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Systematic dependence of $SFE*B(\mathrm{E2})\uparrow$ and $ROTE*B(\mathrm{E2})\uparrow$ on $N_\mathrm{p}N_\mathrm{n}$ in the framework of shape fluctuation model

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Abstract: A systematic dependence of shape fluctuation energy product $SFE*B(\mathrm{E2})\uparrow$ and rotational energy product $ROTE*B(\mathrm{E2})\uparrow$ on the valence nucleon product $N_\mathrm{p}N_\mathrm{n}$ is carried out in the $Z=50-82$, $N=82-126$ major shell space. For the shape transitional nuclei, the product $SFE*B(\mathrm{E2})\uparrow$ drops to zero and becomes negative, indicating direct dependence on $N_\mathrm{p}N_\mathrm{n}$. A relative rise, on the other hand, is observed in the product $ROTE*B(\mathrm{E2})\uparrow$ plotted against $N_\mathrm{p}N_\mathrm{n}$ for all the nuclei in $Z=50-82$ and $N=82-126$. In the $N>104$ region, large positive values of the product $SFE*B(\mathrm{E2})\uparrow$ are observed for the Pt nucleus, which indicates sphericity. A systematic study of the product $SFE*B(\mathrm{E2})\uparrow$ and $ROTE*B(\mathrm{E2})\uparrow$ with atomic number Z is also discussed. The products $SFE*B(\mathrm{E2})\uparrow$ and $ROTE*B(\mathrm{E2})\uparrow$ vary sharply with Z for the $N=88$ isotones. We discuss here for the first time the correlation between $SFE*B(\mathrm{E2})\uparrow$ and $ROTE*B(\mathrm{E2})\uparrow$ with valence nucleon product $N_\mathrm{p}N_\mathrm{n}$.

Keywords: nuclear structure, shape fluctuation energy, rotational energy, $N_\mathrm{p}N_\mathrm{n}$ scheme.

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1 Introduction

In recent years several semi-classical models have been introduced to study deformation in nuclear structure. The Bohr and Mottelson model [1] has been widely used for the study of low energy spectra of deformed nuclei. The Bohr collective Hamiltonian classifies the nucleus as a spherical vibrator and axially symmetric deformed rotor. The ground band energies of a rigid rotator can be expressed through the quantum mechanical rotor formula:

$$E(J)=\frac{\hbar^2}{2\Im}J(J+1), \quad (1)$$

where $\Im$ is the moment of inertia and $J=0,2,4,6...$ is the spin of the nucleus. Later, many nuclear models [2–4] were proposed by expanding the $J(J+1)$ term to study collectivity in even-even nuclei. The variable moment of inertia model of Mariscotti et al. [5] has given a good fit in the spectra of deformed even-even nuclei. However, the model has two limitations: (i) the moment of inertia was chosen proportional to the deformation parameter and (ii) the potential energy was chosen to be harmonic. Satpathy and Satpathy [6] have proposed a three parameter shape fluctuation model which gives a better fit for the energy levels of ground state bands of even-even nuclei. The energy expression for the shape fluctuation model is given by

$$E(J)=B_0J(J+1)+J\phi^{'}E^{'}+J\phi^{'}B^{'}J(J+1), \quad (2)$$

where ϕ is the intrinsic wave function, E is the intrinsic energy of the core and $E^{'}$, $\phi^{'}$ and $B^{'}$ denote the first derivatives. In Eq. (2) the first term gives the energy due to the rotation of the unfluctuated core. The second and third terms give the excess in the intrinsic energy and the rotational energy due to fluctuation of the core, respectively. Based on the assumptions of the shape fluctuation model, Gupta et al. [7] assumed the sum of the last two terms in Eq. (3), the intrinsic energy and interaction term, as the shape fluctuation energy (SFE), and the first term as the rotational energy (ROTE)

$$E(J)=aJ(J+1)+bJ+cJ^2(J+1), \quad (3)$$

and studied the dependence of SFE and ROTE on $N_\mathrm{p}N_\mathrm{n}$. Gupta and Kavathekar [8] extended the search and observed the dependence of rotational content $\dfrac{ROTE}{E(2_1^+)}$ on $N_\mathrm{p}N_\mathrm{n}$. The interacting boson model IBM-1 [9] Hamiltonian, representing O(5) and O(3) symmetries written in terms of Casimir operators, has the same split of energy of $E(2_1^+)$ into vibrational and rotational parts as written in Eq. (3) in their contribution to g-band energies. It was recently suggested [10, 11] that a simplified parametrization of nuclear structure could be obtained

