q|^2 = 1$. The subscript ‘‘H’’ (‘‘L’’) means the heavy (light) mass eigenstates. Here p and q can be parametrized as $\sqrt{1+z}/\sqrt{2}$ and $\sqrt{1-z}/\sqrt{2}$, respectively, where z involves the oscillation mass $\delta m_{\Lambda\bar{\Lambda}}$ appearing in the off-diagonal elements of the effective Hamiltonian used to describe the time evolution of the $\Lambda-\bar{\Lambda}$ system. For details, see Ref. [40] or the general consideration in Ref. [41]. One can define the following mass and width:

$$\begin{aligned} m &\equiv \frac{m_H + m_L}{2}, & \Delta m &\equiv m_H - m_L \\ \Gamma &\equiv \frac{\Gamma_H + \Gamma_L}{2}, & \Delta\Gamma &\equiv \Gamma_H - \Gamma_L, \end{aligned} \quad (17)$$

from which the mixing parameters are defined as

$$x_\Lambda \equiv \frac{\Delta m}{\Gamma}, \quad y_\Lambda \equiv \frac{\Delta\Gamma}{2\Gamma}. \quad (18)$$

For the free Λ case without external magnetic fields, $\Delta m = 2\delta m_{\Lambda\bar{\Lambda}}$. For simplicity, we will confine ourselves to this case. The influence of external magnetic field is discussed in Ref. [40]. The dominant decay mode of Λ is $\Lambda \rightarrow p\pi^-$ with branching ratio 64% [43], and correspondingly $\bar{\Lambda} \rightarrow \bar{p}\pi^+$ assuming CP invariance. In the case that $\Lambda-\bar{\Lambda}$ oscillation happens, the process $\bar{\Lambda} \rightarrow \Lambda \rightarrow p\pi^-$ will be possible. This phenomenon can be probed by counting the number of events \mathcal{N} for $J/\psi \rightarrow \Lambda\bar{\Lambda} \rightarrow (p\pi^-)(p\pi^-)$, the wrong-sign decay, while the right-sign decay would be $J/\psi \rightarrow \Lambda\bar{\Lambda} \rightarrow (p\pi^-)(\bar{p}\pi^+)$. The time-integrated decay rate for the wrong-sign decay relative to the right sign is found to be

$$\mathcal{R} = \frac{\mathcal{N}(J/\psi \rightarrow \Lambda\bar{\Lambda} \rightarrow (p\pi^-)(p\pi^-))}{\mathcal{N}(J/\psi \rightarrow \Lambda\bar{\Lambda} \rightarrow (p\pi^-)(\bar{p}\pi^+))} = \frac{x_\Lambda^2 + y_\Lambda^2}{2}. \quad (19)$$

A discussion of time-dependent observables is presented in Ref. [40], and the BESIII detector is capable of accessing this time information. Assuming $y_\Lambda = 0$ – it is indeed quite a small quantity – one can get an estimate of the mixing mass parameter $\delta m_{\Lambda\bar{\Lambda}}$ from Eq. (19) as

$$\delta m_{\Lambda\bar{\Lambda}} = \frac{1}{\sqrt{2}} \sqrt{\mathcal{R}} \Gamma. \quad (20)$$

Based on the design luminosity of the BEPC-II accelerator in Beijing [44], 10×10^9 J/ψ and 3×10^9 ψ' events

will be collected per year of running. If finally no signal of wrong-sign decay can be detected, one should put an upper limit $\mathcal{R} \leq 4 \times 10^{-7}$ and correspondingly $\delta m_{\Lambda\bar{\Lambda}} \leq 10^{-15}$ MeV at 90% confidence level (C.L.) [40], inferring from the knowledge of interval estimates for very rare signals. To date, a sample of 1.31×10^9 J/ψ events has been collected [45], so the aforementioned upper limit will be increased by a factor of $\sqrt{10/1.31} \approx 2.8$, i.e. $\delta m_{\Lambda\bar{\Lambda}} \leq 3 \times 10^{-15}$ MeV. Consequently, the oscillation time will be bounded by the upper limit as 2.5×10^{-7} s at 90% confidence level. This would be the first such search in the experiment. See also Ref. [40] for more details of this analysis.

