Neighboring azimuthal bin-bin multiplicity correlation as a direct measure for the shear viscosity in relativistic heavy ion collisions *

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Abstract: Neighboring azimuthal bin-bin multiplicity correlation is suggested to be a good measure for internal layer-to-layer interactions of the formed matter in relativistic heavy ion collisions. It is shown to be directly related to the shear viscosity of the formed matter. As an application of this method, the shear viscosity in the samples generated by a multi-phase transport model (AMPT) is estimated. The results are in qualitative agreement with the theoretical calculation from microscopic interactions, i.e., the larger the scattering cross section, the smaller the shear viscosity.

Key words: neighboring azimuthal bin-bin multiplicity correlation, shear viscosity, relativistic heavy ion collision

PACS: 25.75.Ag, 25.75.Gz, 25.75.Ld **DOI:** 10.1088/1674-1137/37/9/094102

1 Introduction

The large collective flow observed in relativistic heavy ion collisions (RHIC) is regarded as one of the most important pieces of evidence of the formation of a new form of matter – quark-gluon plasma (QGP) [1–3]. The behavior of elliptic flow has been successfully described by hydrodynamic models [4, 5]. This shows that the formed matter behaves like an almost-perfect fluid with a very small specific shear viscosity [6].

In recent years, there has been considerable interest in shear viscosity of strongly interacting Quark-Gluon Plasma. Using the string theory, P. K. Kovtun showed that there exists a lowest limit value $\eta/s=\frac{1}{4\pi}$ for a wide class of strong-coupled quantum field systems [7]. In $SU(3)$ gauge theory, an upper bound $\eta/s\leqslant 1$ is given by Harvey B. Meyer [8].

A lot of viscous hydrodynamic models have been performed to reproduce flow measurements from experimental data quite successfully [6, 9]. The shear viscosity can be extracted by comparing the experimental data to the theoretical calculations. However, the precise value of shear viscosity is hard to determine because of the uncertainties, particularly from the poor knowledge of particle production and thermalization in the early stage [10–13]. The unknown initial conditions lead to large uncertainty in eccentricity ε_2 and elliptic flow v_2 [6]. It is the largest source of uncertainty to determine the shear viscosity from experimental data. A combined analysis of v_2 and v_3 could reduce the model uncertainty in the initial deformation of the formed matter and its event-by-event fluctuations [10, 14–17]. Huichao Song and Ulrich Heinz extracted the specific shear viscosity η/s by comparing the experimental data with viscous hydrodynamics and have established a robust upper limit $\frac{1}{4\pi}<\eta/s<2.5\times\frac{1}{4\pi}$ [18]. M. Luzum and J. Y. Ollitrault presented a proposal to extract shear viscosity from a simultaneous fit to p_t-integrated v_n measurements from ultra-central collisions [12]. It can extract the shear viscosity with the smallest uncertainty. Besides, Zhe Xu et al. [19, 20] developed a relativistic perturbative QCD (pQCD) model BAMPS from microscopic transport calculations to extract the shear viscosity coefficient η.

Some attempts were also made to extract the shear viscosity of the matter experimentally. Sean Gavin and Mohamed Abdel-Aziz [21] measured shear viscosity by using transverse momentum correlation in relativistic nuclear collisions. This method has been subsequently used by STAR collaboration in Au+Au collisions at $\sqrt{s_{\rm NN}}$=200 GeV [22, 23].

As there are large uncertainties, a much more precise extraction of shear viscosity is expected. Furthermore, it is necessary to determine the shear viscosity directly from the experimental data, independent of implementa-

Received 24 December 2012, Revised 4 February 2013

* Supported by NSFC (10835005, 11221504), MOE of China (IRT0624, B08033) and Fundamental Research Funds for Central Universities (CUGL 100237)

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tion of hydrodynamic models. From the origin of viscosity, smaller value of shear viscosity means the dense matter is closer to perfect liquid. Perfect fluid means no internal interactions. While for viscous fluid, there is friction between any two layers and the flow velocity is different from layer to layer. A direct probe for friction force between layers will be helpful in getting the value of shear viscosity directly.

