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Dynamical CP violation at finite temperature

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Abstract: By using the generalized Yang-Mills model, CP violation behavior at finite temperature is investigated, and it is shown that dynamical CP violation of the generalized Yang-Mills model at zero temperature can be restored at finite temperature.

Key words: CP violation, dynamical symmetry breaking, finite temperature

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1 Introduction

After the violation of parity symmetry proposed by Lee and Yang [1] was confirmed, it was argued that the elementary electric dipole moments would vanish due to the combined charge conjugation and parity symmetry, i.e., CP symmetry (or time reversal symmetry). However, it was then pointed out by Ramsey [2] and Jackson [3] that T invariance was also an assumption and needed to be checked experimentally. Since then the search for CP violation has been vigorously pursued. The CP violation was eventually discovered in the K^0 decays system by Val Fitch, James Cronin, and collaborators [4] in 1964. Shortly afterwards, in 1967, it was pointed out by Andre Sakharov that CP violation plays an important role in generating the baryon asymmetry in the Universe.

CP violation has two aspects in gauge theories. One is the standard CKM model [5, 6]. The other is the strong CP violation. The first one is a theoretical problem that must be solved in any realistic model of weak CP violation. The second one is the source of CP violation which arises in the strong interaction sector of the theory from the term $\theta(\alpha_s/8\pi)G\tilde{G}$, which is of topological origin. So far, it is well known that the CKM model does not provide a solution to the strong CP violation. And CP violation in the CKM model is not sufficient to generate the desired amount of baryon asymmetry in cosmology. In addition to these, there are other avenues which may reveal the existence of a new source of CP violation that exists in the Standard Model. In the past few decades a significant body of work on CP violation beyond the standard CKM model has appeared. It encompasses left-right symmetric models [7], spontaneous CP violation models [8–11], Timeon model [12] and so on.

In Ref. [13], the authors have constructed a maximally generalized Yang-Mills model (MGYMM). By considering the combination of the MGYMM and the NJL mechanism [14], we have proposed a new mechanism of CP violation in Ref. [15]. Further, in the present paper, for a purely theoretical consideration, we investigate the behavior of our previous CP violation model at finite temperature which will be necessary to explain the desired baryon asymmetry in the Universe. We show that dynamical CP violation at zero temperature can be restored at finite temperature.

2 Dynamical CP violation of the generalized Yang-Mills model

In this section, we will review dynamical CP violation of the generalized Yang-Mills model which we proposed in Ref. [15]. In the usual Yang-Mills theory gauge invariance is assured through the demand that vector gauge transform as $\gamma_\mu V_\mu \to U(\gamma_\mu V_\mu)U^{-1} - (\gamma_\mu \partial_\mu U)U^{-1}$. In Ref. [13], the authors have again studied the Yang-Mills theory and constructed a max-

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imally generalized Yang-Mills model in which an axial-vector field A_μ, a scalar field S, a pseudoscalar field P and a tensor field $T_{\mu\nu}$ are considered to also be acceptable as gauge fields. The main idea of the MGYMM is as follows. Considering a Lagrangian is invariant under a Lie group with N generators; corresponding to each generator of the Lie group there is one gauge field, and it does not matter whether they are vector fields or other fields. One can choose the first $N_{\rm V}$ to be associated with an equal number of vector gauge fields and the last N' to be associated with an equal number of the other fields. Naturally $N_{\rm V}+N'=N$. By taking each of the generators and multiplying it by one of its associated gauge fields and summing them together, the authors construct a maximally generalized Dirac covariant derivative D as

