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Study of entropy in intermediate-energy heavy ion collisions

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Abstract: Using the isospin-dependent quantum molecular dynamics model, the entropy of an intermediate-energy heavy ion collision system after the reaction and the number of deuteronlike and protonlike particles produced in the collision is calculated. In the collision, different parameters are used and the mass number used here is from 40 to 93 at incident energy from 150 MeV to 1050 MeV. We build a new model in which the density distribution of the reaction product is used to calculate the size of the entropy. The entropy calculated with this model is in good agreement with experimental values. Our data reveals that with the increase of the neutron-proton ratio and impact parameter, the entropy of the reaction system decreases, and it increases with the increase of system mass and reaction energy.

Keywords: entropy, neutron-proton ratio, isospin effect, intermediate-energy heavy ion reaction

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1 Introduction

The isospin effect in intermediate-energy heavy ion collisions is an important current research direction of nuclear physics. Many people have studied the equation of state of isospin asymmetric nuclear matter [1–3]. The aim of research into the isospin effect in intermediate-energy heavy ion collisions is to extract information about the isospin-dependent mean field and isospin-dependent cross section, which is important for studying nuclear physics and astrophysics, such as supernova explosions and the cooling rate of neutron stars. In recent years, it has been found that isospin fractionation is a sensitive probe to measure the symmetric potential [4], and nuclear stopping and the number of emitted nucleons is a sensitive probe of nucleon-nucleon cross sections[5].

To study heavy ion reactions experimentally, it is necessary to use a heavy ion accelerator, the cost of which is very high, so we usually use computer simulations of nuclear reactions. Ma Yu-Gang et al. studied the main reaction of viscosity and entropy density ratio by using the Boltzmann-Uehling-Uhlenbeck (BUU) model, from which they found that the ratio decreases with the increase of incident energy [6]. Cai Xiang-Zhou et al. calculated the cross-section of alpha particles in heavy ion reactions by taking into account the Fermi momentum, mean field, nuclear-nuclear function and the Pauli blocking effect in the BUU model [7]. Apart from the BUU model, the quantum molecular dynamics (QMD) model can also be used to simulate the transport of heavy ion reactions, simulating a response to each event and regarding the nucleus as a Gaussian wave packet instead of charged particles, allowing the density and intermediate mass fragments to be calculated after the reaction. In recent years, much research has been done on heavy ion reactions with the QMD model. Zhang Ying-Xun et al. studied the isotopic distribution, neutron-proton ratio and tritium/helium-3 ratio of heavy ion reactions by using an improved QMD model [8], and studied isospin diffusion, symmetry potential, split nuclear effective mass and momentum-dependent effects by using a variety of Skyrme parameters [9]. Feng Zhao-Qing et al. studied antiprotons with heavy ion reactions, the reaction channels of elastic scattering, annihilation, charge exchange, and inelastic collisions have been included in the model. With these methods, they also investigated pions, kaons, antikaons, the production of heavy ions [10], protons and nuclear reactions [11]. Considering the Skyrme energy density function, Li Qing-Feng et al. studied the ^{197}Au + ^{197}Au spin orbit coupling effect by using the ultra relativistic quantum molecular dynamics (UrQMD) model in heavy ion collisions [12]. Zhang Feng-Shou et al. also studied the collisions of ^{197}Au + ^{197}Au [13]. Ma Yu-Gang et al. studied collective flow, extrusion flow and fluctuation of 1A GeV Au + Au reaction [14]. By using an isospin-dependent QMD model, Guo Wen-Jun et al. studied the reaction cross section of ^{12}C and the isotopes of Al [15], the nuclear stopping of the reactions of excited ^{56}Ni nuclei [16], the photons produced by neutron-proton bremsstrahlung and its relationship with the symmetric

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potential in heavy ion collisions [17], the number of emitted nucleons in photonuclear reactions and its relationship with the cross section[18].