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by plotting the Grodzins [12] product $E(2_1^+)*B(\mathrm{E2})\uparrow$ as a function of $N_\mathrm{p}N_\mathrm{n}$. Following Satpathy and Satpathy [6], we have split the energy term in the Grodzins product $E(2_1^+)*B(\mathrm{E2})\uparrow$ as shape fluctuation energy product $SFE*B(\mathrm{E2})\uparrow$ and rotational energy product $ROTE*B(\mathrm{E2})\uparrow$, and studied its systematic dependence on Z and $N_\mathrm{p}N_\mathrm{n}$ for the A = 100–200 mass region. This study may provide a more comprehensive analysis of nuclear structure.

2 Theory

The splitting of energy term in the Grodzins product

$$E(2_1^+)*B(\mathrm{E2})\uparrow \sim \frac{Z^2}{A}, \tag{4}$$

as shape fluctuation energy product $SFE*B(\mathrm{E2})\uparrow$ and rotational product $ROTE*B(\mathrm{E2})\uparrow$, provides detailed information about the nuclear structure. The reduced transition probability product as $SFE*B(\mathrm{E2})\uparrow$ and $ROTE*B(\mathrm{E2})\uparrow$ changes the character of SFE and ROTE. The relative contribution of $SFE*B(\mathrm{E2})\uparrow$ and $ROTE*B(\mathrm{E2})\uparrow$ to $E(2_1^+)*B(\mathrm{E2})\uparrow$ reflects the shape phase changes, gradual or sharp, with Z and $N_\mathrm{p}N_\mathrm{n}$. Rewriting Eq.(3) as,

$$Y_J = E(J)/J = b(J+1)+a+cJ(J+1), \tag{5}$$

we use the least squares fit method to determine the optimum values of constant a,b and c, using the values up to $J^\Pi = 12^+$ i.e., below the backbending region in most of the nuclei. The values of $E(2_1^+)$ are taken from the National Nuclear Data Center (NNDC) website [13]. The reduced quadruple transistion probability, $B(\mathrm{E2})\uparrow$ values are taken from Raman et al. [14]. Since the energy of the $E(2_1^+)$ state descends continuously from values about 1 MeV near closed shells to $\sim 100-200$ keV for well deformed rotational nuclei, both $SFE*B(\mathrm{E2})\uparrow$ and $ROTE*B(\mathrm{E2})\uparrow$ decrease *ipso facto*. On the other hand, their relative contributions decrease and increase in a complementary way and should be particularly informative.

3 Results and discussion

We adopt a grouping based on valence particle and hole pair considerations [15]. Thus, the $Z=50-82$, $N=82-126$ major shell space is partitioned into four quadrants: p-p, h-p, h-h and p-h (p=valence particle, proton or neutron, h = hole). The last quadrant is empty.

3.1 Ba-Gd nuclei, $N>82$ region

This is a particle-particle nucleon sub-region of the $Z=50-82$, $N=82-126$ shell space. A plot of shape fluctuation energy product $SFE*B(\mathrm{E2})\uparrow$ against atomic number Z and valence nucleon product $N_\mathrm{p}N_\mathrm{n}$ for Ba-Gd nuclei is shown in Fig. 1 (a) and (b) respectively.

