# Radii of the bound states in ${ }^{16} \mathrm{~N}$ from the asymptotic normalization coefficients ${ }^{*}$ 

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#### Abstract

The asymptotic normalization coefficients（ANCs）of the virtual decay ${ }^{16} \mathrm{~N} \rightarrow{ }^{15} \mathrm{~N}+\mathrm{n}$ are extracted from the ${ }^{15} \mathrm{~N}\left({ }^{7} \mathrm{Li},{ }^{6} \mathrm{Li}\right){ }^{16} \mathrm{~N}$ reaction populating the ground and first three excited states in ${ }^{16} \mathrm{~N}$ ．The root－mean－square （rms）radii of the valence neutron in these four low－lying ${ }^{16} \mathrm{~N}$ states are then derived by using the ANCs．The probabilities of the valence neutron staying out of the core potentials are found to be $31 \% \pm 8 \%, 58 \% \pm 12 \%, 32 \% \pm$ $8 \%$ ，and $60 \% \pm 12 \%$ ．The present results support the conclusion that a one－neutron halo may be formed in the ${ }^{16} \mathrm{~N}$ first and third excited states，while the ground and second excited states do not have a one－neutron halo structure． However，the core excitation effect has a strong influence on the one－neutron halo structure of the ground and first excited states in ${ }^{16} \mathrm{~N}$ ．


Keywords：neutron halo nuclei，matter radius，asymptotic normalization coefficients（ANCs）
PACS：21．10．Dr，21．10．Gv，27．20．＋n DOI：10．1088／1674－1137／40／11／114104

## 1 Introduction

Considerable efforts have been made experimentally and theoretically to investigate the halo nuclei，which have a tightly bound core surrounded by a diffuse nu－ clear cloud，since their discovery by Tanihata et al $[1,2]$ in 1985．Over the past 30 years，a number of ground state nuclei close to the drip－line have been found to have a halo structure，including the one－neutron halo nuclei ${ }^{11} \mathrm{Be}[3]$ and ${ }^{19} \mathrm{C}$［4］，the two－neutron halo nuclei ${ }^{6} \mathrm{He}[1,2],{ }^{11} \mathrm{Li}[1,2,5],{ }^{14} \mathrm{Be}[6,7]$ and ${ }^{17} \mathrm{~B}[6]$ ，the four－ neutron halo nucleus ${ }^{8} \mathrm{He}[1,2,8]$ ，and the proton halo
nuclei ${ }^{8} \mathrm{~B}[9-11],{ }^{17} \mathrm{Ne}[12],{ }^{20} \mathrm{Mg}[13]$ and ${ }^{26,27,28} \mathrm{P}$［14］． Otsuka et al．$[15,16]$ extended the universality of the halo phenomenon and pointed out that the neutron halo should not be limited to exotic nuclei，and can be ob－ served in excited states of a large number of nuclei on and off the $\beta$ stability line．Later，a number of such nu－ clei were proved to have a halo structure，such as the second excited state in ${ }^{6} \mathrm{Li}$［17］，the first excited state in ${ }^{11} \mathrm{Be}[18]$ ，the second and third excited states in ${ }^{12} \mathrm{~B}$ ［19，20］，and the first excited state in ${ }^{13} \mathrm{C}$［19］．

A neutron halo is basically due to a boundary ef－ fect resulting from the presence of a bound state close to

[^0]the continuum. The valence neutron can tunnel into the space surrounding the nuclear core so that the neutron is present with appreciable probability at distances much larger than the normal nuclear radius [21-23]. Thus, the angular momentum of the valence neutron may have an influence on the halo formation and the $s$-state may have a large probability of being outside the range of the core's potential, while the $p, d$-states have much smaller probabilities to form halo structures because of the large centrifugal barriers.

The ${ }^{16} \mathrm{~N}$ nucleus has a valence neutron beyond the $N=8$ main shell closure. It has four bound states. The valence neutron outside the ${ }^{15} \mathrm{~N}$ core occupies the $1 d_{5 / 2}$ orbit for the ground and second excited states, and the $2 s_{1 / 2}$ orbit for the first and third excited states. The nonlinear relativistic mean-field calculation shows that there is no one-neutron halo in the ground state of ${ }^{16} \mathrm{~N}$ because the centrifugal barriers hinder halo formation, but there exists a one-neutron halo in the first excited state [24]. To date, there is no information about the halo structures of the second and third excited states in ${ }^{16} \mathrm{~N}$.