Note that $\Upsilon(4S)$ also has the same quantum numbers as J/ψ , i.e., $I^G(J^{PC}) = 0^-(1^{--})$, and can also decay to coherent $\Lambda\bar{\Lambda}$ states. The Belle and BaBar detectors could be used to probe this process as well. Currently, there are 772×10^6 $\Upsilon(4S)$ events available from Belle [46] and 471×10^6 from BaBar [47]. Taking into account that the non- $B\bar{B}$ decay mode constitutes less than 4% (at 95% confidence level) of the total decay rate, much fewer $\Lambda\bar{\Lambda}$ events are expected. Otherwise, if most $\Upsilon(4S)$ decayed into $\Lambda\bar{\Lambda}$, the number of events would increase by one or two orders of magnitude, more than BES. Although the above upper limit for $\Lambda-\bar{\Lambda}$ oscillation could be accessed by the collider experiment, we would require an extremely high luminosity to get a comparable bound to the $n-\bar{n}$ case. One can find this point from Eq. (20) – the oscillation mass is proportional to the square root of the luminosity. $n-\bar{n}$ oscillation time is constrained to be $10^7 \div 10^8$ smaller than the $\Lambda-\bar{\Lambda}$ oscillation time, and thus the luminosity should be increased by order 10^{15} , which seems unrealizable in the near future. In other words, extremely huge data samples would be needed to pin down a more stringent bound on such a new-physics (NP) signal. However, we will show that the required energy to generate such an NP phenomenon (e.g., in the exotic instanton model below) can be accessed in the near future. Recently, the Circular Electron Positron Collider (CEPC) has been proposed in China, and aroused great interest in the community [48]. The planned design would allow the electron-positron collider to be converted into a proton-proton collider, with an unprecedented center-of-mass energy of 50–90 TeV, in the second phase. This is the Super proton-proton Collider (SppC) project [49]. At such a high energy, the new physics beyond the LHC discussed in the present paper would be accessible. Future measurements are expected to give valuable hints for this line of research.

4 Parameter space, $n-\bar{n}$ and proton-proton colliders

An operator $\mathcal{O}_{\Lambda\bar{\Lambda}}$ generates not only $\Lambda-\bar{\Lambda}$ transitions but also $NN \rightarrow KK$ transitions. As discussed in the pre-

vious section, $\delta m_{\Lambda\bar{\Lambda}}$ can be constrained up to 10^{-6} eV by the next generation of experiments. However, $NN \rightarrow KK$ is just constraining $\mathcal{O}_{\Lambda\bar{\Lambda}}$ up to $\delta m_{\Lambda\bar{\Lambda}} < 10^{-21}$ eV. $\delta m_{\Lambda\bar{\Lambda}}$ is related to the New Physics scale by

$$\delta m_{\Lambda\bar{\Lambda}} \simeq y_2 \frac{\Lambda_{QCD}^6}{\mathcal{M}_{\Lambda\bar{\Lambda}}^5}, \quad (21)$$

so that $\mathcal{M}_{\Lambda\bar{\Lambda}} \simeq 100$ TeV scale. Furthermore, $\mathcal{O}_{n\bar{n}}$ is actually constrained by $\delta m_{n\bar{n}} < 10^{-23}$ eV, corresponding to $\mathcal{M}_{n\bar{n}} > 300$ TeV [50, 51]. The next generations of experiments will test¹⁾ $\mathcal{M}_{n\bar{n}} \simeq 1000$ TeV [52]. In our model,

$$\frac{\mathcal{M}_{\Lambda\bar{\Lambda}}}{\mathcal{M}_{n\bar{n}}} = \frac{y_2}{y_1} = 10^{-18} \div 10^{18}.$$

