Systematic study of rotational energy formulae for superdeformed bands in La and Ce isotopes

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Abstract: The experimental rotational spectra of superdeformed (SD) bands of ^{130}La, ^{131}Ce(1,2), ^{132}Ce(1,2,3) and ^{133}Ce(1,2,3) in the $A\sim130$ mass region are systematically analyzed with the four parameter formula, power index formula, nuclear softness formula, and VMI model. It is observed that out of all the formulae, the four parameter formula suits best for the study of the ^{130}La, ^{131}Ce(1,2), ^{132}Ce(2,3) and ^{133}Ce(1,2,3) SD bands. The four parameter formula works efficiently in determining the band head spin of the ^{130}La, ^{131}Ce(1,2) ^{132}Ce(2,3) and ^{133}Ce(1,2,3) SD bands. Good agreement is seen between the calculated and observed transition energies whenever the accurate spin is assigned. In ^{132}Ce(1), the power index formula is found to work better than the other three formulae. The dynamic moment of inertia is also calculated for all the formulae and its variation with the rotational frequency is investigated.

Keywords: superdeformed bands, four parameter formula, power index formula, nuclear softness formula, VMI model

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1 Introduction

The presence of superdeformed states with deformation $\beta\approx0.5$–0.6 (prolate ellipsoidal with major axis twice the length of the minor axes) was predicted by Struntinsky [1]. Experimentally, the first evidence of superdeformation was given by Twin et al. [2] in the ^{152}Dy(1) nucleus. Subsequently, Ideguchi et al. [3, 4] observed superdeformation in the $A\sim40$ mass region. The discovery of superdeformation in ^{152}Dy(1) led to study of the phenomenon in different mass regions. The majority of superdeformed (SD) bands are now available in the $A\sim190$, 150, 130, 80, 60 mass regions.

At unknown excitation energies and band head spins, the SD bands are known as hanging bands. This is because the linking transition between the normal deformed (ND) and SD bands is missing. Several different approaches are offered to predict the spin assignments of the SD bands [5–8]. The spins of a few SD bands [9–16] are also available experimentally. The measured spins provide a significant attempt to check the effectiveness of these approaches. The physics of SD bands can be explained by the concept of level spin determination. The different formulae/ models discussed in the literature are the $SU_q(2)$ model, VMI inspired IBM, Lipas Eijiri and its extension, Harris expansion, and three parameter model [5, 17–21]. The maximum formulae in the literature are mostly used only in fitting some of the SD bands in the $A\sim150$, 190 mass regions.

The most remarkable distinction between the superdeformed bands in the $A\sim150$ and $A\sim190$ mass regions is the variation of dynamic moment of inertia $J^{(2)}$ versus rotational frequency. In the $A\sim190$ mass region, $J^{(2)}$ increases with increasing rotational frequency [22, 23]. In the $A\sim150$ mass region, $J^{(2)}$ decreases with increasing rotational frequency [24]. The SD bands in the $A\sim190$ mass region have smaller transitions and are observed at lower spin than the $A\sim150$ mass region ($>20\hbar$).

In constrast to the $A\sim150$ mass region, the spins of yrast SD bands are lower ($\approx25\hbar$ as compared to $\approx45\hbar$) in the $A\sim130$ mass region. Quadrupole deformation is also lower ($\beta\simeq0.4$) in the $A\sim130$ mass region than in the ($\beta\simeq0.6$) $A\sim150$ mass region. The first SD bands were observed in the $A\sim130$ mass region by Kirwan et al. [25], in the ^{132}Ce(1) nucleus. Thereafter, Bazzacco and Deleplanque [26, 27] observed the decay process for the first time in the odd ^{133,135,137}Nd nuclei. Soon after, Petrache et al. [28] observed similar transitions in ^{132,134}Nd. The lifetime measurement of ^{133,137}Nd nuclei in the second minimum was suggested by Mullin et al. [29]. The relative deformation of the SD bands in ^{131,132}Ce was observed by Clark et al. [30]. It was suggested in Ref. [30] that the quadrupole moment of the SD bands in ^{131,132}Ce nuclei is calculated by using the Doppler-shift attenuation method. It was also found that the excited bands

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in ^{131}Ce have large quadrupole moments.

The study of lifetime measurement within the superdeformed minimum of ^{132,133}Ce nuclei was proposed by Hauschild et al. [31]. The phenomenon of triaxiality in the $A\sim130$ mass region was studied by Hauschild et al. [32] in the ^{133}Ce nucleus, in the framework of the cranked Strutinsky Woods-Saxon calculation. Da Li et al. [33] studied the systematics of double odd nuclei of SD bands in the $A\sim130$ mass region. The phenomenon of identical SD bands and $\triangle I=4$ bifurcation in $A\sim130$ mass region was studied by Liu et al. [34]. Santos et al. [35] observed the evidence for identical SD bands and excited bands in ^{132}Ce in the $A\sim130$ mass region. Semple et al. [36] obtained the energy staggering of SD bands in ^{131,132,133}Ce nuclei.

The comparison between the rotational energy formulae for axially symmetric deformed nuclei was proposed by Wu and Lei [37]. It was suggested in Ref. [37] that a simple two-parameter *ab* formula is better to study the rotational spectra of both the ground state bands in even-even nuclei and the odd-odd nuclei. Therefore, this study motivates us to compare the different rotational energy formulae (four parameter formula, power index formula, nuclear softness formula, and VMI model) for the SD bands in La and Ce isotopes in the $A\sim130$ mass region.

The aim of the present work is to study the systematics of the rotational energy formulae for ^{130}La, ^{131}Ce(1,2), ^{132}Ce(1, 2, 3) and ^{133}Ce(1, 2, 3) in the $A\sim130$ mass region. This may provide information about the rotational energy formulae that are best suited to study the SD spectroscopy of ^{130}La, ^{131}Ce(1, 2), ^{132}Ce(1, 2, 3) and ^{133}Ce(1,2, 3) in the $A\sim130$ mass region.

2 Formalism

2.1 Four parameter formula

Under the adiabatic estimation, the rotational energy of an axially symmetric nucleus can be developed in terms of $I(I+1)$ (for $K=0$) [38, 39] as

$$E(I) = A\left((I(I+1))+\frac{B}{A}(I(I+1))^2 +\frac{C}{A}(I(I+1))^3+\frac{D}{A}(I(I+1))^4\right). \tag{1}$$

The extension for the $K\neq0$ band takes a similar form to Eq. (1), but incorporates a term for band head energy, and $I(I+1)$ is replaced by the term $I(I+1)$-K^2. Therefore, the different form in which the energy can be expressed, as suggested in Ref. [40], is

$$E(I) = \frac{1}{2J_0}\left((I(I+1))-\frac{1}{2}\sigma(I(I+1))^2 +\sigma^2(I(I+1))^3-3\sigma^3(I(I+1))^4\right). \tag{2}$$

When a comparison is made between Eq. (1) and Eq. (2), we get

$$A=\frac{1}{2J_0},\quad \frac{B}{A}=-\frac{\sigma}{2},\quad \frac{C}{A}=\sigma^2,\quad \frac{D}{A}=-3\sigma^3, \tag{3}$$

where σ is a softness parameter and J_0 is a moment of inertia.