Many experimental methods have been used for investigating nuclear halo structures, including measurement of the interaction cross section [1, 25], reaction cross section $[26,27]$, proton-nucleus elastic scattering [28, 29], quasielastic scattering [30], electric quadrupole moment [31], nuclear breakup [27, 32], and Coulomb dissociation [33-35]. However, it is difficult to use any of these methods to search for halo nuclei in excited states, due to their very short half-lives. Fortunately, an indirect method, called the asymptotic normalization coefficient (ANC) approach, has been proposed and successfully used to obtain the halo structure information of ${ }^{8} \mathrm{~B}$ $[36,37],{ }^{9} \mathrm{C}[38]$ ground states, and ${ }^{11} \mathrm{Be}[18],{ }^{12} \mathrm{~B}[19]$, ${ }^{13} \mathrm{C}[18,19],{ }^{27} \mathrm{P}[39]$ excited states.

In the present work, the root-mean-square (rms) radii of valence neutrons in the ${ }^{16} \mathrm{~N}$ ground and first three excited states are deduced by using the ANCs for the virtual decay ${ }^{16} \mathrm{~N} \rightarrow{ }^{15} \mathrm{~N}+\mathrm{n}$. The probabilities of the valence neutrons staying out of the core potential are obtained. The present results conclude that the first and third excited states in ${ }^{16} \mathrm{~N}$ may have a one-neutron halo structure, while the ground and second excited states in ${ }^{16} \mathrm{~N}$ cannot form a one-neutron halo. However, the core excitation effect has a big contribution to the ground and first excited states of ${ }^{16} \mathrm{~N}$, and the effect can be ignored in the second and third excited states.

## 2 ANC method

In the ANC approach, the rms radius of the valence neutron for the virtual decay $\mathrm{B} \rightarrow \mathrm{A}+\mathrm{n}$ is defined by

$$
<r^{2}>^{1 / 2}=\left(\int_{0}^{\infty} r^{4}\left[I_{\mathrm{An} l_{\mathrm{B}} j_{\mathrm{B}}}^{\mathrm{B}}(r)\right]^{2} \mathrm{~d} r\right)^{1 / 2}
$$

$$
\begin{equation*}
/\left(\int_{0}^{\infty} r^{2}\left[I_{\mathrm{An} l_{\mathrm{B}} j_{\mathrm{B}}}^{\mathrm{B}}(r)\right]^{2} \mathrm{~d} r\right)^{1 / 2} \tag{1}
\end{equation*}
$$

where $I_{\mathrm{An} l_{\mathrm{B}} j_{\mathrm{B}}}^{\mathrm{B}}(r)$ is the overlap function for $\mathrm{B} \rightarrow \mathrm{A}+\mathrm{n}$, corresponding to the orbital angular momentum $l_{\mathrm{B}}$ and channel spin $j_{\mathrm{B}}$.

The spectroscopic factor for $\mathrm{B} \rightarrow \mathrm{A}+\mathrm{n}$ is the probability of the wave function of the nucleus B being composed of the wave functions of the nucleus A and valence neutron [40], which can be studied by the square norm of the overlap function

$$
\begin{equation*}
S_{\mathrm{An} l_{\mathrm{B}} j_{\mathrm{B}}}^{\mathrm{B}}=\int_{0}^{\infty} r^{2}\left[I_{\mathrm{An}_{\mathrm{B}} j_{\mathrm{B}}}^{\mathrm{B}}(r)\right]^{2} \mathrm{~d} r . \tag{2}
\end{equation*}
$$

Then, Eq. (1) can be replaced by

$$
\begin{equation*}
<r^{2}>^{1 / 2}=\left(S_{\mathrm{An} l_{\mathrm{B}} j_{\mathrm{B}}}^{\mathrm{B}}\right)^{-1 / 2}\left(\int_{0}^{\infty} r^{4}\left[I_{\mathrm{An}_{\mathrm{B}} j_{\mathrm{B}}}^{\mathrm{B}}(r)\right]^{2} \mathrm{~d} r\right)^{1 / 2} . \tag{3}
\end{equation*}
$$