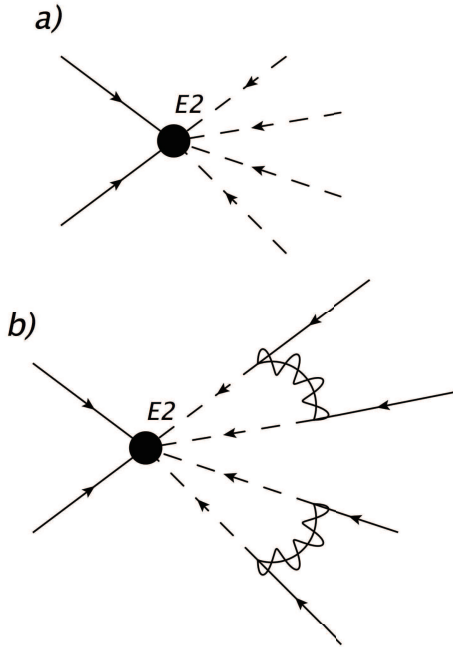


Fig. 3. (a) Four anti-quark production in high energy collisions of two quarks $q_{f_1}q_{f_2} \rightarrow \bar{q}_{f_3}\bar{q}_{f_4}\bar{q}_{f_5}\bar{q}_{f_6}$. (b) Four quark production in high energy collisions of two quarks $q_{f_1}q_{f_2} \rightarrow \bar{q}_{f_3}\bar{q}_{f_4}\bar{q}_{f_5}\bar{q}_{f_6}$. Black vertices are induced by exotic instantons.

Such a large hierarchy range is naturally understood by couplings arising from mixed disk amplitudes. For example, the number 10^{-18} is understood as $\frac{k_1^{(1)}k_1^{(1)}k_1^{(2)}k_1^{(2)}k_2^{(2)}k_2^{(2)}}{k_1^{(1)}k_1^{(1)}k_1^{(2)}k_1^{(2)}k_1^{(2)}k_1^{(2)}} \simeq 10^{-3} \times 10^{-3} \times 10^{-3} \times 10^{-3} \times 10^{-3} \times 10^{-3}$. As a consequence, a 100 TeV-scale test of $\Lambda - \bar{\Lambda}$ is motivated from the theoretical side, independently from $n \leftrightarrow \bar{n}$ limits. A 100 TeV scale scenario for $\Lambda - \bar{\Lambda}$ can be naturally obtained with $e^{+SE_2} \simeq 1$ (small 3-cycles wrapped by the E_2 -brane), $M_S \simeq M_{SUSY} \simeq 100$ TeV. In this case,

a final test-bed for this model can be provided by future proton-proton 100 TeV colliders beyond the LHC. In fact a $uds \rightarrow \bar{u}\bar{d}\bar{s}$ transition will be directly tested, mediated by an exotic instanton in collision. However, a more intriguing scenario can be opened in the case of $10 \text{ TeV} < M_{SUSY} \simeq M_S < 100 \text{ TeV}$. In this case, planned proton-proton colliders can test other flavor amplitudes like $ud \rightarrow \bar{c}\bar{s}\bar{s}, \bar{c}\bar{s}\bar{b}\bar{b}$ that are less constrained by $n - \bar{n}$ or $NN \rightarrow \pi\pi, KK$ processes. In particular, the predicted experimental processes for a high energy proton-proton collider are $pp \rightarrow 4\bar{q}$. This leads to several different channels. This is the case of $pp \rightarrow 4q + 4\chi^0$ leading to four jets and missing transverse energy $pp \rightarrow 4j + E_{M.T}$ or $pp \rightarrow 3j + t + E_{M.T}$ (with standard top decays) and so on. Other interesting channels come from stop production and successive decays like $\tilde{t} \rightarrow W + \tilde{b} \rightarrow W + b + \chi^0$, $\tilde{t} \rightarrow b + \chi^+ \rightarrow b + W + \chi^0$.