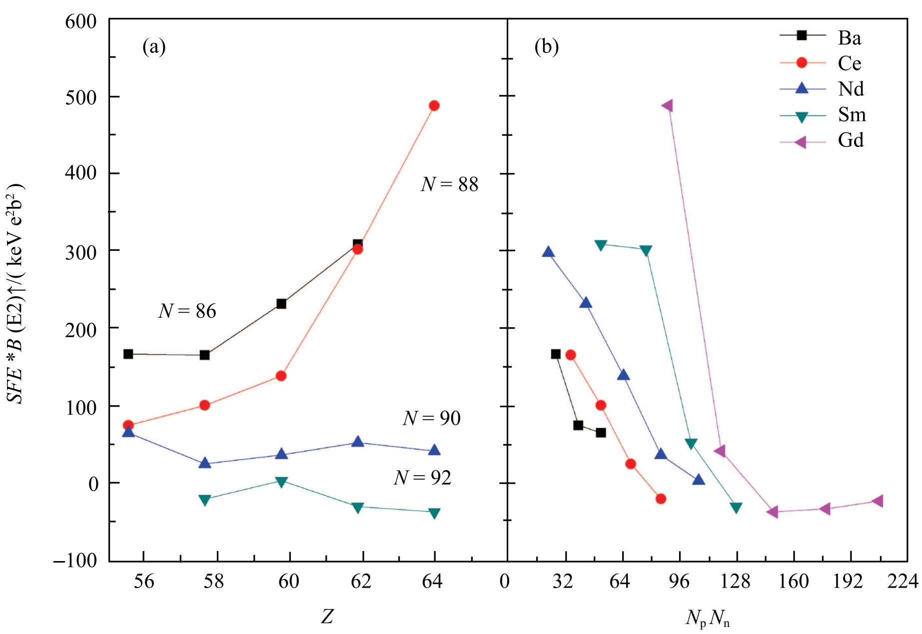

Fig. 1. (color online) Systematics of (a) product $SFE*B(\mathrm{E2})\uparrow$) vs. Z for the $N>82$ region in Ba-Gd nuclei and (b) $SFE*B(\mathrm{E2})\uparrow$) as a function of valence nucleon product $N_\mathrm{p}N_\mathrm{n}$.

In the $N = 88$ isotones the product $SFE * B(\mathrm{E2})\uparrow$ varies sharply with Z due to onset of deformation at $Z = 64$, as supported by many authors [10, 11]. This clearly shows that the fluctuation in the nuclear core increases tremendously for $N = 88$ isotones and sets a platform for deformation. Gupta et al. [7] concluded the same idea that the Grodzins product $E(2_1^+)*B(\mathrm{E2})\uparrow$ increases sharply for $N = 88$ isotones (see Fig. 3 of Ref. [10]) and the dependence of (Z^2/A) is neglected in the plot. Turning now to the $N_\mathrm{p}N_\mathrm{n}$ plot for the Ba and Ce nuclei, the product $SFE * B(\mathrm{E2})\uparrow$ decreases very slowly and remains positive. The decrease in SFE is greater than the rise in $B(\mathrm{E2})\uparrow$ values. A moderate rise in $B(\mathrm{E2})\uparrow$ values is observed for vibrational nuclei. This clearly reflects that in the systematics the Ba and Ce nuclei are more of a particle-particle nature and approach deformed character monotonically. However, in thecase of shape transitional nuclei Nd, Sm and Gd, the product $SFE*B(\mathrm{E2})\uparrow$ drops to zero and becomes negative, indicating a most drastic phase transition for $N = 90$ isotones and the eradication of the $Z = 64$ subshell gap. Casten [16] also observed a similar change as the data points of ground state energy $E(2_1^+)$ shifted from the regular trend (see Fig. 15 of Ref. [17]) plotted as a function of $N_\mathrm{p}N_\mathrm{n}$ for shape transitional nuclei.

The product $ROTE * B(\mathrm{E2})\uparrow$ plotted as a function of atomic number Z and valence nucleon product $N_\mathrm{p}N_\mathrm{n}$ is shown in Fig. 2 (a) and (b) respectively.