In recent years, there has been much research on entropy in heavy ion reactions experimentally [19–21] and theoretically [22–24]. R. K. Puri et al found that the entropy in heavy ion reactions can be calculated by using the ratio of deuteronlike to protonlike particles [23–24], the results of which conform to the experimental values by Doss et al [19]. They also found the entropy will decrease with the increase of neutron-proton ratio of the reaction system. In this paper, we simulate intermediate-energy heavy ion reactions by using the isospin-dependent quantum molecular dynamics (IQMD) model, and when the reaction reaches equilibrium, different fragments (protonlike and deuteronlike) can be calculated according to the density distribution of nucleons. We then study and discuss the entropy of the heavy ion reactions by using the formulas in the literature [19, 23, 24], by which we try to find out the relationship between entropy and reaction energy, the mass of the reaction system, neutron-proton ratio, impact parameters and so on.

2 Model

2.1 Calculation of entropy in heavy ion reactions

Siemens and Kapusta [22] proposed a formula for computing the entropy of nuclear reactions:

$$S_{\mathrm{N}} = 3.945 - \ln R_{\mathrm{dp}} \tag{1}$$

where R_{dp} is the ratio of deuterium in the reaction product with free protons, which can also be expressed by the ratio of the deuteronlike to protonlike particles in the reaction product. We regard the light fragments which can be detected and are close to the nature of free protons as protonlike, and consider the fragments whose nature is close to deuterium as deuteronlike. There are many forms to express protonlike and deutronlike characteristics [23, 25–27], considering the variety of lighter fragments. We can calculate the protonlike and the deuteronlike particles according to the number of these fragments. In formula (1), the deuteronlike and protonlike particles are expressed in a polynomial which has many parameters, and only considers light fragments whose mass number is less than 4. Besides the fragments they considered, the question arises whether other fragments have an effect on the entropy of the nuclear reaction or not. In order to consider all possible light fragments and intermediate mass fragments which influence the entropy, we use different densities to express the protonlike and deutronlike particles:

$$\begin{aligned} p_{\mathrm{like}}: &\quad \rho < 0.02\ \mathrm{fm}^{-3} = \frac{1}{8}\rho_0 \\ d_{\mathrm{like}}: &\quad 0.02\ \mathrm{fm}^{-3} \leqslant \rho < \rho_1 \end{aligned} \tag{2}$$

where ρ_0=0.16 fm^{-3} is the saturation density of the ground state nucleus. In Ref. [28], a nuclear density ρ not greater than $\rho_0/8$ is considered to be a free nucleon, so in this paper, a proton density less than 0.02 fm^{-3} ($\rho_0/8$) is considered to be protonlike. We consider such fragments whose density is greater than 0.02 fm^{-3} but not more than $\rho_1$as deuteronlike. In order to conform to experimental values [19], we find that the calculation results, after much calculation, are nearest to the experimental values when the parameter ρ_1 is equal to 0.07 fm^{-3}. After the weighted average of the impact parameters, the average of the protonlike and the deuteronlike particles can be calculated, then the entropy of the nuclear reaction can also be calculated according to the formula above.

2.2 Isospin-dependent quantum molecular dynamics model

Heavy ion reactions can be simulated by using the QMD model. Considering the influence of isospin effects in the process of heavy ion reactions, the QMD model needs to be modified appropriately. The calculation of density-dependent mean field needs to consider the difference between protons and neutrons (Coulomb potential and symmetric potential), and the calculation of nucleon-nucleon reaction cross section and Pauli blocking should also distinguish the protons and neutrons. Through these changes, the IQMD model was obtained, whose average potential energy expression is

$$U = U^{\mathrm{Sky}} + U^{\mathrm{Yuk}} + U^{\mathrm{Coul}} + U^{\mathrm{MDI}} + U^{\mathrm{Pauli}} + U^{\mathrm{Sym}} \tag{3}$$

where U^{Sky} is density-dependent Skyrme potential, U^{Yuk} is the surface potential, U^{Coul} is the Coulomb potential, U^{MDI} is momentum-dependent potential, U^{Pauli} is the Pauli potential, and U^{Sym} is the symmetry potential.