$$D=\gamma_\mu\partial_\mu-{\rm i}\gamma_\mu V_\mu+\Phi, \tag{1}$$

with $\Phi = S+{\rm i}\gamma_5P-{\rm i}\gamma_\mu A_\mu\gamma_5+\sigma_{\mu\nu}T_{\mu\nu}$ being the generic gauge field in which $S=gS^cT^c$ is a scalar field, $P=gP^bT^b$ a pseudoscalar field, $V_\mu=gV_\mu^aT^a$ a vector field, $A_\mu=gA_\mu^dT^d$ an axial-vector field and $T_{\mu\nu}=gT_{\mu\nu}^eT^e$ a tensor field, the superscript a varies from 1 to $N_{\rm V}$, b varies from $N_{\rm V}+1$ to $N_{\rm V}+N_{\rm P}$, c varies from $N_{\rm V}+N_{\rm P}+1$ to $N_{\rm V}+N_{\rm P}+N_{\rm S}$, d varies from $N_{\rm V}+N_{\rm P}+N_{\rm S}+1$ to $N_{\rm V}+N_{\rm P}+N_{\rm S}+N_{\rm A}$ and the superscript e varies from $N_{\rm V}+N_{\rm P}+N_{\rm S}+N_{\rm A}+1$ to $N_{\rm V}+N_{\rm P}+N_{\rm S}+N_{\rm A}+N_{\rm T}$. By defining the transformation for the gauge fields as

$$-{\rm i}\gamma_\mu V_\mu+\Phi\rightarrow U(-{\rm i}\gamma_\mu V_\mu+\Phi)U^{-1}-(\gamma_\mu\partial_\mu U)U^{-1}, \tag{2}$$

one can obtain that $D\rightarrow UDU^{-1}$. Then the authors build up the Lagrangian which contains only the matter fields and covariant derivatives, and possesses both the Lorentz and gauge invariance

$$L=-\bar\Psi D\Psi+\frac{1}{2g^2}\tilde{\rm Tr}\left(\frac{1}{8}({\rm Tr}D^2)^2-\frac{1}{2}{\rm Tr}D^4\right), \tag{3}$$

where the trace with the tilde is over the gauge (or the Lie group) matrices and the one without the tilde is over matrices of the spinorial representation of the Lorentz group. The expansion of Eq. (3) reads

$$\begin{aligned}L\ =\ &-\bar\Psi D\Psi-\frac{1}{2g^2}\tilde{\rm Tr}(\partial_\mu V_\nu-\partial_\nu V_\mu-{\rm i}[V_\mu,V_\nu])^2\\&-\frac{1}{4g^2}\tilde{\rm Tr}\left[{\rm Tr}(\gamma_\mu\partial_\mu\Phi-{\rm i}\{\gamma_\mu V_\mu,\Phi\})^2\right].\end{aligned} \tag{4}$$

As in the usual Yang-Mills theories, when D acts on the matter field Ψ, its gauge fields are going to be multiplied by constants (the charges) $Q_{\rm V}$, $Q_{\rm S}$, $Q_{\rm P}$, $Q_{\rm A}$ and $Q_{\rm T}$ with the result $D\Psi=(\gamma_\mu\partial_\mu-{\rm i}Q_{\rm V}\gamma_\mu V_\mu+Q_{\rm S}S+{\rm i}Q_{\rm P}\gamma_5P-{\rm i}Q_{\rm A}\gamma_\mu A_\mu\gamma_5+Q_{\rm T}\sigma_{\mu\nu}T_{\mu\nu})\Psi$. From the Standard Model one can conclude that $Q_{\rm V}=1$.

In Ref. [15], by using the above MGYMM with $\Phi=S+{\rm i}\gamma_5P$, we have investigated dynamical CP violation. Taking $\Phi=S+{\rm i}\gamma_5P$ in the covariant derivative Eq. (1), then Eq. (4) changes to be

$$\begin{aligned}L\ =\ &-\bar\Psi\gamma_\mu(\partial_\mu-{\rm i}V_\mu)\Psi-\bar\Psi(Q_{\rm S}S+{\rm i}Q_{\rm P}\gamma_5P)\Psi\\&-\frac{1}{2g^2}\tilde{\rm Tr}(\partial_\mu V_\nu-\partial_\nu V_\mu-{\rm i}[V_\mu,V_\nu])^2\\&-\frac{1}{4g^2}\tilde{\rm Tr}\Big\{{\rm Tr}[\gamma_\mu\partial_\mu(S+{\rm i}\gamma_5P)\\&-{\rm i}\{\gamma_\mu V_\mu,(S+{\rm i}\gamma_5P)\}]^2\Big\}.\end{aligned} \tag{5}$$