In this paper, we first briefly introduce the azimuthal bin-bin multiplicity correlation pattern to measure the internal interaction of the formed matter. It can be used as a good probe for the internal layer-to-layer interactions of formed matter. Then, the relation between the correlation pattern and shear viscosity is derived. Finally, the shear viscosity is estimated with the samples generated by AMPT model for different scattering cross sections.

2 Azimuthal bin-bin multiplicity correlation pattern and shear viscosity

A spatially dependent bin-bin correlation pattern is suggested to explore the internal interaction of the formed matter [24]. It is well known that the general 2-bin correlation is defined as

$$C_{m_1,m_2}=\frac{\langle n_{m_1}n_{m_2}\rangle}{\langle n_{m_1}\rangle\langle n_{m_2}\rangle}-1, \tag{1}$$

where m_1 and m_2 are the positions of the two bins in phase space and n_m is the measured content in the mth bin. If there is no correlation between particles in the observed window, C_{m_1,m_2} vanishes.

We divide the 2π azimuthal angle equally into M bins and specify n_ϕ as the multiplicity in the mth angular bin. If we let $m_1=m$ and $m_2=m+1$, C_{m_1,m_2} is reduced to the neighboring angular-bin correlation pattern, which measures how the nearby particles correlate with each other in different directions of azimuthal space.

$$C_{\phi,\phi+\delta\phi}=\frac{\langle n_\phi n_{\phi+\delta\phi}\rangle}{\langle n_\phi\rangle\langle n_{\phi+\delta\phi}\rangle}-1, \tag{2}$$

$\phi=0$ refers to the direction of the reaction plane in the nuclear collision.

It has been shown by M. Stephanov [25], that if the interaction is small (the system is not far away from non-interaction gas in thermal equilibrium), in the leading order, the joint probability of bin-bin multiplicity can be determined by the interaction energy. The bin-bin multiplicity correlation is,

$$C_{\phi,\phi+\delta\phi}=\mathrm{e}^{-\beta E_{\mathrm{I}}(\phi)}-1\approx-\beta E_{\mathrm{I}}(\phi). \tag{3}$$

Here, $E_{\mathrm{I}}(\phi)$ is the interaction energy and is the function of the azimuthal angle ϕ. β is the inverse of temperature T. This is in fact a very universal expression in microscopic level. For the whole system, the total interaction energy, E_{Corr}, is the integration of $E_{\mathrm{I}}(\phi)$ for the whole azimuthal range,

$$\begin{aligned}E_{\mathrm{Corr}} &= \int_0^{2\pi}E_{\mathrm{I}}(\phi)\mathrm{d}\phi=-\int_0^{2\pi}\frac{C_{\phi,\phi+\delta\phi}}{\beta}\mathrm{d}\phi \\ &= -\int_0^{2\pi}TC_{\phi,\phi+\delta\phi}\mathrm{d}\phi.\end{aligned} \tag{4}$$

In hydrodynamics, the dissipative energy due to the interaction of friction is well estimated. It is usually supposed that the momentum transfer due to viscosity is proportional to the first derivatives of the velocity. So the viscous stress tensor is written as,

$$\sigma'_{ik}=\eta\left(\frac{\partial v_i}{\partial x_k}+\frac{\partial v_k}{\partial x_i}-\frac{2}{3}\delta_{ik}\frac{\partial v_l}{\partial x_l}\right)+\xi\delta_{ik}\frac{\partial v_l}{\partial x_l}, \tag{5}$$

where the $\eta(p,T)$ and $\xi(p,T)$ are shear and bulk viscosities, respectively. In general, they are functions of pressure and temperature. For in-compressive fluid, the last two terms vanish. The dissipative energy due to viscous effects in the whole volume of the fluid is,

$$E_{\mathrm{Diss}}=-\frac{1}{2}\int\eta\left(\frac{\partial v_i}{\partial x_k}+\frac{\partial v_k}{\partial x_i}\right)^2\mathrm{d}V\mathrm{d}t. \tag{6}$$

