Transport of 3D space charge dominated beams *

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Abstract: In this paper we present the theoretical analysis and the computer code design for the intense pulsed beam transport. Intense beam dynamics is a very important issue in low-energy high-current accelerators and beam transport systems. This problem affects beam transmission and beam qualities. Therefore, it attracts the attention of the accelerator physicists worldwide. The analysis and calculation for the intense beam dynamics are very complicated, because the state of particle motion is dominated not only by the applied electromagnetic fields, but also by the beam-induced electromagnetic fields (self-fields). Moreover, the self fields are related to the beam dimensions and particle distributions. So, it is very difficult to get the self-consistent solutions of particle motion analytically. For this reason, we combine the Lie algebraic method and the particle in cell (PIC) scheme together to simulate intense 3D beam transport. With the Lie algebraic method we analyze the particle nonlinear trajectories in the applied electromagnetic fields up to third order approximation, and with the PIC algorithm we calculate the space charge effects to the particle motion. Based on the theoretical analysis, we have developed a computer code, which calculates beam transport systems consisting of electrostatic lenses, electrostatic accelerating columns, solenoid lenses, magnetic and electric quadruples, magnetic sextupoles，octopuses and different kinds of electromagnetic analyzers. The optimization calculations and the graphic display for the calculated results are provided by the code.

Key words: intense 3D beam, nonlinear trajectory, Lie algebraic method，PIC scheme

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1 Introduction

There is growing interest in studying the detailed properties of intense charged particle beams for applications to many scientific research fields. In these applications, intense charged particle beams have to be efficiently transported over long distances. For intense charged particle 3D beams, the beam particles experience both transverse and longitudinal forces, which are the combination of external electromagnetic forces and space charge forces. For intense beam studies, it is important to take special care in determining the self-field potential.

We combine the Lie algebraic method with PIC scheme to solve the equations of particle motion.

In the phase space (x,p_x,y,p_y,z,p_z), the Hamiltonian of the particle motion is

$$H_{\rm t} = [m_0^2c^4+c^2(p_x-qA_x)^2+c^2(p_y-qA_y)^2 +c^2(p_z-qA_z)^2]^{\frac{1}{2}}+q\psi, \tag{1}$$

where x, y and z are the particle coordinates in the real space; p_x, p_y and p_z are the particle canonical momentum components; A_x, A_y and A_z are the magnetic vector components; ψ is the electric potential; q is the charge, m_0 is the rest mass. Now, take $\zeta=(x,x',y,\tau,p_\tau)$ as a new phase space, where $\tau=t-z/v_0$ is the time difference between the arbitrary particle and the reference particle, $p_\tau=p_{\rm t}-p_{\rm t}^0$, $p_{\rm t}=-H_{\rm t}$, $p_{\rm t}^0$ is the value of $p_{\rm t}$ for the reference particle. In this case, the Hamiltonian is

$$H = -[-(p_x-qA_x)^2-(p_y-qA_y)^2-(p_{\rm t}+p_{\rm t}^0+q\psi)^2/c^2 -m_0^2c^2]^{1/2}+qA_z-\frac{p_{\rm t}+p_{\rm t}^0}{v_0}. \tag{2}$$

In the phase space $\zeta=(x,\ x',\ y,\ \tau,\ p_\tau)$, the particle trajectories can be expressed as

$$\zeta_{f_i} = \sum_{j=1}^{6}M_{i,j}\zeta_j+\sum_{j=1}^{6}\sum_{k=1}^{6}S_{i,j,k}\zeta_j\zeta_k+\sum_{j=1}^{6}\sum_{k=1}^{6}\sum_{l=1}^{6}T_{i,j,k,l}\zeta_j\zeta_k\zeta_l +\cdots+\Delta\zeta_i, i=1,\cdots,6, \tag{3}$$

where, $M_{i,j}$, $S_{i,j,k}$ and $T_{i,j,k,l}$ are the first order, second order and the third order coefficients of the particle trajectories contributed by the applied electromagnetic fields, and $\Delta\zeta_i$ is the contribution of the space charge forces to the particle trajectories.

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The particle trajectories in the external fields are analyzed with the Lie algebraic method through third order, and the effects of space charge forces are calculated with the PIC scheme.