This Lagrangian is invariant under CP and T transformations. For convenience, we neglect the interaction terms between the scalar field and the pseudoscalar field. Then the Lagrangian density (5) becomes

$$\begin{aligned}L\ =\ &-\bar\Psi\gamma_\mu(\partial_\mu-{\rm i}V_\mu)\Psi-\bar\Psi(Q_{\rm S}S+{\rm i}Q_{\rm P}\gamma_5P)\Psi\\&-\frac{1}{2g^2}\tilde{\rm Tr}(\partial_\mu V_\nu-\partial_\nu V_\mu-{\rm i}[V_\mu,V_\nu])^2\\&-\frac{1}{g^2}\tilde{\rm Tr}\left[(\partial_\mu S-{\rm i}\{V_\mu,S\})^2+(\partial_\mu P-{\rm i}[V_\mu,P])^2\right].\end{aligned} \tag{6}$$

Looking at Eq. (6), we can find that this model does not include the Higgs potential term V as usual. Despite this, by using the NJL mechanism the symmetry breaking of this model can be realized dynamically.

By substituting the Lagrangian density (6) into the Euler equation, we obtain the equation of motion as

$$\gamma_\mu(\partial_\mu-{\rm i}gV_\mu^aT^a)\Psi+(G_{\rm S}S^cT^c+{\rm i}G_{\rm P}\gamma_5P^bT^b)\Psi=0, \tag{7}$$

$$(\partial_\mu^2-g^2d_{\rm S}V_\mu^aV_\mu^a)S^c-G_{\rm S}\bar\Psi T^c\Psi=0, \tag{8}$$

$$(\partial_\mu^2-g^2f_{\rm P}V_\mu^aV_\mu^a)P^b-{\rm i}G_{\rm P}\bar\Psi\gamma_5T^b\Psi=0, \tag{9}$$

$$\begin{aligned}&(\partial_\mu F_{\mu\nu}^a+gf^{abc}V_\mu^bF_{\mu\nu}^c)+[g^2d_{\rm S}(S^c)^2\\&+g^2f_{\rm P}(P^b)^2]V_\nu^a-{\rm i}g\bar\Psi\gamma_\nu T^a\Psi=0,\end{aligned} \tag{10}$$

in which $G_{\rm S}=gQ_{\rm S}$, $G_{\rm P}=gQ_{\rm P}$, $F_{\mu\nu}^a=\partial_\mu V_\nu^a-\partial_\nu V_\mu^a+gf^{abc}V_\mu^bV_\nu^c$, $d_{\rm S}=d^{abc}d^{abc}$ and $f_{\rm P}=f^{abc}f^{abc}$ (in $d_{\rm S}$ and $f_{\rm P}$, where a varies from 1 to $N_{\rm V}$, b varies from $N_{\rm V}+1$ to $N_{\rm V}+N_{\rm P}$, and c varies from $N_{\rm V}+N_{\rm P}+1$ to $N_{\rm V}+N_{\rm P}+N_{\rm S}$). Multiplying the left- and right-hand side of Eq. (10) by V_ν^a, and then taking the vacuum expectation value of it, to the lowest-order approximation in $\hbar$, we have [16–18]

$$f_{\rm V}\langle V_\mu^aV_\mu^a\rangle=d_{\rm S}\langle(S^c)^2\rangle+f_{\rm P}\langle(P^b)^2\rangle, \tag{11}$$

with $f_{\rm V}=f^{abc}f^{abc}$ (a, b, c vary from 1 to $N_{\rm V}$). Here one can choose the scalar bosons S^{c_1} and P^{b_1} to be associated with the unit generator $T^{c_1}=T^{b_1}=1/\sqrt{2N_{\rm d}}$ times the $N_{\rm d}\times N_{\rm d}$ unit matrix ($N_{\rm d}$: the dimensions of the fundamental representation), and if there is no unit generator in the Lie group one can introduce a unit one to it. We denote the vacuum expectation of the scalar bosons as

$$\langle S^c\rangle=\langle S^{c_1}\rangle\neq 0,\quad \langle P^b\rangle=\langle P^{b_1}\rangle\neq 0. \tag{12}$$

Looking at the Lagrangian (6), we can conclude that the nonzero expectation value of the pseudoscalar field P^b implies CP violation.