If we take the cylindrical coordinates and assume that there is no rotation in the azimuthal direction and ignore the velocity gradient along longitudinal direction at first approximation, the dissipative energy in the whole fluid can be simply written as,

$$\begin{aligned}E_{\mathrm{Diss}}&=-\frac{1}{2}\int\eta\left(\frac{1}{r}\frac{\partial v_{\mathrm{T}}}{\partial\phi}\right)^2\frac{1}{2}r^2\mathrm{d}\phi\mathrm{d}z\mathrm{d}t \\ &=-\frac{1}{4}\int\eta\left(\frac{\partial v_{\mathrm{T}}}{\partial\phi}\right)^2\mathrm{d}\phi\mathrm{d}z\mathrm{d}t.\end{aligned} \tag{7}$$

If the bin-bin correlation is only due to the viscous interaction, the corresponding interaction energy is equal to the dissipative energy. The relation between correlation pattern and shear viscosity can be derived,

$$\eta\approx\frac{4T\int_0^{2\pi}C_{\phi,\phi+\delta\phi}\mathrm{d}\phi}{\int\left(\frac{\partial v_T}{\partial\phi}\right)^2\mathrm{d}\phi\mathrm{d}z\mathrm{d}t}. \tag{8}$$

Here, the temperature is usually estimated with a thermal model, and the integral of velocity gradient in longitudinal and time is roughly replaced by the velocity gradient of final state particles.

To get the value of shear viscosity of the formed matter, we need to calculate the velocity gradient of the radial flow and the azimuthal bin-bin multiplicity correlation separately.

Here, we use the azimuthal distribution of mean radial rapidity of the final state particles to get the velocity gradient of radial flow [26, 27]. In Ref. [27], we suggested a method to measure the anisotropic radial velocity directly. The radial flow is usually described by two parameters. The first one is the isotropic radial velocity (or rapidity, $\nu_{\rm T}=\tanh y_{\rm T}$). It presents the surface profile of isotropic transverse expansion of the source at kinetic freeze-out. The other parameter is the anisotropic radial velocity (i.e., the azimuthal dependent radial velocity). It measures the difference of the radial flow strength in and out of the reaction plane. In Ref. [27], it is demonstrated that the azimuthal amplitude of this suggested distribution characterizes the anisotropic radial flow, and coincides with the parameter of anisotropic radial rapidity extracted from a generalized blast-wave parameterization. So, the velocity gradient of flow can be extracted.

The transverse rapidity of a final state hadron is defined as,

$$y_{\rm T}=\ln\left(\frac{m_{\rm T}+p_{\rm T}}{m_0}\right), \tag{9}$$

where m_0 is the particle mass in the rest frame, $p_{\rm T}$ is transverse momentum, and $m_{\rm T}=\sqrt{m_0^2+p_{\rm T}^2}$ is the transverse mass. The mean transverse rapidity in a given azimuthal angle bin is defined as the summation of all particles' rapidities divided by the total number of particles, i.e.,

$$\langle y_{\rm T}(\phi-\psi_r)\rangle=\frac{1}{N_{\rm event}}\sum_{e=1}^{N_{\rm event}}\frac{1}{N_m^e}\sum_{i=1}^{N_m^e}y_{{\rm T},i}^e(\phi_m-\psi_r), \tag{10}$$

where $y_{{\rm T},i}^e$ is the transverse rapidity of the ith particle and N_m^e is the total number of particles in mth azimuthal angle bin in eth event. Eq. (10) measures the mean transverse motion in the azimuthal direction [26]. It is a periodic function of azimuthal angle and can be well fitted by,

$$\langle y_{\rm T}(\phi)\rangle=y_{\rm T0}+y_{\rm T2}\cos(2\phi). \tag{11}$$

Eq. (11) consists of two parts: an isotropic mean rapidity and a mean azimuthal dependent rapidity amplitude. Furthermore, the thermal motion is isotropic and does not contribute to the anisotropic radial rapidity. So the anisotropic amplitude, $y_{\rm T2}$, should correspond to the parameter of anisotropic radial rapidity.