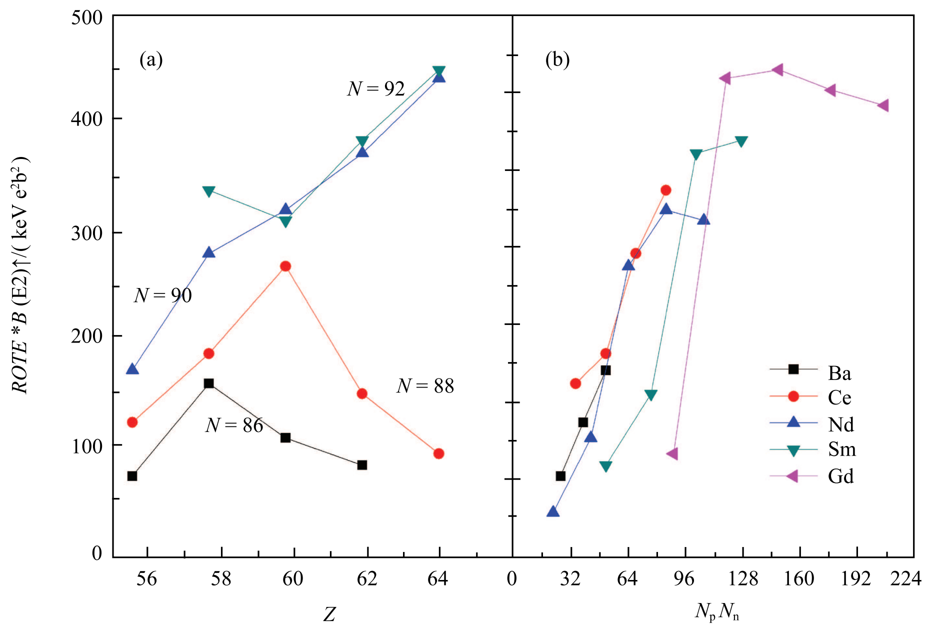

Fig. 2. (color online) Systematics of product $ROTE*B(\mathrm{E2})\uparrow$ for the $N > 82$ region in Ba-Gd nuclei.

For $N \leqslant 88$ isotones, the product $ROTE * B(\mathrm{E2})\uparrow$ decreases as expected because the energy $E(2_1^+)$ is to be minimized towards well deformed rotational nuclei, but the exact inverse of this behavior occurs for $N \geqslant 90$, where the isotones are famous for achieving maximum deformation [17]. The product $ROTE*B(\mathrm{E2})\uparrow$ plotted as a function of $N_\mathrm{p}N_\mathrm{n}$ in Fig. 2(b) increases up to the transition point and gets saturated for these nuclei (Sm and Gd) with the sharp phase transition (see Fig. 2(b)). This is in contradiction with the viewpoint of Gupta et al. [7], in which the rotational energy decreases with increasing deformation (see Fig. 6 of Ref. 7) but the actual rotational energy and $B(\mathrm{E2})\uparrow$ values are in coherence and increasing simultaneously with $N_\mathrm{p}N_\mathrm{n}$, since the $B(\mathrm{E2})\uparrow$ values are correlated to deformation parameter β in the case of rotational nuclei. The contribution of $B(\mathrm{E2})\uparrow$ is highlighted in the rotational product $ROTE*B(\mathrm{E2})\uparrow$, which was hidden in the collective state of $E(2_1^+)*B(\mathrm{E2})\uparrow$ as the Grodzins product.

3.2 Dy-W nuclei, $N < 104$ region

This is the proton-hole and neutron-particle subregion of the $Z = 50$–82, $N = 82$–126 major shell region recognized as quadrant-II. The $SFE*B(\mathrm{E2})\uparrow$ plots are much different from those in the Ba-Gd $N > 82$ region discussed above. The plot of product $SFE * B(\mathrm{E2})\uparrow$ against atomic number Z is shown in Fig. 3(a).

The product $SFE * B(\mathrm{E2})\uparrow$ decreases and finally drops to a minimum, indicating that most of the nuclei in this sub-region are deformed. However, the $N = 90$ isotones are away from the general trend and the product $SFE*B(\mathrm{E2})\uparrow$ first increases and then decreases, indicating sharp shape phase transition in the structure. Turning now to the N_pN_n results, in Fig. 3(b), these are more informative, with the product $SFE{*}B(\mathrm{E2})\uparrow$ dropping to a minimum on a single smooth curve. The smoothness of the N_pN_n curves clearly points to a simple deformation driving mechanism and indicates the direct dependence of the product $SFE*B(\mathrm{E2})\uparrow$ with N_pN_n. This is similar to the curve obtained by Gupta et al. [7] for the shape fluctuation energy plot for Dy-Pt nuclei. On the other hand, in Fig. 4(b) for $N_pN_n > 30$, the product $ROTE*B(\mathrm{E2})\uparrow$ increases and saturation is reached. At large values of N_pN_n, the product $ROTE*B(\mathrm{E2})\uparrow$ starts falling when the nucleus becomes stiff to β-vibration and to centrifugal stretching, when it starts to resemble a rigid rotor.