The spectroscopic factor can be obtained by comparing the experimental and calculated differential cross sections of the transfer reaction. The overlap function $I_{\mathrm{An} l_{\mathrm{B}} j_{\mathrm{B}}}^{\mathrm{B}}(r)$ can be calculated by solving the Schrödinger equation with the optical potential model. Usually, the $I_{\mathrm{An} l_{\mathrm{B}} j_{\mathrm{B}}}^{\mathrm{B}}(r)$ is model dependent and affected by the geometrical radius parameter $r_{0}$ and diffuseness parameter $a$ of the optical potential. At large distance, $r>R_{N}$, the overlap function shows asymptotic behavior,

$$
\begin{equation*}
I_{\mathrm{An} l_{\mathrm{B}} j_{\mathrm{B}}}^{\mathrm{B}}(r)=C_{\mathrm{An} l_{\mathrm{B}} j_{\mathrm{B}}}^{\mathrm{B}} \frac{W_{-\eta, l+1 / 2}\left(2 k_{\mathrm{B}} r\right)}{r} \tag{4}
\end{equation*}
$$

where $C_{\mathrm{An} l_{\mathrm{B}} j_{\mathrm{B}}}^{\mathrm{B}}$ is the nuclear ANC defining the amplitude of the tail of the overlap function; $W_{-\eta, l+1 / 2}\left(2 k_{\mathrm{B}} r\right)$ is the Whittaker function; $k_{\mathrm{B}}=\sqrt{2 \mu_{\mathrm{An}} S_{\mathrm{n}} / \hbar^{2}}$ is the wave number; $\mu_{\mathrm{An}}$ is the reduced mass of the system $\mathrm{A}+$ n; and $S_{n}, R_{N}$ and $\eta$ are the binding energy, nuclear interaction radius and Sommerfield parameter respectively for the corresponding $\mathrm{B} \rightarrow \mathrm{A}+\mathrm{n}$ state. In the case of the neutron $(\eta=0)$, the Whittaker function reduces to $W_{0, l+1 / 2}\left(2 k_{\mathrm{B}} r\right)=\sqrt{k_{\mathrm{B}} r / \pi} K_{l+1 / 2}\left(k_{\mathrm{B}} r\right)$ with $K$ being the modified Bessel function.

Thus, the rms radius can be separated into the contributions of interior and asymptotic regions,

$$
\begin{align*}
<r^{2}>^{1 / 2}= & \left(S_{\mathrm{An} l_{\mathrm{B}} j_{\mathrm{B}}}^{\mathrm{B}}\right)^{-1 / 2}\left[\int_{0}^{R_{N}} r^{4}\left[I_{\mathrm{An} l_{\mathrm{B}} j_{\mathrm{B}}}^{\mathrm{B}}(r)\right]^{2} \mathrm{~d} r\right. \\
& \left.+\left(C_{\mathrm{An} l_{\mathrm{B}} j_{\mathrm{B}}}^{\mathrm{B}}\right)^{2} \int_{R_{N}}^{\infty} r^{2} W^{2}\left(2 k_{\mathrm{B}} r\right) \mathrm{d} r\right]^{1 / 2} . \tag{5}
\end{align*}
$$

The first term in this equation is somehow model dependent, but its contribution is relatively small due to the $r^{4}$
dependence in the integral function. The difference induced by various potential parameters can be regarded as an uncertainty of the calculation. The second term, describing the asymptotic part, is model independent, which presents a major contribution to the rms radius. The ANC for the virtual decay $\mathrm{B} \rightarrow \mathrm{A}+\mathrm{n}$ can be determined through

$$
\begin{equation*}
C_{\mathrm{An} l_{\mathrm{B}} j_{\mathrm{B}}}^{\mathrm{B}}=\left(S_{\mathrm{An} l_{\mathrm{B}} j_{\mathrm{B}}}^{\mathrm{B}}\right)^{1 / 2} \times b_{\mathrm{An} l_{\mathrm{B}} j_{\mathrm{B}}}^{\mathrm{B}}, \tag{6}
\end{equation*}
$$

where $b_{\mathrm{An} l_{\mathrm{B}} j_{\mathrm{B}}}^{\mathrm{B}}$ is the single particle ANC of the bound state $B$ which can be deduced by the single particle wave functions with the optical potential model. Consequently, the radius of the valence neutron in $B$ can be extracted by the measurement of the ANC for $\mathrm{B} \rightarrow \mathrm{A}+$ n.