In order to exhibit more clearly the CP-violating character of the model, we may perform a unitary transformation under which P^b is unchanged, but $\Psi\rightarrow {\rm e}^{-{\rm i}\frac{1}{2}\gamma_5\alpha}\Psi$. Hence, in Eq. (7), by choosing

$$\tan\alpha=\frac{G_{\rm P}\langle P^b\rangle T^b}{G_{\rm S}\langle S^c\rangle T^c}, \tag{13}$$

we have

$$\bar{\Psi}(G_{\rm S}\langle S^c\rangle T^c+{\rm i}G_{\rm P}\gamma_5\langle P^b\rangle T^b)\Psi\rightarrow\bar{\Psi}M_{\Psi}\Psi, \tag{14}$$

in which the fermion mass

$$M_{\Psi}=\left[\frac{1}{2N_{\rm d}}(G_{\rm s}\langle S^{c_1}\rangle)^2+\frac{1}{2N_{\rm d}}(G_{\rm P}\langle P^{b_1}\rangle)^2\right]^{\frac{1}{2}}. \tag{15}$$

Then after taking the vacuum expectation values of Eq. (8) and Eq. (9) to the lowest-order approximation in $\hbar$, we obtain the self-consistency equations as

$$M_{\rm S}^2\langle S^{c_1}\rangle = -G_{\rm S}\sqrt{\frac{1}{2N_{\rm d}}}\langle\bar{\Psi}\Psi\rangle={\rm i}G_{\rm S}\sqrt{\frac{1}{2N_{\rm d}}}{\rm Tr}S_{\rm F}(0), \tag{16}$$

$$\begin{aligned}M_{\rm P}^2\langle P^{b_1}\rangle &= -{\rm i}G_{\rm P}\sqrt{\frac{1}{2N_{\rm d}}}\langle\bar{\Psi}\gamma_5\Psi\rangle\\ &= -G_{\rm P}\sqrt{\frac{1}{2N_{\rm d}}}{\rm Tr}[\gamma_5 S_{\rm F}(0)],\end{aligned} \tag{17}$$

in which $M_{\rm S}^2=g^2d_{\rm S}\langle V_\mu^aV_\mu^a\rangle$, $M_{\rm P}^2=g^2f_{\rm P}\langle V_\mu^aV_\mu^a\rangle$.

Comparing Eq. (16) with Eq. (17), we have

$$\langle S^{c_1}\rangle=-\frac{{\rm i}f_{\rm P}G_{\rm S}{\rm Tr}S_{\rm F}(0)}{d_{\rm S}G_{\rm P}{\rm Tr}[\gamma_5S_{\rm F}(0)]}\langle P^{b_1}\rangle=-\frac{f_{\rm P}G_{\rm S}}{d_{\rm S}G_{\rm P}\tan\alpha}\langle P^{b_1}\rangle. \tag{18}$$

By substituting Eq. (18) into Eq. (15), we have

$$M_{\Psi}=\eta\langle P^{b_1}\rangle, \tag{19}$$

with

$$\eta=\frac{1}{d_{\rm S}G_{\rm P}\tan\alpha}\sqrt{\frac{f_{\rm P}^2G_{\rm S}^4+d_{\rm S}^2G_{\rm P}^4\tan^2\alpha}{2N_{\rm d}}}.$$