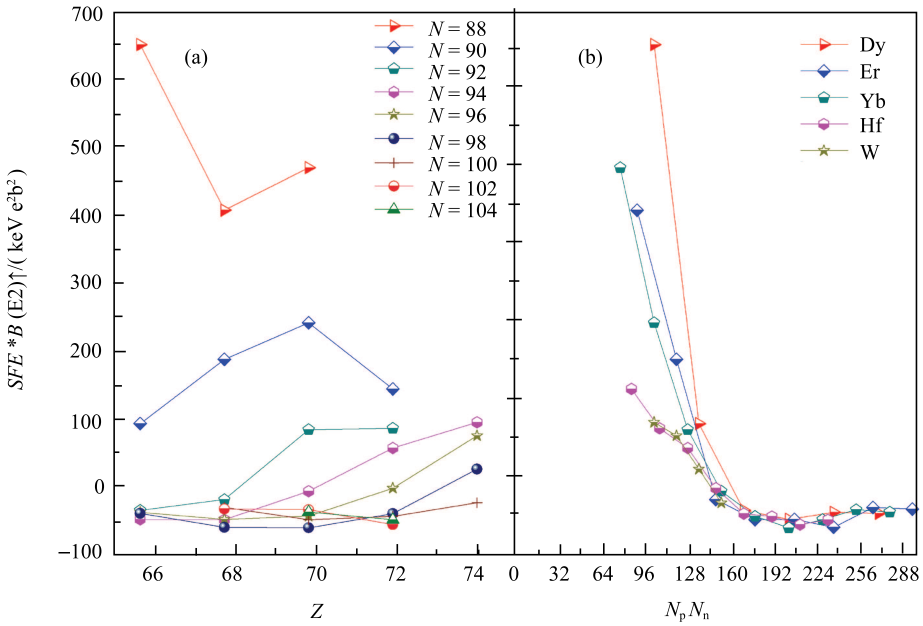

Fig. 3. (color online) Systematics of product $SFE*B(\mathrm{E2})\uparrow$ for the $N<104$ region in Dy-W nuclei.

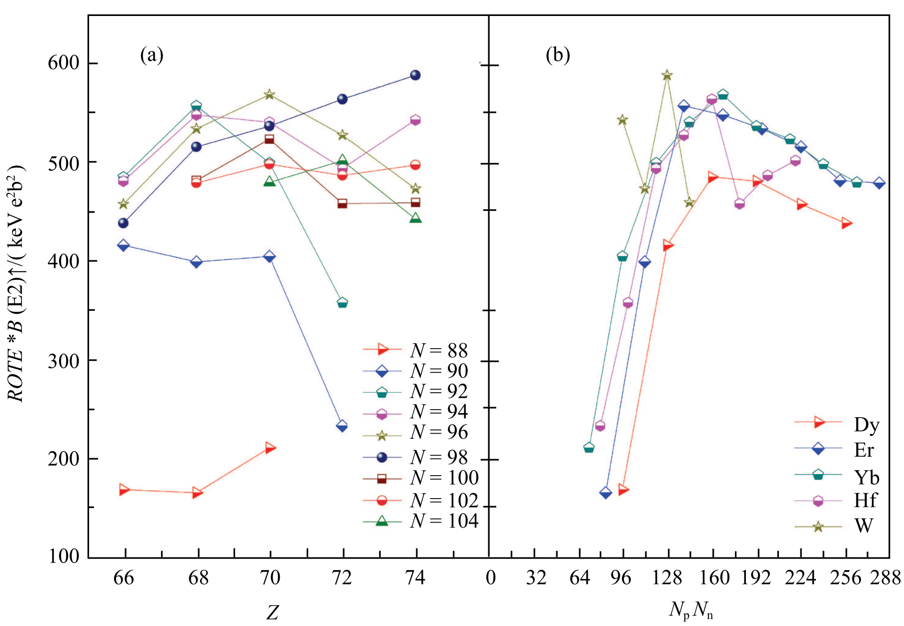

Fig. 4. (color online) Systematics of product $ROTE*B(\mathrm{E2})\uparrow$ for the $N<104$ region in Dy-W nuclei.