3 A rough estimation for shear viscosity in AMPT

As an application of this method, the shear viscosity in the sample generated by a multi-phase transport model (AMPT) [28] with string melting is estimated. The AMPT is a multi-phase transport model, which contains four main components: the initial conditions, partonic interactions, conversion from the partonic to the hadronic matter and hadronic interactions. The initial conditions are based on the HIJING model [29]. The time evolution of partons is then treated by the Zhang’s Parton Cascade (ZPC) model [30]. After partons stop interacting, a combined coalescence and string fragmentation model is used for the hadronization process. After hadronization, scatterings among the resulting hadrons are described by a relativistic transport (ART) model [31] which includes baryon-baryon, baryon-meson and meson-meson elastic and inelastic scatterings.

Our previous work [32, 33] has shown that the initial conditions for the geometric anisotropy of the overlapping collision region affects the observed azimuthal bin-bin multiplicity correlation pattern. So in order to get real internal interactions of the observed sample, we should be careful to choose the same kind of events, i.e., the sample with fixed multiplicity and fixed impact parameter. We generate two samples for Au+Au collisions at $\sqrt{s_{\rm NN}}=200$ GeV for σ=3 mb and σ=6 mb (σ is the scattering cross section). Here, the multiplicity $N_{\rm ch}$ and impact parameter are fixed.

First we study the case for the sample with σ=3 mb. Here we choose the rapidity range $y\in[-2.0, 2.0]$ and fix the multiplicity $N_{\rm ch}=2793\pm20$ and the impact parameter b=7 fm. The azimuthal bin-bin multiplicity correlation pattern and the mean transverse rapidity distribution are shown in Fig. 1(a), (b), which are both in-plane like.

They are typical periodic functions of the azimuthal angle. It is easy to express them in the form of the Fourier expansion (here we keep to the second order terms approximately), which is as follows: $C_{\phi,\phi+\delta\phi}\approx 0.00246+0.00059\cos(2\phi)$, $\langle y_{\rm T}(\phi)\rangle\approx1.311+0.0356\cos(2\phi)$.

As we know, the anisotropic contribution is mainly from the second harmonic term. Here, we will ignore the flat part of the azimuthal bin-bin multiplicity correlation pattern. From the equation $v_{\rm T}=\tanh y_{\rm T}$, we can get that: $\langle v_{\rm T}(\phi)\rangle\approx0.865+0.0356\cos(2\phi)$. The corresponding velocity gradient of radial flow along the azimuthal direction is shown in Fig. 1(c). If we approximate the temperature to $T_{\rm c}$=154 MeV [34], using Eq.(8), the value of shear viscosity is about 0.15 corresponding to the case for the sample with σ=3 mb.

We measure the shear viscosity in the sample of σ=6 mb in a similar way. The results are shown in Fig. 2. If we fit them by the 2nd order Fourier expansion: $C_{\phi,\phi+\delta\phi}\approx0.00347+0.00068\cos(2\phi)$, $\langle y_{\rm T}(\phi)\rangle\approx 1.316+0.0402\cos(2\phi)$. Then we get the value of shear viscosity for σ=6 mb is about 0.13.

Finally, we compare the values of the shear viscosity extracted from the AMPT model with the theoretical calculation [35]. On the microscopic side, η is related to