As discussed in Section 2.1, the cross sections of these processes have a peculiar behavior not common to gauge models because of their exponential decrease up to Λ for $s \gg \Lambda^2$.

5 Conclusions

In this paper, we have discussed phenomenological implications of a new class of instantons known as exotic instantons. They can generate $\Delta B = 2$ violating transitions as $n \leftrightarrow \bar{n}, \Lambda \leftrightarrow \bar{\Lambda}$ and $\Delta B = 2$ high energy collisions like $qq \rightarrow \bar{q}\bar{q}\bar{q}\bar{q}, \bar{q}\bar{q}\bar{q}\bar{q}$ in hypercharge-preserving combinations. We have explored the possibility of detecting exotic instantons in future colliders, in comparison with present low energy limit channels like $n \leftrightarrow \bar{n}, NN \rightarrow \pi\pi, KK$.

We summarize our main conclusions as follows.

- 1) Contrary to other non-perturbative solutions like electroweak gauge instantons, exotic instantons can induce effective operators with a high coupling. As a consequence, their effects can be seen in low energy observables as well as in high energy colliders.
- 2) A neutron-antineutron transition can be generated by exotic instantons and it can provide an indirect test-bed for a class of models mentioned in this paper²⁾.
- 3) Our model predicts $\Lambda - \bar{\Lambda}$ transitions. The possibility of testing these in future high luminosity electron-positron colliders seems very far from our present technological capability, if compared to the actual related limits from $NN \rightarrow KK$ transitions.
- 4) $\Delta B = 2$ exotic instantons can be reached in the next generation of high energy proton-proton colliders beyond LHC, well compatible with neutron-antineutron limits, $NN \rightarrow \pi\pi, KK$ and so on. We have stressed how

1) See Ref. [53] for a discussion on perturbative renormalization corrections of $n - \bar{n}$ operators.

2) This is an example of a different UV completion of a six-quark operator from a non-perturbative classical configurations, rather than extra heavy fields. This has intriguing analogies with classicalization [54–58, 60]. Reference [58] is related to discussions in Ref. [59].

cross-section running with center-of-mass energy cannot be reproduced by any quantum field theory model. In fact an exponential softening of the cross-section cannot be reproduced from any other UV completion of the six-quark effective operator in the context of quantum field theories. This is a feature distinguishing our string theory model from other quantum field theory models.

5) The cosmological impact of exotic instantons revealed in Ref. [8, 10] provides additional constraints on their parameters. Different from the case of electroweak gauge instantons, due to the enhancement of the effect of exotic instantons in high energy collisions, a direct quantitative relationship between cosmological and physical consequences of the model considered is possible.

The class of models suggested here strongly motivates two directions for future experimental physics: neutron-antineutron experiments and high-energy col-

liders beyond the LHC. Eventually, detection of exotic instanton-mediated processes could motivate the construction of technologically challenging high luminosity collider in order to detect a $\Lambda-\bar{\Lambda}$ transition or new rare physics experiments searching for $NN\rightarrow KK$. These measurements could constrain the exotic instantons' geometry and their intersections (with ordinary D-branes) and their wrapped 3-cycles on the CY_3 . Future beyond-LHC colliders (such as the proposed CEPC+SppC) might bring us exciting new surprises at the higher-energy and higher-luminosity frontiers.