2 The Lie algebraic method

Expanding the Hamiltonian H into power series gives

$$H=H_0+H_1+H_2+H_3+H_4+\cdots. \quad (4)$$

In the phase space ζ, the final coordinate ζ_f and the initial coordinate ζ of a particle are related by a map M:

$$\varsigma_f=M\varsigma. \quad (5)$$

According to the Refs. [1] and [2], M is

$$M=\cdots M_4M_3M_2, \quad (6)$$

where

$$M_2=\exp(:f_2:),M_3=\exp(:f_3:),M_4=\exp(:f_4:),\cdots\cdots \quad (7)$$

and

$$f_2=-\int_0^{\ell}H_2(\varsigma,z)\mathrm{d}z, \quad (8)$$

$$f_3=-\int_0^{\ell}h_3^{\mathrm{int}}(\varsigma,z)\mathrm{d}z, \quad (9)$$

$$f_4=-\int_0^{\ell}h_4^{\mathrm{int}}(\varsigma,z)\mathrm{d}z, \quad (10)$$

$$h_n^{\mathrm{int}}(\varsigma,z)=M_2H_n. \quad (11)$$

The first order, second order, and third order terms of the orbit solutions are

$$\varsigma^1=f_2\varsigma, \quad (12)$$

$$\varsigma^2=f_3\varsigma^1\cdots\cdots, \quad (13)$$

$$\varsigma^3=:f_4:\varsigma^1+\frac{1}{2}:f_3:^2\varsigma^1=:f_4:\varsigma^1+\frac{1}{2}:f_3:\varsigma^2. \quad (14)$$

3 The PIC algorithms [3, 4]

The beam self-fields are calculated with the PIC scheme. The initial particle distributions in the 3D phase space are generated randomly. Two types of distribution can be chosen: uniform distribution and Gaussian distribution.

Generally speaking, to simulate the particle motion in the beam self-field with the PIC method, the following steps would be taken (see Fig. 1):

a) Randomly generate the particle distribution in the 3D phase space.

b) Suppose the beam moves along the straight line (ignoring the curvilinear orbit due to the short time step. Δt); divide the beam into cubic volumes; let h_x ,h_y , and h_z be the grid widths in the x, y and z directions respectively; the grid point numbers are N_x, N_y and N_z in the x, y and z directions.

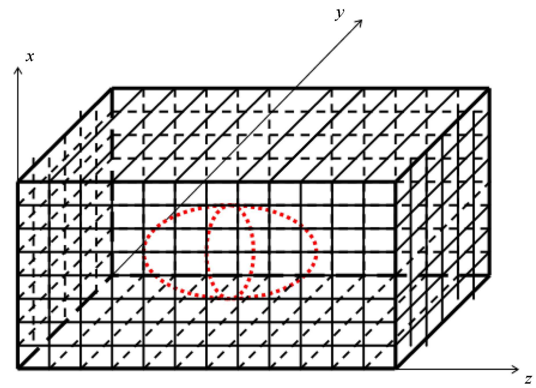

Fig. 1. Mesh generation.

c) The charge q of every macroparticle in a mesh cell is assigned to the nearest 8 grid points.

d) The potentials $\phi_{i.j,k}$ at the mesh points are calculated using the Green's function.

e) The electric fields at the mesh points are calculated with the central interpolation method:

$$E_x=-\frac{\phi_{i+1,j,k}-\phi_{i-1,j,k}}{2h_x},\ E_y=-\frac{\phi_{i,j+1,k}-\phi_{i,j-1,k}}{2h_y}, \quad E_z=-\frac{\phi_{i,j,k+1}-\phi_{i,j,k-1}}{2h_z}. \quad (15)$$

The electric fields at all particle positions are calculated by interpolating from the electric field at the grid points using the same weighting scheme as for the calculation of the charge density deposition.

f) From Newton's law, the contributions of the self-fields to the particle trajectories are:

$$\Delta x=\frac{1}{2}qE_x\Delta t^2/m,\Delta y=\frac{1}{2}qE_y\Delta t^2/m,\Delta z=\frac{1}{2}qE_z\Delta t^2/m, \quad (16)$$

where m is the particle mass, $\Delta t=\Delta z/v_0$, Δz is the step length along the reference orbit, v_0 is the velocity of the reference particle. The particle velocity changes are

$$\Delta v_x=qE_x\Delta t/m,\Delta v_y=qE_y\Delta t/m,\Delta v_z=qE_z\Delta t/m. \quad (17)$$

4 Computer code design

Combining the results obtained from the PIC scheme and the Lie algebraic method, we designed a computer code based on the program LEADS [5]. Many optical elements are contained in the code, such as electrostatic gap lens, Einzel lens, dc accelerating column, magnetic dipole, magnetic/electric quadrupole, sextupole, solenoid, cylindrical/spherical electrostatic analyzer, $\boldsymbol{E}\times\boldsymbol{B}$ Wien filter and drift space. Each element is divided into several small segments of length Δz. So, the time step is $\Delta z/v_0$, v_0 is the reference particle velocity. The optimization procedures are incorporated in the code to configure the system parameters and the graphical plotting is provided to show the simulation results.