In the calculation of this paper, two main kinds of isospin effect are considered: the isospin-dependent nucleon-nucleon cross section and the density-dependent symmetry energy, where the symmetry energy (potential) is

$$U^{\mathrm{Sym}} = 32\frac{\rho_{\mathrm{n}} - \rho_{\mathrm{p}}}{\rho_0}\tau_{\mathrm{z}}, \tag{4}$$

where $\tau_{\mathrm{z}} = \pm 1$ is the isospin quantum number of nucleons, and ρ_{n}, ρ_{p} are the density of neutron and proton respectively. In order to calculate the nucleon-nucleon cross section, we use the piecewise function formula of cross section in the process of the nucleus collisions [29]. Considering the effect of a medium on the cross section, the cross section in free space is multiplied by a coefficient which is a function of the density of nuclear matter.

According to the results of the collective flow phenomenon of intermediate-energy heavy ion reactions [30] in recent years, $\gamma=-0.2$. As you can see, when the energy per nucleon is no more than 400 MeV, the n-p section is about three times the n-n (p-p) section. If the reaction energy is very high, however, the two cross-sections are close in size, which leads to the isospin effect not being obvious. Because γ is negative, the medium effect reduces the nucleon-nucleon cross section.

2.3 Density calculation

In order to find the protonlike and deuteronlike particles, it is necessary to calculate each nuclear density which is determined by the distance between nuclei after the nuclear reaction. The farther away the other nuclei are from one nucleus, the lower its density is. In the IQMD model, if there were A nuclei in the reaction system, one can sum to the density of A nuclei for calculating the density near to the i^{th} nucleon. The nucleon density is

$$\rho(r_i)=\sum_{j=1}^{A}\frac{1}{(2\pi L)^{3/2}}\exp\left[-\frac{(r_i-r_j)^2}{2L}\right], \tag{5}$$

where L=3.8 fm is the width of the Gaussian wave packet.

3 Results and discussion

We calculated the reaction with 400 MeV and 1050 MeV per nucleon reaction energy in the ^{40}Ca + ^{40}Ca system, and 400 MeV and 650 MeV per nucleon reaction energy in the ^{93}Nb + ^{93}Nb system. The number of protonlike and deuteronlike particles was then calculated according to the density of the nucleon after the balance of the reaction. After the weighted mean of the collision parameters, their entropy can be calculated according to Equation (1), as shown in Fig. 1.

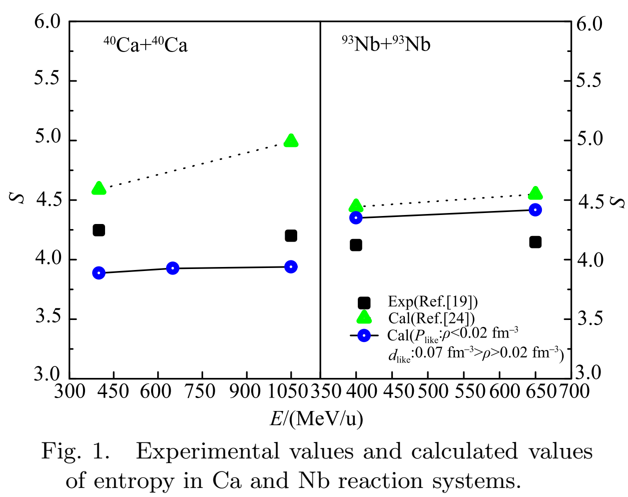

Fig. 1. Experimental values and calculated values of entropy in Ca and Nb reaction systems.

Figure 1 shows the entropy of the collision with 400 MeV and 650 MeV per nucleon energy in ^{40}Ca+^{40}Ca system and 1050 MeV per nucleon energy in the ^{93}Nb+^{93}Nb system. The solid square points are experimental values, the dashed line shows the results of the calculation of Vermani and Kaur et al and the solid line shows the results of our calculation. As you can see in the figure, for both the ^{40}Ca + ^{40}Ca and ^{93}Nb + ^{93}Nb systems, the calculated results are near to the experimental values.