By substituting Eq. (11) and Eq. (18) into Eq. (17), we can rewrite the self-consistency equation as

$$\begin{aligned}&(g^2f_{\rm P}^3G_{\rm S}^2+g^2d_{\rm S}f_{\rm P}^2G_{\rm P}^2\tan^2\alpha)\langle P^{b_1}\rangle^3\\ &= -f_{\rm V}d_{\rm S}G_{\rm P}^3\tan^2\alpha\sqrt{\frac{1}{2N_{\rm d}}}{\rm Tr}\left[\gamma_5S_{\rm F}(0)\right]\\ &= -f_{\rm V}d_{\rm S}G_{\rm P}^3\tan^2\alpha\sqrt{\frac{1}{2N_{\rm d}}}{\rm Tr}\int\frac{{\rm d}^4p}{(2\pi)^4}\frac{\gamma_5}{-{\rm i}\gamma_\mu p_\mu-(G_{\rm S}\langle S^{c_1}\rangle T^{c_1}+{\rm i}G_{\rm P}\gamma_5\langle P^{b_1}\rangle T^{b_1})}\\ &= -f_{\rm V}d_{\rm S}G_{\rm P}^3\tan^2\alpha\sqrt{\frac{1}{2N_{\rm d}}}{\rm Tr}\int\frac{{\rm d}^4p}{(2\pi)^4}\frac{\gamma_5[{\rm i}\gamma_\mu p_\mu-(G_{\rm S}\langle S^{c_1}\rangle T^{c_1}+{\rm i}G_{\rm P}\gamma_5\langle P^{b_1}\rangle T^{b_1})]}{p^2+M_{\Psi}^2}\\ &= -\frac{2\langle P^{b_1}\rangle}{N_{\rm d}}f_{\rm V}d_{\rm S}G_{\rm P}^4\tan^2\alpha\int\frac{{\rm d}^4p}{(2\pi)^4}\frac{\rm i}{p^2+M_{\Psi}^2}.\end{aligned} \tag{20}$$

From Eq. (18), Eq. (19) and Eq. (20) we can finally obtain that the non-vanishing vacuum expectation values of the scalar field S and the pseudoscalar field P are completely determined by the self-energy of the fermions. Note that Eq. (20) and Eq. (18) determine the magnitude and the direction of the vacuum state $C_0=(\langle S^c\rangle,\ \langle P^b\rangle)$. Evidently, the CP and T symmetry is broken dynamically, but the product CPT symmetry remains intact. The amplitude of the CP violation [19] is

$$A_-=\frac{G_{\rm P}^2\sin\alpha\cos\alpha}{2N_{\rm d}(k^2+M_{\rm P}^2)}, \tag{21}$$

where k denotes the 4-momentum transfer. From Eq. (21) we see that the CP violation amplitude A_- depends on the vacuum expectation value $\langle P^{b_1}\rangle$ and we can also see that the CP and T symmetry can be restored when $\langle P^{b_1}\rangle\rightarrow 0$.

3 *CP* violation at finite temperature

In this section we will investigate the behavior of CP violation when we take temperature into account. Let us now assume that the system is at finite temperature $T=1/b$. In this case we can use Matsubara formalism which consists of taking $p_0=(2n+1)\pi/b$ (n integer for fermions) and changing $(1/2\pi)\int \mathrm{d}p_0 \rightarrow \frac{\mathrm{i}}{\beta}\sum_{n=-\infty}^{\infty}$ [20]. At the same time we also change the system from Minkowski space to Euclidean space. Then the self-consistency Eq. (20) at finite temperature can be written as

$$\begin{aligned}
&[g^2 f_{\mathrm{P}}^3 G_{\mathrm{S}}^2+g^2 d_{\mathrm{S}} f_{\mathrm{P}}^2 G_{\mathrm{P}}^2 \tan^2\alpha(\beta)]\langle P^{b_1}(\beta)\rangle^2 \\
&= \frac{2 f_{\mathrm{V}} d_{\mathrm{S}} G_{\mathrm{P}}^4 \tan^2\alpha(\beta)}{N_{\mathrm{d}}} \frac{1}{\beta} \sum_{n=-\infty}^{\infty} \int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3} \frac{1}{\vec{p}^{\,2}+(2n+1)^2\pi^2/\beta^2+M_{\Psi}^2(\beta)} \\
&= \frac{f_{\mathrm{V}} d_{\mathrm{S}} G_{\mathrm{P}}^4 \tan^2\alpha(\beta)}{4N_{\mathrm{d}}\pi^3} \frac{1}{\beta} \sum_{n=-\infty}^{\infty} \int \frac{\mathrm{d}^3\vec{p}}{(2n+1)^2\pi^2/\beta^2+\vec{p}^{\,2}+\eta^2\langle P^{b_1}(\beta)\rangle^2}.
\end{aligned} \tag{22}$$