As the band quantum number is not known for the SD bands, Eq. (1) can be expressed as

$$\begin{aligned}E(I) = {} & E_0+A(I(I+1)-I_0(I_0+1))+B((I(I+1))^2 \\ & -(I_0(I_0+1))^2)+C((I(I+1))^3-(I_0(I_0+1))^3 \\ & +D((I(I+1))^4-(I_0(I_0+1))^4),\end{aligned} \tag{4}$$

where E_0 and I_0 are the band head energy and band head spin respectively. As the band head energies and band head spins remain unknown for the SD bands, one may apply fitting of the E_γ transitions as

$$E_\gamma(I)=E(I)-E(I-2). \tag{5}$$

By using Eqs. (4) and (5), we get

$$\begin{aligned}E_\gamma(I\rightarrow I-2) = {} & A(I(I+1)-(I-2)(I-1)) \\ & +B((I(I+1))^2-((I-2)(I-1))^2) \\ & +C((I(I+1))^3-((I-2)(I-1))^3) \\ & +D((I(I+1))^4-((I-2)(I-1))^4),\end{aligned} \tag{6}$$

where A, B, C and D are the model parameters which can be resolved by fitting the E_γ transitions for the SD bands.

2.2 Power index formula

Gupta et al. [41] studied the single term expression for the ground band level energies of a soft rotor. The arithmetic mean approach of the two terms used in the Bohr- Mottelson formula is replaced by the geometric mean and is called the power index formula:

$$E(I)=aI^b. \tag{7}$$

In the power index formula, a and b are the fitting parameters. The only spectroscopic information available for the SD bands is the intraband transition energies and intensities, so one may apply the fitting of the E_γ transitions as

$$E_\gamma(I)=E(I)-E(I-2). \tag{8}$$

Using Eq. (7) and Eq. (8), we get

$$E_\gamma(I)=a\left(I^b-(I-2)^b\right). \tag{9}$$

The parameters a and b are obtained by least squares fitting of the observed transition energies.

2.3 Variable moment of inertia model

Marriscotti et al. [42] suggested the VMI model, which calculates the energy level with angular momentum as the sum of a potential energy term and a rotational energy term. The VMI model describes each nucleus by its band head moment of inertia ($\Im_0$) and restoring force constant (C).

$$E_I(\Im)=\frac{1}{2}C(\Im_I-\Im_0)^2+\frac{1}{2}\left[\frac{I(I+1)}{\Im_I}\right]. \quad (10)$$

The band head energy level of the SD rotational bands (I_0) is expressed as

$$E_I(\Im)=E_0+\frac{1}{2\Im_I}[I(I+1)-I_0(I_0+1)]+\frac{1}{2}C(\Im_I-\Im_0)^2, \quad (11)$$

where E_0 and $\Im_0$ are the band head energy and the ground moment of inertia of the SD bands respectively. In an equilibrium condition the variable moment of inertia can be resolved as

$$\frac{\partial E(\Im_I)}{\partial \Im_1}=0, \quad (12)$$

which leads to

$$\Im_I^3-\Im_I^2\Im_0-[I(I+1)-I_0(I_0+1)]/2C=0. \quad (13)$$

The above eqnarray has one real root for any positive value of $\Im_0$ and C so, combining Eqs. (11) and (13), we get

$$E_I = E_0+\left[\frac{I(I+1)-I_0(I_0+1)}{2\Im_0}\right] \times\left[1+\frac{I(I+1)-I_0(I_0+1)}{4C(\Im_0)^3}\right]. \quad (14)$$

The E_γ transition energies for the SD bands can be expressed as

$$E_\gamma=E(I)-E(I-2). \quad (15)$$

Therefore, using Eqs. (14) and (15), we get

$$E_\gamma(I\to I-2) = \frac{[I(I+1)-(I-2)(I-1)]}{2\Im_0} +\frac{[I(I+1)]^2-[(I-2)(I-1)]^2}{8C(\Im_0)^4}, \quad (16)$$

where $\Im_0$ and C are model parameters, which can be found using fitting techniques.

2.4 Nuclear softness formula

To calculate the energy levels of ground state bands in even- even nuclei, the nuclear softness formula was proposed by Gupta [43]. In this work the variation of moment of inertia with spin was taken into account. An identical expression called the soft rotor formula (SRF) was given by Brentano et al. [44] for transitional and well deformed nuclei. In the soft rotor formula the moment of inertia ($\Im$) depends upon the excitation energy ($\Im=\Im_0(1+\sigma I+\beta E)$. Therefore, the rigid rotor formula is given by

$$E=\frac{\hbar^2}{2\Im}I(I+1). \quad (17)$$

When the variation of moment of inertia with spin is taken into account, the rigid rotor formula is modified as

$$E=\frac{\hbar^2}{2\Im_I}I(I+1). \quad (18)$$

Applying the Taylor series expansion to the $\Im_I$ term about its ground state ($\Im_0$ for $I=0$), we get

$$E_I = \frac{\hbar^2}{2}\left(\left[\frac{1}{\Im_0}-\left(\frac{1}{\Im_I^2}\frac{\partial\Im_I}{\partial I}\right)\right]_{I=0} I +\left[\frac{2}{\Im_I^3}\left(\frac{\partial\Im_I}{\partial I}\right)^2-\frac{1}{\Im_I^2}\frac{\partial^2\Im_I}{\partial I^2}\right]_{I=0}\frac{I^2}{2!}+\cdots\right)I(I+1). \quad (19)$$

The nuclear softness parameter [45] describes the increase of rigidity of the nucleus with the increase of deformation, so Eq. (19) can be expressed as

$$E_I = \frac{\hbar^2 I(I+1)}{2\Im_0}\frac{1}{1+\sigma_1 I} \times\left(1-\frac{\sigma_2 I^2}{(1+\sigma_1 I+\sigma_2 I^2)}-\frac{\sigma_3 I^3}{(1+\sigma_1 I+\sigma_3 I^3)}+\cdots\right), \quad (20)$$

where

$$\sigma_1=\frac{1}{\Im_0}\frac{\triangle\Im_0}{\triangle I},\sigma_2=\frac{1}{2!\Im_0}\frac{\partial^2\Im_0}{\partial I^2},\sigma_3=\frac{1}{3!\Im_0}\frac{\partial^3\Im_0}{\partial I^3}\cdots. \quad (21)$$

For the first order nuclear softness (putting $\sigma_{2,3}$=0), Eq. (20) becomes

$$E=\frac{\hbar^2}{2\Im_0}\times\frac{I(I+1)}{(1+\sigma I)}. \quad (22)$$

For the SD bands the transition energies can be expressed as

$$E_\gamma=E(I)-E(I-2). \quad (23)$$

Using Eq. (22) and Eq. (23) we get

$$E=\frac{\hbar^2}{2\Im_0}\times\left[\frac{I(I+1)}{(1+\sigma I)}-\frac{(I-2)(I-1)}{1+\sigma(I-2)}\right]. \quad (24)$$