Figure 2 shows the entropy of the collisions of nuclei such as ^{93}Rb, ^{93}Y, ^{93}Nb, ^{93}Tc, and ^{93}Rh with 400 MeV and 650 MeV per nucleon energy. We find that more entropy is produced when the per nucleon reaction energy is 650 MeV than when it is 400 MeV, due to the high energy making the nucleus more fragile, producing more light fragments and protonlike particles, and increasing the temperature of the nucleus and degree of disorder. We also find that the larger the neutron-proton ratio of the reaction system is, the less entropy can be produced, because the collision of a neutron-rich reaction system will produce more free neutrons and less free protons and reduce the number of protonlike particles, but the impact on the deuteronlike particles is not very strong, so neutron-rich reaction systems have a smaller entropy than neutron-deficient reaction systems. This is close to the conclusion calculated by Kaur et al [24]. In our calculation, due to the nuclides having the same mass, we can rule out the impact produced by the mass of the reaction system. The neutron-proton ratio in Fig. 2 ranges from 1.06 to 1.51, because the response system that we use exists in the nuclide tables and is not far from the β stable line, so the range of neutron-proton ratios we give is not very big.

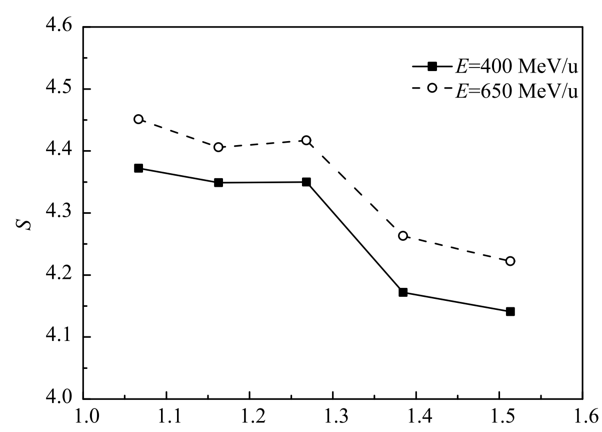

Fig. 2. Entropy in the reaction systems of different neutron-proton ratios with A=93.

Figure 3 shows the entropy of the collision in reaction systems with different masses (^{40}Ca + ^{40}Ca, ^{60}Ni + ^{60}Ni,

^{82}Kr + ^{82}Kr and ^{93}Nb + ^{93}Nb) with 400 MeV and 650 MeV per nucleon energy, in which we find that reaction systems with a high reaction energy produce more entropy, similar to Fig. 2. When we study the relationship of entropy with the change of the mass of the reaction system, in order to eliminate the effects of the freedom of the isospin degree, we choose some relatively stable nuclides in the nuclide chart which exist in nature. The reason why the entropy of the reaction increases with the increase of the mass of the reaction system is that the heavier nuclei have larger volumes and more nucleons, so under the condition of invariable nuclear energy, the heavier nucleus has a corresponding larger overall reaction energy and diameter. In the process of reaction, the average collision number per nucleon is larger with the increase of the diameter of the nucleus, so a heavier nucleus under the condition of same reaction energy per nucleon will produce more light fragments and proton-like particles after the collision, so reaction products have higher disorder which increases the reaction entropy.

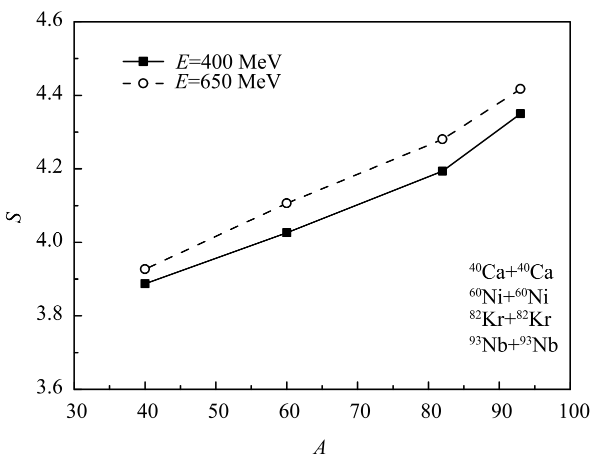

Fig. 3. Entropy in reaction systems with different masses.