In order to determine the probability $(P)$ of the valence neutron being outside the radius $\left(R_{N}\right)$ of the interaction potential, one can calculate by

$$
\begin{equation*}
P=\frac{\left(C_{\mathrm{An}_{\mathrm{B}} j_{\mathrm{B}}}^{\mathrm{B}}\right)^{2} \int_{R_{N}}^{\infty} W^{2}\left(2 k_{\mathrm{B}} r\right) \mathrm{d} r}{\int_{0}^{\infty} r^{2}\left[I_{\mathrm{An} l_{\mathrm{B}} j_{\mathrm{B}}}^{\mathrm{B}}(r)\right]^{2} \mathrm{~d} r} \tag{7}
\end{equation*}
$$

where $R_{N}$ can be calculated by the radius of the core $\left.<r^{2}\right\rangle_{\text {core }}[23,41]$

$$
\begin{equation*}
R_{N}^{2}=\frac{5}{3} \times\left(<r^{2}>_{\mathrm{core}}+4\right) \tag{8}
\end{equation*}
$$

The radius of ${ }^{15} \mathrm{~N}$ is 2.42 fm [42], so the radius of interaction potential between the neutron and ${ }^{15} \mathrm{~N}$ nucleus is derived to be 4.05 fm . According to the halo definition of Hansen et al. [23], the probability of finding the valence nucleon is larger than $50 \%$ outside the range of the interaction potential, that is $P>50 \%$.

## 3 Experiment and DWBA calculations

Most recently, the angular distributions of the ${ }^{15} \mathrm{~N}\left({ }^{7} \mathrm{Li},{ }^{6} \mathrm{Li}\right){ }^{16} \mathrm{~N}$ reaction populating the ground state and the first three excited states at $E_{\mathrm{x}}=0.120,0.298$, and 0.397 MeV in ${ }^{16} \mathrm{~N}$ were measured, as shown in Fig. 1 together with the DWBA calculations. The detail of the setup, the data analysis procedures and DWBA calculations have been published elsewhere [43, 44], where the angular distributions were used to extract the astrophysical ${ }^{15} \mathrm{~N}(\mathrm{n}, \gamma){ }^{16} \mathrm{~N}$ reaction rate $[43]$ and the ${ }^{16} \mathrm{~F}$ proton widths [44].

The dependence of the spectroscopic factors and ANCs on the geometric parameters of the Woods-Saxon potential for the single-particle bound states in ${ }^{16} \mathrm{~N}$ were investigated, as shown in Fig. 2. In the calculations, the geometrical radius parameter $r_{0}$ and diffuseness parameter $a$ were varied on a grid of 49 points for $r_{0}=1.10-1.40$
fm and $a=0.40-0.80 \mathrm{fm}$ with steps of 0.05 fm , independently. While the well depths were adjusted to reproduce the binding energies corresponding to the ground and first three excited states. The results show that the spectroscopic factors of those states are $0.99 \pm 0.26,0.67$ $\pm 0.11,0.85 \pm 0.22$ and $0.63 \pm 0.10$, respectively. The errors result from the uncertainty of the differential cross sections $(9 \%, 13 \%, 9 \%, 12 \%)$ and the radius and diffuseness parameters $(25 \%, 11 \%, 24 \%, 9 \%)$. The squares of the ANCs for the virtual decay ${ }^{16} \mathrm{~N} \rightarrow{ }^{15} \mathrm{~N}+\mathrm{n}$ were derived to be $0.19 \pm 0.02,3.45 \pm 0.51,0.12 \pm 0.01$ and 2.69 $\pm 0.38 \mathrm{fm}^{-1}$, respectively. The errors result from the uncertainty of the differential cross sections ( $9 \%, 13 \%, 9 \%$, $12 \%$ ) and the radius and diffuseness parameters (3\%, $7 \%$, $3 \%, 7 \%)$. This indicates that the ANC method can be used to calculate the halo structure of ${ }^{16} \mathrm{~N}$, because the ANCs vary slowly under reasonable variation of the single particle potential.


Fig. 1. Angular distributions of the ${ }^{15} \mathrm{~N}\left({ }^{7} \mathrm{Li}\right.$, $\left.{ }^{6} \mathrm{Li}\right){ }^{16} \mathrm{~N}$ reaction leading to the ground and first three excited states in ${ }^{16} \mathrm{~N}$.


Fig. 2. (color online) Dependence of the spectroscopic factors (S) and the square of the ANCs ( $\mathrm{ANC}^{2}$ ) on the single-particle ANCs of the ground and first three excited states in ${ }^{16} \mathrm{~N}$.

In order to further check the ANC method's suitability, the overlap function $I_{15}^{16} \mathrm{~N}+\mathrm{n}$ multiplied by the square of $r$ is compared with the ANCs $C_{15}^{16} \mathrm{~N}+\mathrm{n}$ multiplied by the Whittaker function and $r$, as shown in Fig. 3. It illustrates the coincidence of the asymptotic behavior and Whittaker functions beyond the radius of the interaction potential, which proves a correct asymptotic normalization of the model overlap wave functions.