2.5 Dynamic moment of inertia $J^{(2)}$

Theoretically, when the exact spin assignments are assigned to the SD bands, one can easily calculate the value of dynamic moment of inertia $J^{(2)}$ by using the calculated transition energies as follows [46]:

$$J^{(2)}=4000/[E_\gamma(I+2)-E_\gamma(I)]. \quad (25)$$

Experimentally, $J^{(2)}$ is independent of spin, but theoretically, $J^{(2)}$ can be calculated from the single particle energy level spectrum as a function of spin. In the present work we have calculated theoretically the $J^{(2)}$ of SD bands by least squares fitting of the E2 transition energies in Eq. (25). This approach is known as the best fit method (BFM). In this approach the spins of the SD bands are provided by least squares fitting of the calculated $J^{(2)}$ and also a comparison is made with their respective experimental results. Theoretically, another approach can be the variation of moment of inertia ($J^{(1)}$, $J^{(2)}$)with angular momentum. Both these approaches may be useful guidelines for the spin assignments.

3 Results and discussion

The E2 transition energies of superdeformed rotational bands can be identified by the following series: $E_\gamma(I_0+2n)$, $E_\gamma(I_0+2n-2)$, ..., $E_\gamma(I_0+4)$ and $E_\gamma(I_0+2)$. These E2 transition energies of SD bands are fitted by using the best fit method (BFM) in Eqs. (6), (9), (16) and (24). The root mean deviation can be defined as

$$\chi=\left[\frac{1}{n}\sum_{n=1}^{n}\left(\frac{E_\gamma^{\rm cal}(I_i)-E_\gamma^{\rm exp}(I_i)}{E_\gamma^{\rm exp}(I_i)}\right)^2\right]^{1/2}, \qquad (26)$$

where n is the total number of transitions used in the fitting procedure. The calculated results of the root mean deviation with the experimental results depends upon the determined spin. Good agreement between the calculated and observed transition energies is observed whenever the exact band head spin (I_0) is assigned. However, the values of root mean deviation show large changes whenever the value of (I_0) is increased or decreased by ±1.

The experimental data for ^{130}La, ^{131}Ce(1,2), ^{132}Ce(1,2,3), ^{133}Ce(1,2,3) has been taken from the SD tables of Singh et al. [47] and continuously updated data from Ref. [48]. This data has been fitted to the rotational energy formulae, viz. the four parameter formula, power index formula, nuclear softness formula and VMI model. In this work a comparison is made by applying the four parameter formula, power index formula, nuclear softness formula and VMI model to the ^{130}La, ^{131}Ce(1,2), ^{132}Ce(1,2,3), ^{133}Ce(1,2,3) SD bands in the $A\sim130$ mass region. It includes comparisons of band head spin assignments with the experimental data, calculated transition energies with the experimental transition energies, and the trends of dynamic moment of inertia with the rotational frequency. The band head spins (I_0) determined from the above-stated formulae are compared with the experimental data (see Table 1).

Table 1. The band head spin (I_0) obtained for ^{130}La, ^{131}Ce(1,2), ^{132}Ce(1,2,3), ^{133}Ce(1,2,3) by using different formulae. Here 1, 2 and 3 in parenthesis represent bands 1, 2, and 3 respectively.

SD band	$E_\gamma(I_0+2\to I_0)$	power index formula	4 parameter formula	nuclear softness formula	VMI model	Ref. [48]
^{130}La	762	14	16	13	22	16
^{131}Ce(1)	591.5	15	14	17	21	14.5
^{131}Ce(2)	847.4	28.5	26.5	31.5	28.5	23.5
^{132}Ce(1)	770.8	20	24	21	37	20
^{132}Ce(2)	724.4	18	19	21	24	19
^{132}Ce(3)	890.2	21	25	26	30	24
^{133}Ce(1)	748.3	21	21	23	25.5	21.5
^{133}Ce(2)	720.3	26.5	16.5	22.5	23.5	18.5
^{133}Ce(3)	956.9	23.5	21.5	26.5	31.5	22.5

3.1 ^{130}La SD band

The band head spins obtained by the VMI model diverge widely from the values given experimentally [48]. However, the band head spins obtained by the power index formula, four parameter formula and the nuclear softness formula are close to the experimental data (see Table 1). An example of least squares fitting of the ^{130}La SD band obtained from the four parameter formula, and a comparison with the other three formulae, is given in Table 2. At a particular band head spin a smaller fixed minimum value of root mean deviation is obtained by the four parameter formula. Thus, the four parameter formula is more reliable than the other three formulae in determination of spin assignments.

To support our results, the band head spin of the ^{130}La SD band obtained from the χ plot using the four parameter formula is shown in Fig. 1. Even the intraband energies are reproduced extremely well by the four parameter formula (see Table 2). The values of $J^{(2)}$ for the ^{130}La SD band from the four parameter formula, power index formula, nuclear softness formula and the VMI model are calculated using Eq. (25).

The calculated and experimental results of $J^{(2)}$ are shown in Fig. 3. When the graph of $J^{(2)}$ versus rotational frequency is plotted for the $A\sim130$ mass region, $J^{(2)}$ shows a smooth decrease with the increase in rotational frequency. It is observed from Fig. 3 that this trend is shown more efficiently by the four parameter formula than the other formulae. Therefore, it is suggested that the four parameter formula is the most efficient formula to study the ^{130}La SD band in the $A\sim130$ mass region.

Table 2. Spin determination of ^{130}La using different formulae. I_0 corresponds to band head spin. $\delta = E_\gamma^{\rm exp}(I) - E_\gamma^{\rm cal}(I)$, where E_γ is in keV. χ is the rms deviation given by Eq. (26).

$E_\gamma^{\rm exp}(I)$	I_0=16 (four parameter formula)			I_0=13 (nuclear softness formula)			I_0=14 (power index formula)		
	I	$E_\gamma^{cal}(I)$	δ	I	$E_\gamma^{cal}(I)$	δ	I	$E_\gamma^{cal}(I)$	δ
762	18	766.3	-4.3	15	746.0	16	16	754.8	7.2
852	20	845.3	6.7	17	835.5	16.5	18	839.0	13
921	22	922.3	-1.3	19	922.1	-1.1	20	921.5	-0.5
998	24	998.0	0	21	1005.9	-7.9	22	1002.8	-4.8
1073	26	1073.3	-0.3	23	1087.2	-14.2	24	1082.8	-9.8
1148	28	1149.8	-1.8	25	1165.9	-17.9	26	1161.8	-13.8
1229	30	1229.6	-0.6	27	1242.3	-13.3	28	1239.7	-10.7
1319	32	1316.0	3.0	29	1316.3	2.7	30	1316.8	2.2
1412	34	1413.1	-1.1	31	1388.2	23.8	32	1393.1	18.9
χ	0.0000103422			0.000110415			0.0013109691		

3.2 ^{131}Ce(1) SD band

The band head spins obtained by the four parameter formula and power index formula are in close agreement with the experimental data [48] as compared to the other formulae for ^{131}Ce(1)(see Table 1). The minimum value of root mean deviation is obtained by the four parameter formula. Therefore, the four parameter formula is reliable in assigning the spins for the ^{131}Ce(1) SD band. The band head spin of the ^{131}Ce(1) SD band obtained from the χ plot using the four parameter formula is shown in Fig. 1.