Figure 4 shows the entropy produced in collisions in the ^{93}Nb + ^{93}Nb system with different reaction energies. It is obvious that the entropy increases with the reaction energy: when the reaction energy is higher, larger entropy be produced, but the increase amplitude of the entropy becomes smaller because the nucleus will be more broken and there will be more light fragments such as free protons when the low reaction energy (150 MeV) increases to secondary energy (400 MeV), which increases the entropy. At high reaction energy (650 MeV), the nucleus is almost broken and reaction products are close to the biggest disordered state. If we go on increasing the reaction energy (900 MeV), the entropy will become larger, but the extent of change is very small, so there will be saturation at high energy, which means the entropy will not increase indefinitely.

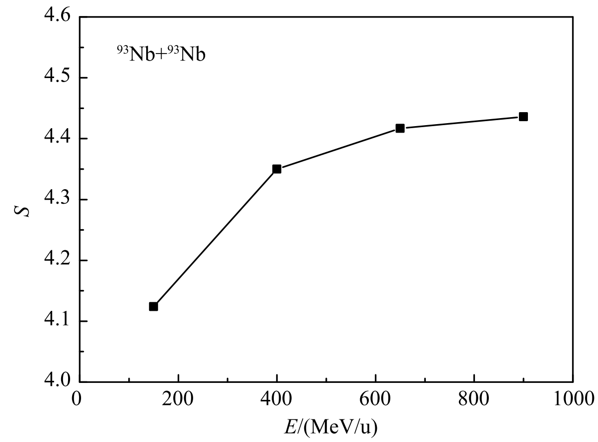

Fig. 4. Entropy of different reaction energies in Nb-Nb system.

Figure 5 shows the entropy in ^{40}Ca+^{40}Ca and ^{93}Nb+^{93}Nb system reactions with 400 MeV per nucleon energy as a function of change of collision parameters. Because the nuclear diameter of ^{93}Nb is bigger than ^{40}Ca, we also calculated the collision of ^{93}Nb with b=10 fm. It can be seen that whether the system is lighter or heavier, the entropy is very high in a central collision, because 400 MeV per nucleon reaction energy is greater than 8 MeV nucleus binding energy, so the nucleus is broken in a central collision and a large number of light fragments and a small amount of intermediate mass fragments are produced. When the impact parameter is larger, however, the light fragments such as free protons are fewer because only a small number of nucleons are involved in collisions, so the entropy in heavy ion reactions decreases with the increase of the impact parameter.

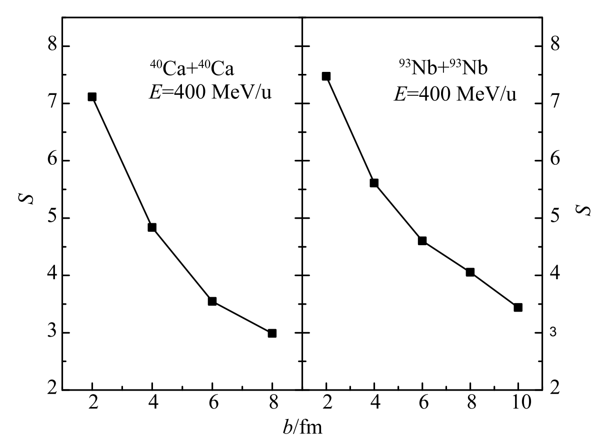

Fig. 5. Variation of entropy in Ca and Nb systems with change of impact parameters.