3.3 Hf-Pt nuclei, $N > 104$ region

This is the hole-hole sub-space of the $Z = 50$–82, $N = 82$–126 major shell space recognized as quadrant-III. The plot of shape fluctuation energy product $SFE * B(\mathrm{E2})\uparrow$ shows strong dependence on atomic number Z for the Hf-Pt nuclei, as shown in Fig. 5(a). Here, the product $SFE * B(\mathrm{E2})\uparrow$ is negative for $Z < 76$, indicating deformation, and suddenly starts increasing as $Z > 76$, which clearly manifests a loss of collectivity. This is a similar observation to that in the previous section, with the elements most spherical for $N < 90$ and becoming more deformed at $N > 90$. Casten et al. [16] made the same observation about this similarity in the structure, because as Z increases beyond 76 the last protons enter strongly upsloping $\frac{11}{2}[505]$ and $\frac{7}{2}[402]$ orbits. On the other hand, in Fig. 5(b), for large N_pN_n values the product $SFE * B(\mathrm{E2})\uparrow$ is low and negative for Hf-W-Os nuclei, indicating a trend towards deformation. However, large positive values of the product are observed for the Pt nucleus, which indicates loss of collectivity. Gupta et al. [7] also observed the same positive values of shape fluctuation energy for the Pt nucleus in the $N > 104$

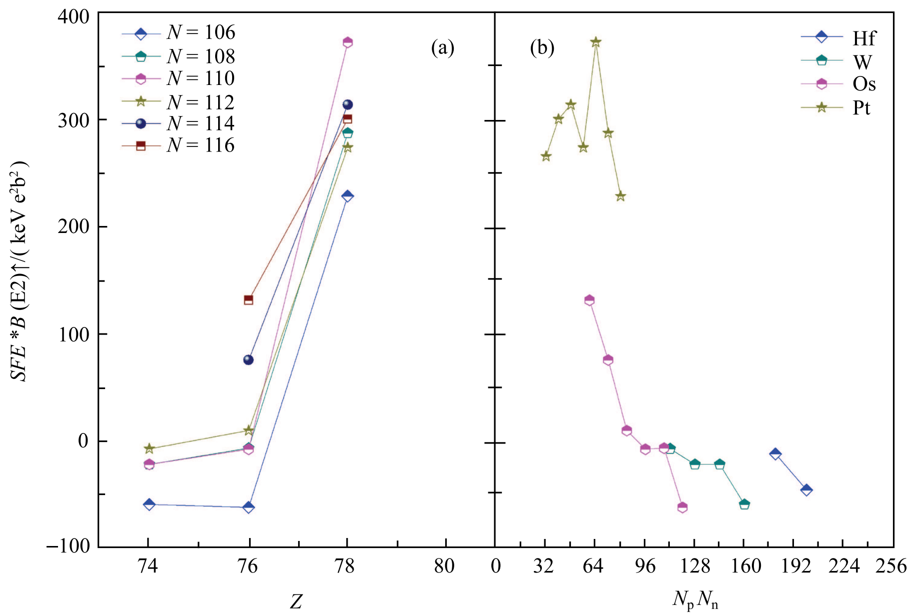

Fig. 5. (color online) Systematics of product $SFE * B(\mathrm{E2})\uparrow$ for the $N > 104$ region in Hf-Hg nuclei.