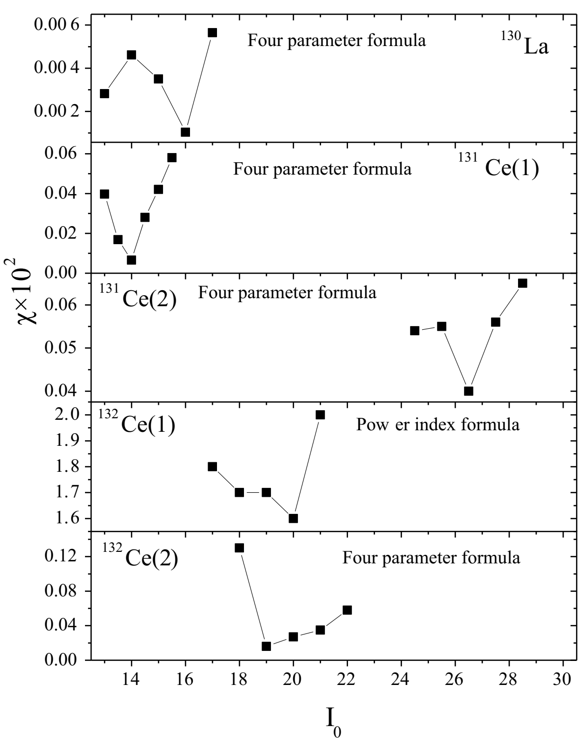

Fig. 1. χ plot to obtain I_0 for SD bands in La and Ce isotopes using different formulae.

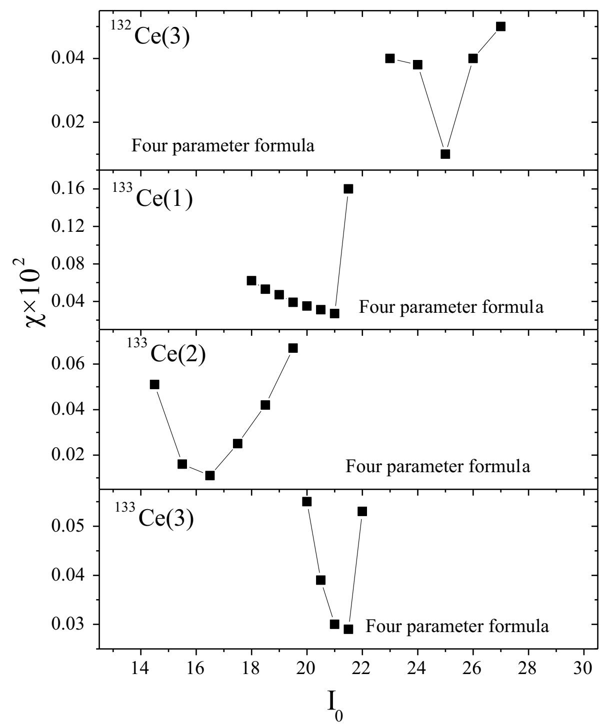

Fig. 2. χ plot to obtain I_0 for SD bands in La and Ce isotopes using different formulae.

The intraband energies of the ^{131}Ce(1) SD band are well reproduced by the four parameter formula (see Table 3). It is clear from Table 3 that the minimum root mean deviation is obtained at I_0=14. However, if I_0=13.5 or 14.5, a large change can be observed in the values of root mean deviation. Hence, both the values have been neglected. The values of $J^{(2)}$ for the ^{131}Ce(1) SD band from the four parameter formula, power index formula, nuclear softness formula and VMI model are calculated by using Eq. (25).

A comparison between the calculated and experimental results of $J^{(2)}$ is shown in Fig. 3. It is observed from Fig. 3 that the calculated results of $J^{(2)}$ using the four parameter formula show a general trend of $J^{(2)}$ versus the rotational frequency and agrees well with the experimental data. Hence, the four parameter formula is an efficient tool in assigning band head spin and in reproducing the experimental trend of $J^{(2)}$ versus rotational frequency in the ^{131}Ce(1) SD band.

3.3 ^{131}Ce(2) SD band

The band head spins obtained by the power index formula, nuclear softness formula and VMI model for the ^{131}Ce(2) SD band show large variation from the values given experimentally [48]. The band head spin obtained by the four parameter formula is closer to the experimental values (see Table 1). This is because the minimum root mean deviation is obtained by the four parameter formula. The root mean deviation values depend upon the transition number used in fitting.

The values of root mean deviation for two or more spin determinations may be close to each other, which leads to the uncertainty in spin proposition. Thus, this may be the main cause for the lack of agreement between the three formulae (the power index formula, nuclear softness formula and VMI model) and the experimental data.

An example for the least squares fitting of the ^{131}Ce(2) SD band obtained by the four parameter formula is given in Table 4. From Table 4, at I_0 =26.5, root mean deviation is minimum. However, if I_0 is assumed to be 25.5 or 27.5, root mean deviation shows large changes, so both the values have been neglected.

Hence, it is suggested that the four parameter formula is reliable for the spin prediction in the ^{131}Ce(2) SD band. The band head spins of the ^{131}Ce(2) SD band obtained from χ plot using the four parameter formula are shown in Fig. 1. Even the intraband energies are reproduced very well by the four parameter formula (see Table 4). The values of $J^{(2)}$ for the ^{131}Ce(2) SD band from the four parameter formula, power index formula, nuclear softness formula and VMI model are calculated using Eq. (25).

Figure 3 shows a comparison between the calculated and experimental results of $J^{(2)}$. It is observed from Fig. 3 that the calculated result of $J^{(2)}$ using the four parameter formula, nuclear softness formula and VMI model produce a general trend of $J^{(2)}$ versus rotational frequency and agrees well with the experimental data.

The above facts indicate that the four parameter formula is an effective tool in assigning the band head spin for the ^{131}Ce(2) SD band in the $A \sim$130 mass region. The four parameter formula, nuclear softness formula and VMI model are powerful tools in reproducing the experimental trend of $J^{(2)}$ versus rotational frequency.

3.4 ^{132}Ce(1) SD band

The band head spins obtained by the four parameter formula and VMI model differ broadly from the given experimental values [48] (see Table 1). This lack of agreement may be due to the occurrence of band crossing in the transitions involved in the best fit method (BFM).