Fig. 3. (color online) Comparison of the $r^{2} I_{15}^{16} \mathrm{~N}+\mathrm{n}$ and $r C_{15}^{16_{\mathrm{N}+\mathrm{n}}} W\left(2 k_{\mathrm{B}} r\right)$ for the ground and first three excited states of ${ }^{16} \mathrm{~N}$. The radius of the ${ }^{15} \mathrm{~N}$ +n interaction potential is also indicated.

## 4 Results

According to Eq. (5), the quantity [36]

$$
\begin{equation*}
\chi^{2}=\sum_{r=R_{N}}^{80 f m}\left(I_{\mathrm{An}_{\mathrm{B}} j_{\mathrm{B}}}^{\mathrm{B}}(r)-C_{\mathrm{An} l_{\mathrm{B}} j_{\mathrm{B}}}^{\mathrm{B}} \frac{W\left(2 k_{\mathrm{B}} r\right)}{r}\right)^{2} \tag{9}
\end{equation*}
$$

is used to estimate the difference between the interior part and asymptotic part. The values of $\chi^{2}$ are parameter dependent and are calculated by varying radius and diffuseness. Figure 4 shows the dependence of $\chi^{2}$ on the radius of the valence neutron, and Fig. 5 presents the dependence of $\chi^{2}$ on the probability of the valence neutron being outside the nuclear interaction radius. Then, in terms of experimental ANCs, one can calculate the radius of valence neutron $R_{\mathrm{rms}}^{\mathrm{vn}}$ and the probability of it staying outside the range of the interaction radius $P$ by

$$
\begin{equation*}
R_{\mathrm{rms}}^{\mathrm{vn}}=\sum_{i} w_{i}<r^{2}>_{i}^{1 / 2} \tag{10}
\end{equation*}
$$

and

$$
\begin{equation*}
P=\sum_{i} w_{i} P_{i} \tag{11}
\end{equation*}
$$

with the weight of

$$
\begin{equation*}
w_{i}=\frac{\left(\chi_{i}^{2}\right)^{-1}}{\sum_{i}\left(\chi_{i}^{2}\right)^{-1}} \tag{12}
\end{equation*}
$$

The sum runs over the the 49 points which are calculated by varying the radius and diffuseness parameters. Then the valence neutron rms radii are calculated to be (3.85 $\pm 0.31) \mathrm{fm},(4.82 \pm 0.42) \mathrm{fm},(3.91 \pm 0.32) \mathrm{fm}$ and $(4.96$ $\pm 0.41) \mathrm{fm}$. The errors result from the uncertainty of the differential cross sections $(5 \%, 7 \%, 5 \%, 6 \%)$ and the radius and diffuseness parameters $(7 \%, 6 \%, 7 \%, 6 \%)$. The valence neutron rms radii are shown in Table 1 together with the separation energies, valence neutron configurations, spectroscopic factors, squares of the ANCs, halo radii and $P$ corresponding to the ground and first three excited states of ${ }^{16} \mathrm{~N}$. It is evident that a neutron halo is absent in the $1 d_{5 / 2}$ ground and second excited states, while the first and third excited states of ${ }^{16} \mathrm{~N}$ have a halo structure according to the halo definition that the halo nucleon spends about $50 \%$ of the time outside the range of the core potentials [41].


Fig. 4. Dependence of $\chi^{2}$ on the radius of the valence neutron for the ground and first three excited states in ${ }^{16} \mathrm{~N}$. Symbols with the same radius parameters are connected by lines.


Fig. 5. Dependence of $\chi^{2}$ on the probability of the valence neutron being outside the nuclear interaction radius for the ground and first three excited states in ${ }^{16} \mathrm{~N}$. Symbols with the same radius parameters are connected by lines.