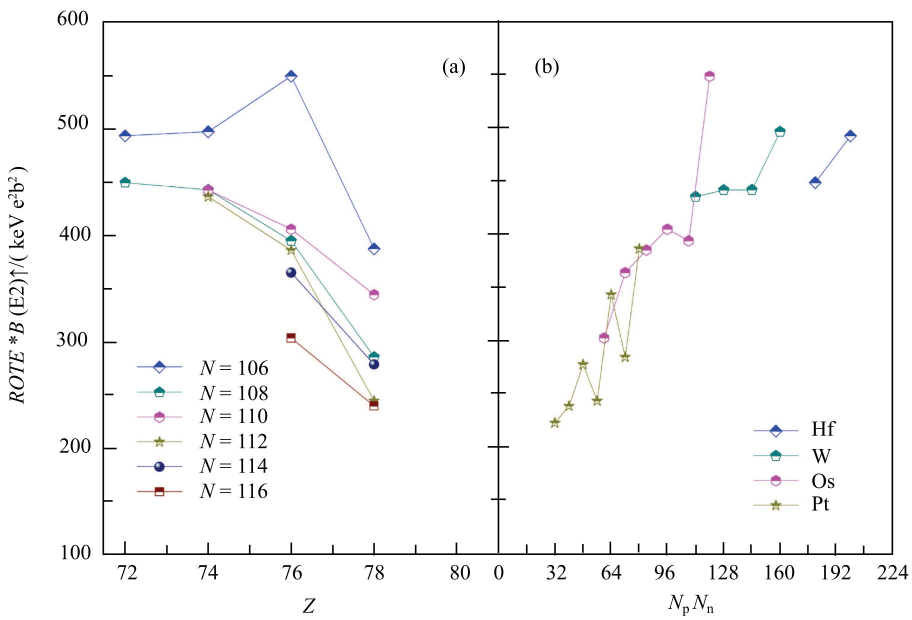

Fig. 6. (color online) Systematics of product $ROTE * B(\mathrm{E2})\uparrow$ for the $N > 104$ region in Hf-Pt nuclei.

region. The Pt nucleus becomes more evident if we observe the plot of product $ROTE*B(\mathrm{E2})\uparrow$ versus $N_\mathrm{p}N_\mathrm{n}$ in Fig. 6(b). The data points of Pt overlaps with those of Os. This shows that the Pt nucleus has the possibility to deform for large $N_\mathrm{p}N_\mathrm{n}$. The plot of product $ROTE*B(\mathrm{E2})\uparrow$ against atomic number Z is shown in Fig. 6(a). As Z increases beyond 76, the product $ROTE*B(\mathrm{E2})\uparrow$ start decreasing and the nuclei start to be driven towards the spherical configuration.

4 Conclusion

The shape fluctuation energy product $SFE*B(\mathrm{E2})\uparrow$ gives a good measure of vibrational component along with transitional probability. Its relative contribution to the Grodzins product $E(2_1^+)*B(\mathrm{E2})\uparrow$ vividly highlighted the shape phase transition with Z and $N_\mathrm{p}N_\mathrm{n}$. The systematics of the product $SFE*B(\mathrm{E2})\uparrow$ and rotational product $ROTE*B(\mathrm{E2})\uparrow$, plotted as function of Z, indicate a sharp rise and fall for $N=88$ isotones, complementary to each other. The plots also highlight the dual reversal of the roles of protons and neutrons and of holes and particles. The nuclei which are most deformed beyond $Z<76$ become most spherical as $Z>76$. The $N_\mathrm{p}N_\mathrm{n}$ product dependence on $SFE*B(\mathrm{E2})\uparrow$ and $ROTE*B(\mathrm{E2})\uparrow$ is visible in all the quadrants. In all the well deformed regions studied above, the product $SFE*B(\mathrm{E2})\uparrow$ decreases and give negative values, becoming independent of the $N_\mathrm{p}N_\mathrm{n}$ product. The product $ROTE*B(\mathrm{E2})\uparrow$ dependence on $N_\mathrm{p}N_\mathrm{n}$ product is important here. The product $ROTE*B(\mathrm{E2})\uparrow$ rises with $N_\mathrm{p}N_\mathrm{n}$ in all the quadrants. Gupta et al. [7] suggested that the rotational energy decreases with increase in deformation (see Fig. 6 of Ref. 7) so that a realistic picture of rotational energy in collective state $E(2_1^+)$ is not justified. Here, the plots vividly highlight the rotational contribution along with transition probability $B(\mathrm{E2})\uparrow$ as both N_p and N_n increase simultaneously. This indicates that the rotational content and excitation strength represent coherent motion.

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