This may lead the root mean deviation to show irregularities and make spin determination more difficult. The band head spins obtained by the power index formula and the nuclear softness formula are close to the experimental data. The smaller root mean deviation is obtained by the power index formula (see Table 5). From Table 5, at I_0 =20 the root mean deviation is minimum.

However, if I_0 is assumed to be 19 or 21, the root mean deviation shows large changes, so both the values have been neglected. This shows that the power index formula is the more reliable in spin determination for the ^{132}Ce(1) SD band. Even the intraband energies are reproduced very well by the power index formula (see Table 5). The band head spin of the ^{132}Ce(1) SD band obtained from the χ plot using the power index formula is shown in Fig. 1.

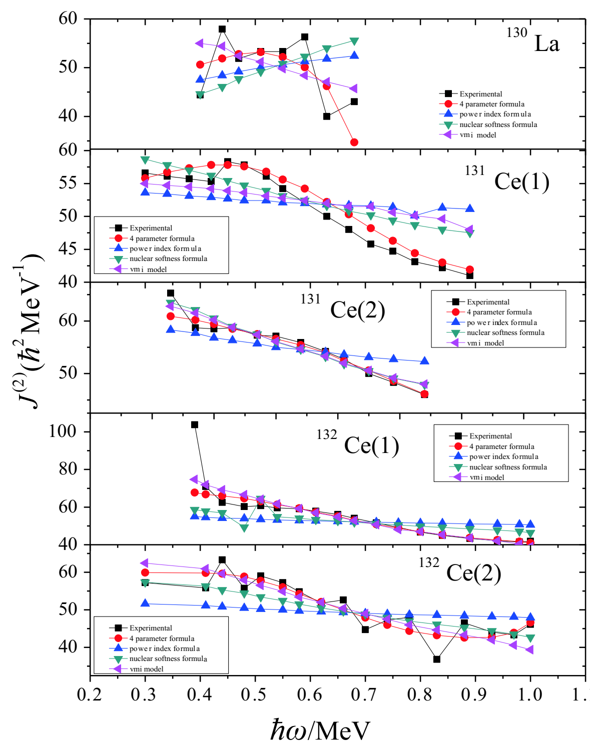

Fig. 3. (color online) Variation of calculated results of dynamic moment of inertia $J^{(2)}$ with rotational frequency for SD bands in La and Ce isotopes, and comparison with experimental data.

The values of $J^{(2)}$ for the ^{132}Ce(1) SD band from the four parameter formula, power index formula, nuclear softness formula, and VMI model are calculated using Eq. (25). A comparison between the calculated and experimental results of $J^{(2)}$ are shown in Fig. 3.

Table 3. Spin determination of ^{131}Ce(1) using four parameter formula. I_0 corresponds to band head spin. $\delta = E_\gamma^{\rm exp}(I) - E_\gamma^{\rm cal}(I)$, where E_γ is in keV. χ is the rms deviation given by Eq. (26).

$E_\gamma^{\rm exp}(I)$	I_0=13.5			I_0=14			I_0=14.5		
	I	$E_\gamma^{cal}(I)$	δ	I	$E_\gamma^{cal}(I)$	δ	I	$E_\gamma^{cal}(I)$	δ
591.5	15.5	590.6	0.9	16	592.1	-0.6	16.5	593.5	-2.0
662.1	17.5	663.3	-1.2	18	663.7	-1.6	18.5	664.0	-1.9
733.3	19.5	734.5	-1.2	20	734.2	-0.9	20.5	733.9	-0.6
805.1	21.5	804.6	0.5	22	803.9	1.2	22.5	803.2	1.9
874.9	23.5	873.9	1.0	24	873.1	1.8	24.5	872.4	2.5
944	25.5	942.8	1.2	26	942.2	1.8	26.5	941.6	2.4
1012.5	27.5	1012.0	0.5	28	1011.6	0.9	28.5	1011.4	1.1
1081.6	29.5	1082.0	-0.4	30	1082.0	-0.4	30.5	1082.1	-0.5
1152.9	31.5	1153.5	-0.6	32	1153.9	-1.0	32.5	1154.2	-1.3
1226.7	33.5	1227.2	-0.5	34	1227.7	-1.0	34.5	1228.3	-1.6
1303.4	35.5	1303.6	-0.2	36	1304.2	-0.8	36.5	1304.7	-1.3
1383.3	37.5	1383.2	0.1	38	1383.6	-0.3	38.5	1384.0	-0.7
1466.6	39.5	1466.4	0.2	40	1466.5	0.1	40.5	1466.5	0.1
1553.9	41.5	1553.1	0.8	42	1552.8	1.1	42.5	1552.5	1.4
1643.2	43.5	1643.3	-0.1	44	1642.7	0.5	44.5	1642.1	1.1
1735.8	45.5	1736.1	-0.3	46	1735.6	0.2	46.5	1735.2	0.6
1830.5	47.5	1830.4	0.1	48	1831.0	-0.5	48.5	1831.5	-1.0
χ		0.000168535			0.0000667745			0.000286815	

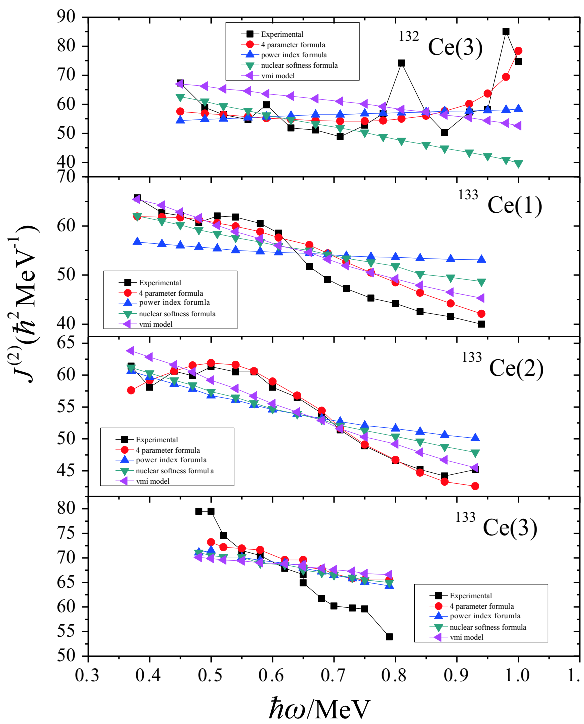

Fig. 4. (color online) Variation of calculated results of dynamic moment of inertia $J^{(2)}$ with rotational frequency for SD bands in La and Ce isotopes, and comparison with experimental data.

From Fig. 3, the calculated results of $J^{(2)}$ using the four parameter formula and the VMI model show a general trend of $J^{(2)}$ versus the rotational frequency. The consistent deviation of $J^{(2)}$ versus the rotational frequency is obtained by the power index formula and the nuclear softness formula.

It is suggested that the power index formula is an effective tool in assigning the band head spin for the ^{132}Ce(1) SD band in the $A\sim130$ mass region. The four parameter formula and the VMI model show a general trend of $J^{(2)}$ versus rotational frequency.