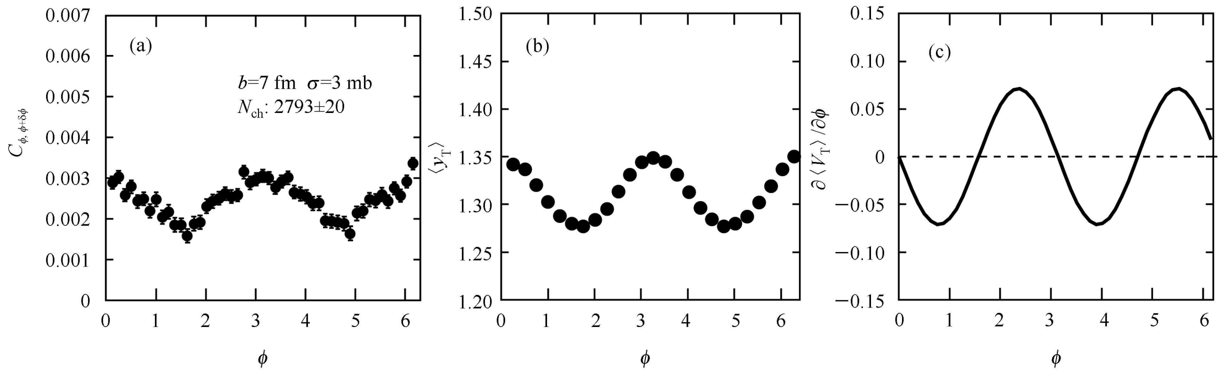

Fig. 1. The azimuthal bin-bin multiplicity correlation pattern (a), the azimuthal distribution of mean radial rapidity(b), and the velocity gradient of the radial flow (c), for Au+Au collisions at $\sqrt{s_{\mathrm{NN}}}=200$ GeV with σ=3 mb, b=7 fm and N_{ch}=2793±20 generated by AMPT with string melting model.

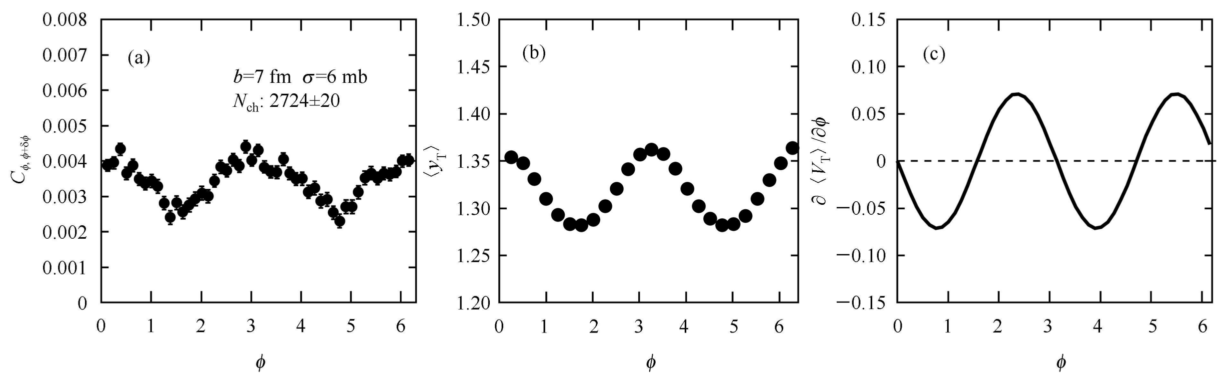

Fig. 2. The azimuthal bin-bin multiplicity correlation pattern (a), the azimuthal distribution of mean radial rapidity(b), and the velocity gradient of the radial flow (c), for Au+Au collisions at $\sqrt{s_{\mathrm{NN}}}=200$ GeV with σ=6 mb, b=7 fm and N_{ch}=2724±20 generated by AMPT with string melting model.

the scattering cross section σ. Ideal hydrodynamics assumes that the transport mean free path is so small that viscous terms can be ignored. In fact, in kinetic theory for a relativistic fluid [36], the shear viscosity can be written as $\eta\approx\frac{4}{15}n\overline{p}\lambda$, where n is the particle density, $\overline{p}$ is the average momentum of a fluid particle, λ is the mean free path. The mean free path is $\lambda\sim 1/(n\sigma)$, where σ is the transport cross section, thus $\eta\sim T/\sigma$. In a classical gas of massless particles with isotropic differential cross section, the shear viscosity is given by $\eta\approx 1.264T/\sigma$ [37, 38]. Our estimates for the shear viscosity are 0.15 for σ=3mb and 0.13 for σ=6 mb respectively. It is interesting to note that our simple estimates are in qualitative agreement with the theoretical expectation, i.e., the larger the scattering cross section, the smaller the shear viscosity.

4 Summary and conclusions

In this paper, the azimuthal bin-bin multiplicity correlation pattern is suggested, which is used as a good probe for the internal layer-to-layer interaction of the formed matter at RHIC. Then, the relation between correlation pattern and shear viscosity is derived. As an example, we use the AMPT model to estimate the shear viscosity. Two samples for different cross sections σ=3 mb and σ=6 mb have been analyzed. Our results are in qualitative agreement with the theoretical calculation from microscopic interactions, i.e., the larger the scattering cross section, the smaller the shear viscosity. Furthermore, we are looking forward to applying this suggested measure of shear viscosity in current relativistic heavy ion collision experiments.

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