5 Simulation example

With the new code, we calculated a beam transport system (see Fig. 2).

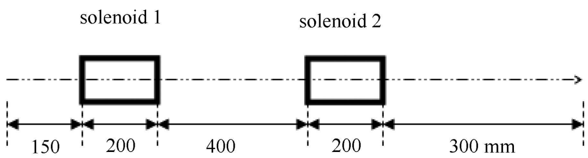

Fig. 2. Layout of the LEBT.

The system consists of two solenoid lenses and some drift spaces. The radii of the solenoids are 50 mm, the lengths of them are 200 mm, the longitudinal magnetic fields in the two solenoids are 4.77 kG.

The D^+ beam energy is 50 keV, the average bunched beam current is 6 mA, the initial parameters of the beam in the phase space are 1×40 mm·mrad both in the x- and y-directions. The pulse repetition is 9 MHz, and the initial longitudinal phase space is 1 keV×60°. The calculated beam envelopes and phase space diagrams at the end of the optical system are shown in Figs. 3 and 4 with the space charge effects off and on.

We can see from Figs. 3 and 4 that the beam envelopes in the cases (linear approximation and zero current, nonlinear approximation and zero current) have little difference. At the end of the system, they are 0.1 mm and 0.11 mm respectively. But if we take the space charge effect (6 mA beam current) and nonlinear effect into account, the beam envelope at the end of the system is 0.19 mm. The difference is ~90%. Also, the phase space diagram in the transverse direction for the 6 mA beam is strongly twisted.

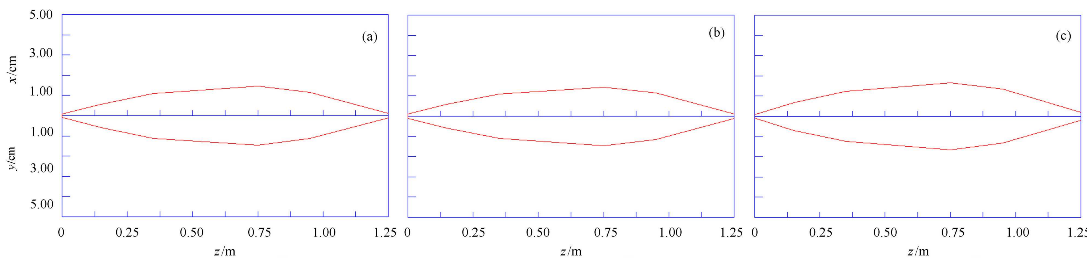

Fig. 3. (color online) Beam envelopes in the system. (a) Linear calculation and with space charge off; (b) Nonlinear calculation and with space charge off; (c) Nonlinear calculation and with space charge on.

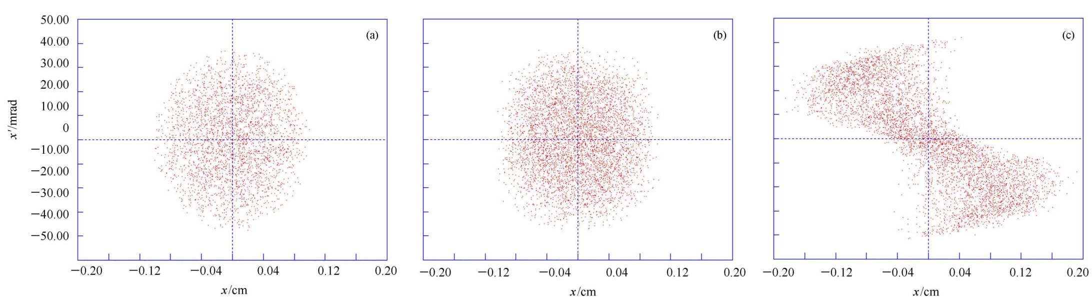

Fig. 4. (color online) Phase diagrams at the end of the system. (a) Linear calculation and with space charge off; (b) Nonlinear calculation and with space charge off; (c) Nonlinear calculation and with space charge off.

6 Conclusions

Space charge dominated 3D beams are analyzed with both the Lie algebraic and PIC methods. A computer code is developed based on the analysis. Many optical lenses are contained in the code. So, intense beam dynamics in the transport systems and in the high voltage accelerators can be simulated.

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