3.5 ^{132}Ce(2,3) SD band

The band head spins obtained by the VMI model differ broadly from the given experimental values [48] (see Table 1) for the ^{132}Ce(2) SD bands. This disagreement may be due to the significant band mixing occurring in the E2 transition used in fitting the ^{132}Ce(2) SD bands. This may lead the root mean deviation to show irregularities and make spin determination more difficult. The band head spins obtained by the four parameter formula, nuclear softness formula and power index formula are close to the experimental data. The least squares fitting of the ^{132}Ce(2) SD band obtained from the four parameter formula, and a comparison with the other three formulae, are shown in Table 6. The smallest fixed root mean deviation is obtained by the four parameter formula.

Thus, the four parameter formula is the most reliable in spin determination for the ^{132}Ce(2) SD band. The intraband energies are also reproduced very well by the four parameter formula (see Table 6). The band head spin of the ^{132}Ce(2) SD band obtained from the χ plot using the four parameter formula is shown in Fig. 1. In the ^{132}Ce(3) SD band, the band head spins obtained by the four parameter formula are close to the experimental data (see Table 1). To support our results, a χ plot using the four parameter formula is shown in Fig. 2. Intraband energies are also reproduced well (see Table 7). The values of $J^{(2)}$ for the ^{132}Ce(2,3) SD band from the four parameter formula, power index formula, nuclear softness formula and VMI model are calculated using Eq. (25). A comparison between the calculated and experimental results of $J^{(2)}$ is shown in Figs. 3 and 4. It is observed from Figs. 3 and 4 that the calculated results of $J^{(2)}$ using the four parameter formula, nuclear softness formula and the VMI model show a general trend for $J^{(2)}$ versus the rotational frequency for the ^{132}Ce(2) SD band. The experimental trend of $J^{(2)}$ versus the rotational frequency is obtained by the four parameter formula for the ^{132}Ce(3) SD band.

3.6 ^{133}Ce(1,2,3) SD band

The band head spin obtained by the four parameter formula is in close agreement with the experimentaldata [48] as compared to the other three formulae (see Table 1) for the ^{133}Ce(1,2,3) SD band. The minimum value of root mean deviation is obtained by the four parameter formula for all the SD bands of ^{133}Ce(1,2,3). Therefore,

Table 4. Spin determination of ^{131}Ce(2) using four parameter formula. I_0 corresponds to band head spin. $\delta = E_\gamma^{exp}(I) - E_\gamma^{cal}(I)$, where E_γ is in keV. χ is the rms deviation given by Eq. (26).

$E_\gamma^{\exp}(I)$	I_0=25.5			I_0=26.5			I_0=27.5		
	I	$E_\gamma^{cal}(I)$	δ	I	$E_\gamma^{cal}(I)$	δ	I	$E_\gamma^{cal}(I)$	δ
847.4	27.5	845.1	2.3	28.5	845.3	2.1	29.5	845.4	2.0
908.6	29.5	910.9	-2.3	30.5	910.9	-2.3	31.5	910.8	-2.2
976.7	31.5	977.4	-0.7	32.5	977.3	-0.6	33.5	977.2	-0.5
1045.0	33.5	1044.7	0.3	34.5	1044.6	0.4	35.5	1044.5	0.5
1113.1	35.5	1113.0	0.1	36.5	1112.9	0.2	37.5	1112.9	0.2
1182.9	37.5	1182.3	0.6	38.5	1182.4	0.5	39.5	1182.4	0.5
1252.9	39.5	1252.9	0	40.5	1253.0	0.9	41.5	1253.1	-0.2
1324.4	41.5	1325.9	-0.7	42.5	1325.2	-0.8	43.5	1325.3	-0.9
1398.1	43.5	1399.1	-1.0	44.5	1399.2	-1.1	45.5	1399.2	-1.1
1474.0	45.5	1475.4	-1.4	46.5	1475.4	-1.4	47.5	1475.4	-1.4
1553.9	47.5	1554.4	-0.5	48.5	1554.3	-0.4	49.5	1554.2	-0.3
1636.6	49.5	1636.7	-0.1	50.5	1636.6	0	51.5	1636.5	0.1
1723.5	51.5	1723.1	0.4	52.5	1723.2	0.3	53.5	1723.2	0.3
χ		0.000553068			0.000385124			0.000561882	

Table 5. Spin determination of ^{132}Ce(1) using power index formula. I_0 corresponds to band head spin. $\delta = E_\gamma^{exp}(I) - E_\gamma^{cal}(I)$, where E_γ is in keV. χ is the rms deviation given by Eq. (26).

$E_\gamma^{\exp}(I)$	I_0=19			I_0=20			I_0=21		
	I	$E_\gamma^{cal}(I)$	δ	I	$E_\gamma^{cal}(I)$	δ	I	$E_\gamma^{cal}(I)$	δ
770.8	21	694.6	76.2	22	698.0	72.8	23	701.1	69.7
809.3	23	768.4	40.9	24	770.6	38.7	25	772.7	36.6
865.7	25	842.6	23.1	26	843.9	21.8	27	845.1	20.6
929.6	27	917.1	12.5	28	917.6	12	29	918.0	11.6
995.9	29	992.0	3.9	30	991.8	4.1	31	991.6	4.3
1061.7	31	1067.2	-5.5	32	1066.5	-4.8	33	1065.8	-4.1
1128.8	33	1142.7	-13.9	34	1141.6	-12.8	35	1140.6	11.8
1196.4	35	1218.5	-22.1	36	1217.1	-20.7	37	1215.8	-19.4
1265.6	37	1294.6	-29	38	1293.0	-27.4	39	1291.6	-26
1336.8	39	1370.9	-34.1	40	1369.3	-32.5	41	1367.8	-31
1410.7	41	1447.4	-36.7	42	1445.9	-35.2	43	1444.4	-33.7
1488.1	43	1524.2	-36.1	44	1522.8	-34.7	45	1521.5	-33.4
1569.4	45	1601.2	-31.8	46	1600.1	-30.7	47	1599.1	-29.7
1654.9	47	1678.4	-23.5	48	1677.7	-22.8	49	1677.0	-22.1
1743.9	49	1755.7	-11.8	50	1755.5	-11.6	51	1755.3	-11.4
1836.1	51	1833.3	2.8	52	1833.7	2.4	53	1833.9	2.2
1931	53	1911.1	19.9	54	1912.1	18.9	55	1913.0	18
2027.2	55	1989.0	38.2	56	1990.7	36.5	57	1992.3	34.9
2122.8	57	2067.2	55.6	58	2069.7	53.1	59	2072.0	50.8
χ		0.0170720239			0.01633508494			0.02051021697	

Table 6. Spin determination of ^{132}Ce(2) using different formulae. I_0 corresponds to band head spin. $\delta = E_\gamma^{exp}(I) - E_\gamma^{cal}(I)$, where E_γ is in keV. χ is the rms deviation given by Eq. (26).