We first calculate the summation in Eq. (22):

$$\sum_{n=-\infty}^{\infty} \frac{1}{(2n+1)^2\pi^2/\beta^2+\vec{p}^{\,2}+\eta^2\langle P^{b_1}(\beta)\rangle^2} = \frac{\beta^2}{\pi^2} \sum_{n=-\infty}^{\infty} \frac{1}{(2n+1)^2+a^2}, \tag{23}$$

with $a^2=\frac{\beta^2}{\pi^2}\left(\vec{p}^{\,2}+\eta^2\langle P^{b_1}(\beta)\rangle^2\right)$. The summation in Eq. (23) can be simplified to

$$\sum_{n=-\infty}^{\infty} \frac{1}{(2n+1)^2+a^2} = \sum_{n=0}^{\infty} \frac{2}{(2n+1)^2+a^2} = 2\sum_{n=1}^{\infty} \frac{1}{n^2+a^2} - \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n^2+(a/2)^2}. \tag{24}$$

By using

$$\sum_{n=1}^{\infty} \frac{1}{n^2+a^2} = \frac{\pi}{2a} \coth \pi a - \frac{1}{2a^2}, \tag{25}$$

and

$$\coth \pi a = \frac{1}{2}[\tanh(\pi a/2)+\coth(\pi a/2)], \tag{26}$$

one gets

$$\sum_{n=-\infty}^{\infty} \frac{1}{(2n+1)^2+\mathrm{a}^2} = \frac{\pi}{2\mathrm{a}} \tanh(\pi a/2) = \frac{\pi}{2\mathrm{a}}\left[1-\frac{2}{\exp(\pi a)+1}\right]. \tag{27}$$

Thus, the self-consistency equation at finite temperature, Eq.(22), can be reduced to

$$\begin{aligned}
&[g^2 f_{\mathrm{P}}^3 G_{\mathrm{S}}^2+g^2 d_{\mathrm{S}} f_{\mathrm{P}}^2 G_{\mathrm{P}}^2 \tan^2\alpha(\beta)]\langle P^{b_1}(\beta)\rangle^2 \\
&= \frac{f_{\mathrm{V}} d_{\mathrm{S}} G_{\mathrm{P}}^4 \tan^2\alpha(\beta)}{4N_{\mathrm{d}}\pi^3} \frac{1}{\beta} \sum_{n=-\infty}^{\infty} \int \frac{\mathrm{d}^3\vec{p}}{(2n+1)^2\pi^2/\beta^2+\vec{p}^{\,2}+\eta^2\langle P^{b_1}(\beta)\rangle^2} \\
&= \frac{f_{\mathrm{V}} d_{\mathrm{S}} G_{\mathrm{P}}^4}{8N_{\mathrm{d}}\pi^3} \int \mathrm{d}^3\vec{p} \left(\frac{\tan^2\alpha(\beta)}{\left(\vec{p}^{\,2}+\eta^2\langle P^{b_1}(\beta)\rangle^2\right)^{1/2}} - \frac{2\tan^2\alpha(\beta)}{\left(\vec{p}^{\,2}+\eta^2\langle P^{b_1}(\beta)\rangle^2\right)^{1/2} \left\{\exp\left[\beta\left(\vec{p}^{\,2}+\eta^2\langle P^{b_1}(\beta)\rangle^2\right)^{1/2}\right]+1\right\}} \right) \\
&= \frac{f_{\mathrm{V}} d_{\mathrm{S}} G_{\mathrm{P}}^4 \tan^2\alpha(\beta)}{8N_{\mathrm{d}}\pi^3}(A-B),
\end{aligned} \tag{28}$$

with

$$A = \int_0^{\infty} \frac{4\pi\left|\vec{p}\right|^2 \mathrm{d}\left|\vec{p}\right|}{\left(\vec{p}^{\,2}+\eta^2\langle P^{b_1}(\beta)\rangle^2\right)^{1/2}}, \tag{29}$$

$$B = \int_0^{\infty} \frac{8\pi\left|\vec{p}\right|^2 \mathrm{d}\left|\vec{p}\right|}{\left(\vec{p}^{\,2}+\eta^2\langle P^{b_1}(\beta)\rangle^2\right)^{1/2} \left\{\exp\left[\beta\left(\vec{p}^{\,2}+\eta^2\langle P^{b_1}(\beta)\rangle^2\right)^{1/2}\right]+1\right\}}. \tag{30}$$