Table 1. Summary of the separation energies $\left(S_{n}\right)$, valence neutron configurations (conf.), spectroscopic factors $\left(S_{15}^{16} \mathrm{~N} n\right.$ ), the square of ANCs $\left(\left(C_{15}^{16} \mathrm{~N} n\right)^{2}\right)$, halo radii $\left(R_{\mathrm{rms}}^{\mathrm{vn}}\right), P$ and rms matter radii $\left(R_{\mathrm{m}}^{16} \mathrm{~N}\right)$ for the ground and first three excited states in ${ }^{16} \mathrm{~N}$.

| state | $S_{n} / \mathrm{MeV}$ | conf. | $S_{15}^{16} \mathrm{~N} n$ | $\left(C_{15}^{16} \mathrm{~N} n\right)^{2} / \mathrm{fm}^{-1}$ | $R_{\mathrm{rms}}^{\mathrm{vn}} / \mathrm{fm}$ | $P(\%)$ | $R_{\mathrm{m}}^{16} \mathrm{~N} / \mathrm{fm}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| G.S. | 2.490 | $1 d_{5 / 2}$ | $0.99 \pm 0.26$ | $0.19 \pm 0.02$ | $3.85 \pm 0.31$ | $31 \pm 8$ |  |
| Ex1 | 2.370 | $2 s_{1 / 2}$ | $0.67 \pm 0.11$ | $3.45 \pm 0.51$ | $4.82 \pm 0.42$ | $58 \pm 12$ | $2.52 \pm 0.03$ |
| Ex2 | 2.192 | $1 d_{5 / 2}$ | $0.85 \pm 0.22$ | $0.12 \pm 0.01$ | $3.91 \pm 0.32$ | $32 \pm 8$ |  |
| Ex3 | 2.093 | $2 s_{1 / 2}$ | $0.63 \pm 0.10$ | $2.69 \pm 0.38$ | $4.96 \pm 0.41$ | $60 \pm 12$ | $2.53 \pm 0.03$ |

These findings also allow us to estimate the matter radius of the ${ }^{16} \mathrm{~N}$ nucleus $R_{\mathrm{m}}^{16} \mathrm{~N}$ according to [45]

$$
\begin{equation*}
\left.\left(R_{\mathrm{m}}^{16}\right)^{\mathrm{N}}\right)^{2}=\frac{A}{A+1}\left(R_{\mathrm{m}}^{15} \mathrm{~N}\right)^{2}+\frac{A}{(A+1)^{2}}\left(R_{\mathrm{rms}}^{\mathrm{vn}}\right)^{2} \tag{13}
\end{equation*}
$$

where $A=15$ is the mass number and $R_{\mathrm{m}}^{15} \mathrm{~N}=2.42 \mathrm{fm}$ [42] is the matter radius of the ${ }^{15} \mathrm{~N}$ nucleus. The matter radii of the ${ }^{16} \mathrm{~N}$ ground and first three excited states are then deduced as listed in Table 1. The matter radius of the ${ }^{16} \mathrm{~N}$ ground state $R_{\mathrm{m}}^{16} \mathrm{~N}=(2.52 \pm 0.03) \mathrm{fm}$ is in good agreement with the measured value ( $2.50 \pm 0.10$ ) fm [42]. The density distributions of the ${ }^{15} \mathrm{~N}$ nucleus [46] and valence neutron of the ${ }^{16} \mathrm{~N}$ ground and first three excited states are shown in Fig. 6, from which one can also see that the density distributions of valence neutrons in the $s$ orbit have longer tails expanding to the outside of the core nucleus than those in the $d$ orbit do.


Fig. 6. (color online) The density distributions of the ${ }^{15} \mathrm{~N}$ nucleus and valence neutron for the ${ }^{16} \mathrm{~N}$ ground and first three excited states. The density distributions of the ${ }^{15} \mathrm{~N}$ nucleus are calculated by a formula in Ref. [46].

To estimate the effect of core excitation on the valence neutron radius [36], we calculate the neutron spectroscopic factors of the ground and first three excited states in ${ }^{16} \mathrm{~N}$ where the neutron orbits the ground and excited states of the ${ }^{15} \mathrm{~N}$ core by using the shell model computer code NuShell [47]. In the calculation, the interaction ZBMI [48] is used, assuming a closed ${ }^{12} \mathrm{C}$ core and populating the shells $1 p_{1 / 2}, 1 d_{5 / 2}$ and $2 s_{1 / 2}$. The
shell model results are listed in Table 2. According to the table, the contributions of the core excitation are about $S_{15 \mathrm{~N}_{\text {ex }} n}^{16_{\mathrm{N}}} /\left(S_{15 \mathrm{~N}_{\mathrm{Ns}} n}^{1 \mathrm{~N}^{2}}+S_{15 \mathrm{~N}_{\mathrm{e} x} n}^{16} \mathrm{~N}^{2}\right)=28 \%, 30 \%, 6 \%$ and $7 \%$, respectively. When the neutron orbits the excited states of the ${ }^{15} \mathrm{~N}$ core, the rms radii of the valence neutron in the four low-lying ${ }^{16} \mathrm{~N}$ states are calculated to be $3.06 \mathrm{fm}, 3.07 \mathrm{fm}, 3.09 \mathrm{fm}$ and 3.09 fm , and the probabilities staying outside the range of the interaction radius are $12 \%, 12 \%, 12 \%$ and $13 \%$, respectively. The results are entirely due to the large excited energy of ${ }^{15} \mathrm{~N}$.