$E_\gamma^{\rm exp}(I)$	I_0=18 Power index formula			I_0=19 Four parameter formula			I_0=20 Nuclear softness formula		
	I	$E_\gamma^{cal}(I)$	δ	I	$E_\gamma^{cal}(I)$	δ	I	$E_\gamma^{cal}(I)$	δ
724.4	20	682.8	-24.9	21	728.7	-4.3	22	708.0	16.4
794.3	22	760.3	-28.4	23	795.4	-1.1	24	777.8	16.4
865.9	24	838.5	-33.5	25	862.2	3.7	26	848.8	17.1
929	26	917.1	-25.5	27	929.3	-0.3	28	921.0	8.0
1000.8	28	996.2	-22.8	29	997.3	3.5	30	994.6	6.2
1068.5	30	1075.8	-21.6	31	1066.8	1.7	32	1069.5	-1.0
1138.4	32	1155.8	4.7	33	1138.2	0.2	34	1145.7	-7.3
1211.3	34	1236.2	8.1	35	1212.0	-0.7	36	1223.3	-12.0
1288.5	36	1316.9	15.7	37	1288.7	-0.2	38	1302.4	-13.9
1364.5	38	1398.0	41.6	39	1368.6	-4.1	40	1382.9	-18.4
1453.9	40	1479.4	34	41	1452.0	1.9	42	1464.8	-10.9
1538.3	42	1561.1	27.4	43	1538.8	-0.5	44	1548.4	-10.1
1621.5	44	1643.1	11.9	45	1628.7	-7.2	46	1633.4	-11.9
1730.1	46	1725.4	4.6	47	1721.1	9	48	1720.1	10.0
1816.1	48	1808.0	-7.3	49	1814.9	1.2	50	1808.5	7.6
1906.6	50	1890.9	-17.4	51	1908.5	-1.9	52	1898.5	8.1
1998.9	52	1974.0	24.9	53	1999.6	-0.7	54	1990.2	8.7
2085.6	54	2057.4	28.2	55	2085.4	0.2	56	2083.8	1.8
χ		0.0131341589			0.000162943			0.00816426359	

Table 7. Spin determination of ^{132}Ce(3) using four parameter formula. I_0 corresponds to band head spin. $\delta = E_\gamma^{exp}(I) - E_\gamma^{cal}(I)$, where E_γ is in keV. χ is the rms deviation given by Eq. (26).

$E_\gamma^{\rm exp}(I)$	I_0=24			I_0=25			I_0=26		
	I	$E_\gamma^{cal}(I)$	δ	I	$E_\gamma^{cal}(I)$	δ	I	$E_\gamma^{cal}(I)$	δ
890.2	26	880.2	10	27	881.1	9.1	28	881.8	8.4
949.6	28	950.4	-0.8	29	950.6	-1.0	30	950.8	-1.2
1017.5	30	1021.1	-3.6	31	1020.9	-3.4	32	1020.7	-3.2
1088.4	32	1092.3	-3.9	33	1091.8	-3.4	34	1091.5	-3.1
1161.4	34	1164.0	-2.6	35	1163.5	-2.1	36	1163.1	-1.7
1228.2	36	1236.2	-8.0	37	1235.9	-7.7	38	1235.6	-7.4
1305.3	38	1309.0	-3.7	39	1308.8	-3.5	40	1308.6	-3.3
1383.5	40	1382.3	1.2	41	1382.2	1.3	42	1382.2	1.3
1465.4	42	1455.8	9.6	43	1456.0	9.4	44	1456.1	9.3
1541.1	44	1529.4	11.7	45	1529.7	11.4	46	1530.0	11.1
1611.5	46	1602.8	8.7	47	1603.2	8.3	48	1603.5	8.0
1665.4	48	1675.6	-10.3	49	1675.9	-10.5	50	1676.1	-10.7
1735.6	50	1747.1	-11.5	51	1747.3	-11.7	52	1747.4	-11.8
1815.2	52	1816.7	-1.5	53	1816.7	-1.5	54	1816.7	-1.5
1885.0	54	1883.5	1.5	55	1883.2	1.8	56	1883.0	2.0
1953.7	56	1946.3	7.4	57	1945.9	7.8	58	1945.6	8.1
2000.7	58	2003.7	-3.0	59	2003.5	-2.8	60	2003.3	-2.6
2054.2	60	2054.1	0.1	61	2054.5	-0.3	62	2054.7	-0.5
χ		0.000388346			0.00032446879			0.00047007	

Table 8. Spin determination of ^{133}Ce(1) using four parameter formula. I_0 corresponds to band head spin. $\delta = E_\gamma^{exp}(I) - E_\gamma^{cal}(I)$, where E_γ is in keV. χ is the rms deviation given by Eq. (26).

$E_\gamma^{exp}(I)$	I_0=20.5			I_0=21			I_0=21.5		
	I	$E_\gamma^{cal}(I)$	δ	I	$E_\gamma^{cal}(I)$	δ	I	$E_\gamma^{cal}(I)$	δ
748.3	22.5	743.7	4.6	23	744.2	4.1	23.5	744.8	3.5
809.1	24.5	808.7	0.4	25	808.8	0.3	25.5	808.9	0.2
872.8	26.5	873.6	-0.8	27	873.5	-0.7	27.5	873.3	-0.5
937.2	28.5	938.6	-1.4	29	938.3	-1.1	29.5	938.0	-0.8
1003.0	30.5	1003.8	-0.8	31	1003.5	-0.5	31.5	1003.2	-0.2
1067.5	32.5	1069.6	-2.1	33	1069.4	-1.9	33.5	1069.2	-1.7
1132.2	34.5	1136.2	-4.0	35	1136.1	-3.9	35.5	1136.0	-3.8
1198.3	36.5	1204.0	-5.7	37	1204.1	-5.8	37.5	1204.1	-5.8
1266.6	38.5	1273.3	-6.7	39	1273.5	-6.9	39.5	1273.6	-7.0
1377.4	40.5	1344.6	32.8	41	1344.8	32.6	41.5	1345.0	32.4
1411.3	42.5	1418.0	-6.7	43	1418.3	-7.0	43.5	1418.4	-7.1
1488.6	44.5	1494.2	-5.6	45	1494.3	-5.7	45.5	1494.5	-5.9
1570	46.5	1573.4	-3.4	47	1573.4	-3.4	47.5	1573.4	-3.4
1654.7	48.5	1656.0	-1.3	49	1655.8	-1.1	49.5	1655.7	-1.0
1743	50.5	1742.3	0.7	51	1742.0	1.0	51.5	1741.8	1.2
1833.4	52.5	1832.6	0.8	53	1832.4	1.0	53.5	1832.3	1.1
1927.5	54.5	1927.1	0.4	55	1927.3	0.2	55.5	1927.5	0
χ		0.000311629			0.000275173			0.00106941	

Table 9. Spin determination of ^{133}Ce(2) using four parameter formula. I_0 corresponds to band head spin. $\delta = E_\gamma^{exp}(I) - E_\gamma^{cal}(I)$, where E_γ is in keV. χ is the rms deviation given by Eq. (26).