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Appendix A

Superpotential calculus

In our paper, we have considered an instanton with a rigid $O(1)$ symmetry in context of IIA superstring theory. This instanton has the universal zero mode structure $dx^4 d^2\theta$, which yields a new term to the holomorphic F-terms in the effective supergravity action. The computation of these effects can be done from the Conformal Field Theory perspective. In particular, one can compute the superpotential of an M-point correlator in a string instantonic background $\langle \Phi_{a_1 b_1} \dots \Phi_{a_M b_M} \rangle_{\mathcal{B}}$, where \mathcal{B} denotes the instantonic background and Φ are generic physical fields in the bi-fundamental representations of two generic N' and M' stacks of D6-branes

The correlator is related to the effective supergravity quantities in the action as follows [64, 65]:

$$\langle \Phi_{a_1 b_1} \dots \Phi_{a_M b_M} \rangle_{\mathcal{B}} = \frac{e^{\mathcal{K}/2} Y_{\Phi_{a_1 b_1}, \dots, \Phi_{a_M b_M}}}{\sqrt{K_{a_1 b_1, \dots, a_M b_M}}}, \quad (A1)$$

where \mathcal{K} is the non-holomorphic Kähler potential, K is the Kähler metric and Y is the holomorphic superpotential coupling. The following generic formula for the computation of the correlation function in the semiclassical approximation is [64, 65]:

$$\begin{aligned} \langle \Phi_{a_1 b_1}, \dots, \Phi_{a_M b_M} \rangle_{\mathcal{B}} \simeq & \int d^4 x d^2 \theta \sum_{conf} \prod_a \left(\prod_{i=1}^{I^+} d\lambda_a^i \right) \left(\prod_{i=1}^{I^-} d\bar{\lambda}_a^i \right) \\ & \times \exp(-S^{(0)}) \exp \left(\sum_b C_{\mathcal{E}D_b^*} + C_{\mathcal{E}O^*} \right) \\ & \times \langle \hat{\Phi}_{a_1 b_1}[\mathbf{x}_1] \rangle_{\lambda_{a_1}, \bar{\lambda}_{b_1}} \dots \langle \hat{\Phi}_{a_L b_L}[\mathbf{x}_L] \rangle_{\lambda_{a_L}, \bar{\lambda}_{b_L}}, \quad (A2) \end{aligned}$$

where $\hat{\Phi}_{a_k b_k}$ is the chain-product of all the the vertex operators at fixed values of k

$$\hat{\Phi}_{a_k b_k} = \Phi_{a_k x_{k,1}} \cdot \Phi_{x_{k,1} x_{k,2}} \dots \Phi_{x_{k,n-1}, x_{k,n}} \cdot \Phi_{x_{k,n}, b_k}$$

while

$$\langle \hat{\Phi}_{a_1 b_1}[\mathbf{x}_1] \rangle_{\lambda_{a_1}, \bar{\lambda}_{b_1}}$$

a CFT disk correlator for $\hat{\Phi}$ and for the charged zero modes $\lambda_{a_1}, \bar{\lambda}_{b_1}$ inserted in the boundary of the mixed disk amplitudes. $C_{\mathcal{E}D_b^*}, C_{\mathcal{E}O^*}$ denote the sum all over the annulus diagrams with and without one cross-cap, respectively.

From Eqs. (A1)-(A2), the holomorphic superpotential coupling can be entirely re-expressed in terms of holomorphic couplings in the CFT amplitudes:

$$\begin{aligned} Y_{\hat{\Phi}_{a_1 b_1}, \dots, \Phi_{a_M b_M}} = & \\ \sum_{conf} \exp(-S_0) Y_{\lambda_{a_1} \hat{\Phi}_{a_1 b_1}[\mathbf{x}_1], \bar{\lambda}_{b_1}} \dots Y_{\lambda_{a_L} \hat{\Phi}_{a_L b_L}[\mathbf{x}_L], \bar{\lambda}_{b_L}}, & \quad (A3) \end{aligned}$$

where S_0 corresponds to the vacuum disk amplitude for the E2-brane:

$$S_{\mathcal{E}}^{(0)} = -\langle 1 \rangle_{Disc} = \frac{1}{g_s} \frac{V_{E2}}{l_s^3},$$

with V_{E2} the volume of the 3-cycles wrapped by the E2-instanton. This formula generalizes the prescription given in Section II, in the particular case of brane intersections described above. In particular, the instanton can reproduce the six-quark operator if and only if with the number of brane intersections considered above.

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