Table 2. The calculated neutron spectroscopic factors of the four low-lying ${ }^{16} \mathrm{~N}$ states where the neutron orbits the ground and excited states of the ${ }^{15} \mathrm{~N}$ core.

| state | $S_{15}^{16} \mathrm{~N}$ | $S_{\mathrm{gs}}^{16} \mathrm{~N}$ | $S_{15 \mathrm{~N}_{\mathrm{ex} 1} n}^{16 \mathrm{~N}}$ |
| :--- | :---: | :---: | :---: |
| G.S. | 0.93 | 0.37 | 0.00 |
| Ex1 | 0.95 | 0.00 | 0.40 |
| Ex2 | 0.87 | 0.05 | 0.00 |
| Ex3 | 0.96 | 0.00 | 0.07 |

## 5 Summary and conclusion

The nuclear halo is an interesting threshold phenomenon and the ANC method is an effective tool to study the halo structure in the ground states, especially in the excited states. The ANCs of the virtual decay ${ }^{16} \mathrm{~N}$ $\rightarrow{ }^{15} \mathrm{~N}+\mathrm{n}$ are extracted from the ${ }^{15} \mathrm{~N}\left({ }^{7} \mathrm{Li},{ }^{6} \mathrm{Li}\right){ }^{16} \mathrm{~N}$ reaction populating the ground and first excited states in ${ }^{16} \mathrm{~N}$. They are used to derive the rms radii of the valence neutron and the probability of the valence neutron being outside the radius of the interaction potential of ${ }^{15} \mathrm{~N}+$ n for these states. The rms radii of the valence neutron in two $s$-states of ${ }^{16} \mathrm{~N}$ are found to be significantly larger than those of the valence neutron in two $d$-states. In addition, the probabilities of the valence neutron being outside the radius of the interaction potential for the two $s$-states are larger than $50 \%$, while the probabilities for the two $d$-states are significantly smaller. The present results support the conclusion that the two $s$-states of ${ }^{16} \mathrm{~N}$ have neutron halo structure, while the two $d$-states do not. However, core excitation effects should be considered for the ground and first excited states of ${ }^{16} \mathrm{~N}$.

## References

1 I. Tanihata, H. Hamagaki, O. Hashimoto et al, Phys. Rev. Lett. 55: 2676 (1985)
2 I. Tanihata, H. Hamagaki, O. Hashimoto et al, Phys. Lett. B, 160: 380 (1985)
3 M. Fukuda, T. Ichihara, N. Inabe et al, Phys. Lett. B, 268: 339 (1991)
4 D. Bazin, B.A. Brown, J. Brown et al, Phys. Rev. Lett., 74: 3569 (1995)
5 I. Tanihata, T. Kobayashi, T. Suzuki et al, Phys. Lett. B, 287 : 307 (1992)
6 T. Suzuki, R. Kanungo, O. Bochkarev et al, Nucl. Phys. A, 658: 313 (1999)
7 M. Labiche, N.A. Orr, F. M. Marqués et al, Phys. Rev. Lett., 86: 600 (2001)
8 M. V. Zhukov, A. A. Korsheninnikov, M. H. Smedberg, Phys. Rev. C, 50: 1R (1994)
9 M. H. Smedberg, T. Baumann, T. Aumann et al, Phys. Lett. B, 452: 1 (1999)
10 M. Fukuda, M. Mihara, T. Fukao et al, Nucl. Phys. A, 656: 209 (1999)
11 V. Guimaraes, J. J. Kolata, D. Peterson, Phys. Rev. Lett., 84: 1862 (2000)
12 A. Ozawa, T. Kobayashi, H. Sato et al, Phys. Lett. B, 334: 18 (1994)