$E_\gamma^{\rm exp}(I)$	I_0=15.5			I_0=16.5			I_0=17.5		
	I	$E_\gamma^{cal}(I)$	δ	I	$E_\gamma^{cal}(I)$	δ	I	$E_\gamma^{cal}(I)$	δ
720.3	17.5	716.0	4.3	18.5	718.7	1.6	19.5	720.8	-0.5
785.4	19.5	787.5	-2.1	20.5	788.1	-2.7	21.5	788.6	-3.2
854.4	21.5	856.3	-2.1	22.5	855.6	-1.4	23.5	855.0	-0.8
920.2	23.5	923.0	-2.8	24.5	921.6	-1.4	25.5	920.5	-0.3
986.9	25.5	988.0	-1.1	26.5	986.6	0.3	27.5	985.4	1.5
1052.1	27.5	1052.3	-0.2	28.5	1051.2	0.9	29.5	1050.3	1.8
1118.2	29.5	1116.6	1.6	30.5	1116.1	2.1	31.5	1115.6	2.6
1184.3	31.5	1181.9	2.4	32.5	1182.0	2.3	33.5	1182.1	2.2
1253.1	33.5	1249.0	4.1	34.5	1249.7	3.4	35.5	1250.3	2.8
1323.8	35.5	1319.0	4.8	36.5	1320.1	3.7	37.5	1320.9	2.9
1397.8	37.5	1392.5	5.3	38.5	1393.6	4.2	39.5	1394.5	3.3
1475.5	39.5	1470.2	5.3	40.5	1470.9	4.6	41.5	1471.5	4
1557.2	41.5	1552.2	5.0	42.5	1552.3	4.9	43.5	1552.4	4.8
1642.9	43.5	1638.5	4.4	44.5	1637.9	5.0	45.5	1637.3	5.6
1731.3	45.5	1728.4	2.9	46.5	1724.2	7.1	47.5	1726.2	4.8
1821.6	47.5	1820.3	1.3	48.5	1819.5	2.1	49.5	1818.8	2.8
1910	49.5	1912.2	-2.2	50.5	1913.3	-3.3	51.5	1914.2	-4.2
χ		0.0170720239			0.01633508494			0.02051021697	

Table 10. Spin determination of ^{133}Ce(3) using four parameter formula. I_0 corresponds to band head spin. $\delta = E_\gamma^{exp}(I) - E_\gamma^{cal}(I)$, where E_γ is in keV. χ is the rms deviation given by Eq. (26).

$E_\gamma^{\rm exp}(I)$	I_0=21			I_0=21.5			I_0=22		
	I	$E_\gamma^{cal}(I)$	δ	I	$E_\gamma^{cal}(I)$	δ	I	$E_\gamma^{cal}(I)$	δ
956.9	23	926.7	30.2	23.5	927.4	29.5	24	927.9	29
985.9	25	987.1	-1.2	25.5	987.2	-1.3	26	987.2	-1.3
1020.5	27	1044.8	-24.3	27.5	1044.6	-24.1	28	1044.4	-23.9
1082.1	29	1100.5	-18.4	29.5	1100.2	-18.1	30	1099.9	-17.8
1148.5	31	1155.1	-6.6	31.5	1154.8	-6.3	32	1154.5	-6.0
1207.6	33	1209.2	-1.6	33.5	1209.1	-1.5	34	1209.0	-1.4
1274.6	35	1263.8	10.9	35.5	1263.9	10.8	36	1263.9	10.8
1334.7	37	1319.5	15.2	37.5	1319.7	15	38	1319.8	14.9
1388.3	39	1376.9	11.4	39.5	1377.1	11.2	40	1377.3	11.0
1438.6	41	1436.0	2.6	41.5	1436.1	2.5	42	1436.3	2.3
1488.9	43	1496.7	-7.8	43.5	1496.8	-7.9	44	1496.8	-7.9
1547.8	45	1558.3	-10.5	45.5	1558.2	-10.4	46	1558.1	-10.3
1612.6	47	1619.4	-6.8	47.5	1619.2	-6.6	48	1619.1	-6.5
1686.7	49	1677.7	9.0	49.5	1677.8	8.9	50	1677.8	8.9
χ		0.00040365			0.000297973			0.000314899	

the four parameter formula is reliable in assigning the spins for the ^{133}Ce(1,2,3) SD band. The band head spin of the ^{133}Ce(1,2,3) SD band, obtained from the χ plot using the four parameter formula, is shown in Fig. 2.

The intraband energies of the ^{133}Ce(1,2,3) SD band are reproduced well by the four parameter formula (see Tables 8, 9, 10). The values of $J^{(2)}$ for the ^{133}Ce(1,2,3) SD band from the four parameter formula, power index formula, nuclear softness formula and VMI model are calculated using Eq. (25). A comparison between the calculated and experimental results of $J^{(2)}$ is shown in Fig. 4.

It is observed from Fig. 4 that the calculated results of $J^{(2)}$ using the four parameter formula show a general trend for $J^{(2)}$ versus the rotational frequency and agree well with the experimental data for the ^{133}Ce(2,3) SD band. In the ^{133}Ce(1) SD band, the four parameter formula, nuclear softness formula and VMI model show a general trend for $J^{(2)}$ versus the rotational frequency. The power index formula shows a constant behaviour.

4 Conclusion

In this work, we have analyzed the ^{130}La, ^{131}Ce(1,2), ^{132}Ce(1,2,3), and ^{133}Ce(1,2,3) SD bands of the $A \sim 130$ mass region with the four parameter formula, power index formula, nuclear softness formula and VMI model. The formula which gives a minimum root mean deviation value at a particular band head spin is considered best for studying that SD band in La-Ce isotopes in the $A \sim 130$ mass region. The four parameter formula has been used to study ^{130}La, ^{131}Ce(1,2), ^{132}Ce(2, 3) and ^{133}Ce(1,2,3) SD bands in the $A \sim 130$ mass region in assigning the band head spin and in reproducing the intraband energies. The general trend of $J^{(2)}$ versus the rotational frequency in ^{130}La, ^{131}Ce(1), ^{132}Ce(3), and ^{133}Ce(2, 3) SD bands also proves to be effective with the four parameter formula. In the ^{131}Ce(2), ^{132}Ce(2), and ^{133}Ce(1) SD bands, the four parameter, nuclear softness formula and VMI model justify the effectiveness of $J^{(2)}$. A constant behaviour is shown by the power index formula for the ^{133}Ce(1) SD bands. The power index formula works better for ^{132}Ce(1) in the $A \sim 130$ mass region in assigning the band head spin and in reproducing the intraband energies. To obtain the general trend of $J^{(2)}$ versus the rotational frequency in the ^{132}Ce(1) SD band, the four parameter formula and the VMI model prove to be effective whereas the power index formula and the nuclear softness formula show a constant behaviour of $J^{(2)}$.

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