4 Conclusions

The light fragments produced in heavy ion collisions are studied with the energy per nucleon ranging from 150 MeV to 1050 MeV in Ca, Ni, Kr, and Nb systems using the IQMD model, through which a new model is established for computing entropy. The protonlike and deuteronlike particles can be distinguished according to the density of the reaction products, and the calculation of entropy by this method conforms to the experimental results. We find that the entropy decreases with the increase of the neutron-proton ratio in the reaction system of different nuclides with mass number 93. Through the nuclide collisions of reaction systems of different masses near the β stable line, we find that the entropy increases with the increase of mass of the reaction system. In the reaction system of ^{93}Nb + ^{93}Nb, the entropy increases with the increase of reaction energy, but saturation occurs at large energy, so entropy will not increase indefinitely with energy. The entropy is smaller when the impact parameter is larger.

References

1 I. Vidaña and I. Bombaci, Phys. Rev. C, **66**: 045801 (2002)
2 D. V. Shetty, S. J. Yennello, and G. A. Souliotis, Phys. Rev. C, **75**: 034602 (2007)
3 B. A. Li, C. M. Ko, and Z. Z. Ren, Phys. Rev. Lett., **78**: 1644 (1997)
4 J. Y. Liu, W. J. Guo et al, Phys. Rev. C, **70**: 034610 (2004)
5 J. Y. Liu, W. J. Guo et al, Phys. Rev. Lett., **86**: 975 (2001)
6 S. X. Li, D. Q. Fang, Y. G. Ma et al, Phys. Rev. C, **84**: 024607 (2011)
7 C. Zhong, X. Z. Cai, W. Q. Shen et al, Chin. Phys. Lett., **20**: 629 (2003)
8 Y. X. Zhang, Z. X. Li, and C. S. Zhou, Phys. Rev. C, **85**(5): 051602
9 Y. X. Zhang, M. B. Tsang, and Z. X. Li et al, Phys. Lett. B, **732**: 186–190
10 Z. Q. Feng and H. Lenske, Phys. Rev. C, **89**(4): 04461
11 Z. Q. Feng, W. J. Xie, and G. M. Jin, Phys. Rev. C, **90**(6): 064604
12 C. C. Guo, Y. J. Wang, Q. F. Li et al, Phys. Rev. C, **90**(3): 034606
13 W. J. Xie and F. S. Zhang, Phys. Lett. B, **735**: 250–255
14 J. Wang, Y. G. Ma, G. Q. Zhang et al, Phys. Rev. C, **90**(5): 054601
15 L. M. Yang, W. J. Guo, F. Zhang et al, Chin. Phys. C, **37**(10): 104101
16 A. Zhang, W. J. Guo, J. W. Huang et al, Chin. Phys. C, **38**(5): 054103
17 L. M. Yang, W. J. Guo, Y. P. Zhang et al, Chin. Phys. C, **38**(7): 074105
18 W. J. Guo, J. W. Huang, G. C. Yong et al, Chin. Phys. Lett., **31**(10): 102501
19 K. G. R. Doss, et al, Phys. Rev. C, **32**: 116 (1985)
20 K. G. R. Doss et al, Phys. Rev. C, **37**: 163 (1988)
21 Y. G. Ma, Phys. Rev. Lett., **83**: 3617–3620 (1999)
22 P. J. Siemens and J. I. Kapusta, Phys. Rev. Lett., **43**: 1486 (1979)
23 Y. K. Vermani and R. K. Puri, Nucl. Phys. A, **847**: 243–252 (2010)
24 S. Kaur and R. K. Puri, Phys. Rev. C, **89**: 057603 (2014)
25 G. Bertsch and J. Cognon, Phys. Rev. C, **24**(6): 2514 (1981)
26 G. Bertsch, Nucl. Phys. A, **400**: 221 (1983)
27 G. Peilert et al. Phys. Rev. C, **39**:1402 (1989)
28 B. A. Li, C. M. Ko, and W. Bauer, Int. J. Mod. Phys. E, **7**: 147–230 (1998)
29 K. Chen, Z. Fraenkel, Friedlander D et al, Phys. Rev., **166**: 949 (1968)
30 D. Klakow, G. Welke, and W. Bauer, Phys. Rev. C, **48**: 1982 (1993)