13 T. Suzuki, H. Geissel, O. Bochkarev et al, Nucl. Phys. A, 616: 286c (1997)
14 A. Navin, D. Bazin, B. A. Brown et al, Phys. Rev. Lett., 81: 5089 (1998)
15 T. Otsuka, N. Fukunishi, H. Sagawa, Phys. Rev. Lett., 70: 1385 (1993)
16 T. Otsuka, M. Ishihara, N. Fukunishi, T. Nakamura, M. Yokoyama, Phys. Rev. C, 49: 2289R (1994)
17 Z. H. Li, B. Guo, W. P. Liu et al, Phys. Lett. B, 527: 50 (2002)
18 T. L. Belyaeva, R. Perez-Torres, A. A. Ogloblin et al, Phys. Rev. C, 90: 064610 (2014)
19 Z. H. Liu, C. J. Lin, H. Q. Zhang et al, Phys. Rev. C, 64: 034312 (2001)
20 C. J. Lin, Z. H. Liu, H. Q. Zhang et al, Chin. Phys. Lett., 18: 1183 (2001)
21 P.G. Hansen B. Jonson, Europhys. Lett., 4: 409 (1987)
22 I. Tanihata, J. Phys. G: Nucl. Part. Phys., 22: 157 (1996)
23 P. G. Hansen A. S. Jensen, Annu. Rev. Nucl. Part. Sci., 45: 591 (1995)

24 Z. Z. Ren, B. Q. Chen, Z. Y. Ma, G. O. Xu, Z. Phys. A, 357: 137 (1997)
25 B. Blank, C. Marchand, M.S. Pravikoff et al, Nucl. Phys. A, 624: 242 (1997)
26 R. E. Warner, J. H. Kelley, P. Zecher et al, Phys. Rev. C, 52: 1166R (1995)
27 F. Negoita, C. Borcea, F. Carstoiu et al, Phys. Rev. C, 54: 1787 (1996)
28 G. D. Alkhazov, M. N. Andronenko, A. V. Dobrovolsky et al, Phys. Rev. Lett., 78: 2313 (1997)
29 P. Egelhof, G. D. Alkhazov, M. N. Andronenko et al, Eur. Phys. J. A, 15: 27 (2002)

30 I. Pecina, R. Anne, D. Bazin et al, Phys. Rev. C, 52: 191 (1995)

31 T. Minamisono, T. Ohtsubo, I. Minami et al, Phys. Rev. Lett., 69: 2058 (1992)
32 J. H. Kelley, Sam M. Austin, A. Azhari et al, Phys. Rev. Lett., 77: 5020 (1996)
33 T. Motobayashi, N. Iwasa, Y. Ando et al, Phys. Rev. Lett., 73: 2680 (1994)
34 B. Davids, D. W. Anthony, Sam M. Austin et al, Phys. Rev. Lett., 81: 2209 (1998)
35 N. Iwasa, F. Boué, G. Surówka et al, Phys. Rev. Lett., 83: 2910 (1999)
36 F. Carstoiu, L. Trache, C. A. Gagliardi, R. E. Tribble, A. M. Mukhamedzhanov, Phys. Rev. C, 63: 054310 (2001)
37 Z. H. Liu, J. D. Bao, Chin. Phys. Lett., 21: 457 (2004)
38 Z. H. Li, B. Guo, W. P. Liu et al, Chin. Phys. Lett., 22: 1870 (2005)

39 B. Guo, Z. H. Li, W. P. Liu, X. X. Bai, Chin. Phys. Lett., 23: 52 (2006)
40 E. T. Li, Z. H. Li, Y. J. Li et al, Phys. Rev. C, 90: 067601 (2014)

41 B. Jonson, Phys. Rep., 389: 1 (2004)
42 A. Ozawa, O. Bochkarev, L. Chulkov et al, Nucl. Phys. A, 691: 599 (2001)
43 B. Guo, Z. H. Li, Y. J. Li et al, Phys. Rev. C, 89: 012801R (2014)

44 Z. D. Wu, B. Guo, Z. H. Li et al, Phys. Rev. C, 89: 054315 (2014)

45 J. A. Tostevin, J. S. Al-Halili, Nucl. Phys. A, 616: 418 (1997)
46 J. W. Negele, Phys. Rev. C, 1: 1260 (1970)
47 B.A. Brown, W.D.M. Rae, MSU-NSCL report (2007)
48 A. P. Zuker, B. Buck, J. B. McGrory, Phys. Rev. Lett., 21: 39 (1968)


[^0]:    Received 18 January 2016
    ＊Supported by National Natural Science Foundation of China（11505117，11490560，11475264，11321064，11375269），Natural Science Foundation of Guangdong Province（2015A030310012）， 973 program of China（2013CB834406）National key Research and Development Province（2016YFA0400502）

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