The integration in A is divergent. Introducing an invariant momentum cut-off Λ^2one can make the integration finite. The result is

$$A=2\pi\left[\Lambda^2-\eta^2(\beta)\langle P^{b_1}(\beta)\rangle^2\ln\left(\frac{\Lambda^2}{\eta^2(\beta)\langle P^{b_1}(\beta)\rangle^2}+1\right)\right]. \tag{31}$$

Here we are interested in the CP symmetry behavior at finite temperature and wish to find the critical temperature T_{C} at which the vacuum expectation value $\langle P^{b_1}(\beta_{\mathrm{C}})\rangle$ tends to zero. So the integration in B can be calculated in the approximation $\langle P^{b_1}(\beta_{\mathrm{C}})\rangle=0$ as

$$B=\int_0^\infty\frac{8\pi\left|\vec{p}\right|\mathrm{d}\left|\vec{p}\right|}{\exp\beta\left|\vec{p}\right|+1}=\frac{2\pi^3}{3\beta^2}. \tag{32}$$

In the approximation $\langle P^{b_1}(\beta_{\mathrm{C}})\rangle=0$, A changes to be

$$A=2\pi\Lambda^2. \tag{33}$$

By substituting Eq. (32) and Eq. (33) into Eq. (28), we have

$$T_{\mathrm{c}}=\frac{1}{b_{\mathrm{c}}}=\frac{\sqrt{3}L}{p}. \tag{34}$$

Thus, we have obtained the critical temperature T_{c} at which the vacuum expectation value $\langle P^{b_1}(\beta_{\mathrm{C}})\rangle$ tends to zero, which means that the dynamical breaking of the CP and T symmetry at zero temperature can be restored at finite temperature. This result is the same as those of Ref. [17] and Refs. [21–23], which shows us that the critical temperature of the phase transformation in dynamical symmetry breaking model is only related to the momentum cut-off Λ.

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References

1 LEE T D, YANG C N. Phys. Rev., 1956, **104**: 254
2 Ramsey N F. Phys. Rev., 1958, **109**: 222
3 Jackson J D et al. Phys. Rev., 1957, **106**: 517
4 Christenson J H et al. Phys. Rev. Lett., 1964, **13**: 138
5 Cabbibo C. Phys. Rev. Lett., 1963, **10**: 531
6 Kobayashi M, Maskawa T. Pro. Theor. Phys., 1973, **49**: 652
7 Pati J C, Salam A. Phys. Rev. D, 1974, **10**: 275
8 Branco G C. Phys. Rev. Lett., 1980, **44**: 504
9 LIU J, Wolfenstein L. Nucl. Phys. B, 1987, **289**: 1
10 WU Y L, Wolfenstein L. Phys. Rev. Lett., 1994, **73**: 1762
11 WU Y L, ZHOU Y F. Phys. Rev. D, 2000, **61**: 096001
12 Friedberg R, LEE T D. arXiv: 0809.3633vl [hep-ph]
13 WANG D F, SONG H S. Comm. Theor. Phys., 2006, **45**: 297
14 Nambu Y, Jona-Lasinio G. Phys. Rev, 1961, **122**: 345
15 WANG D F et al. Chin. Phys. C (HEP & NP), 2011, **35**(2): 130
16 WANG D F, SONG H S, MI D. Comm. Theor. Phys., 2004, **41**: 729
17 WANG D F, SONG H S, MI D. Comm. Theor. Phys., 2004, **42**: 385
18 WANG D F, SONG H S, AN Y. Chin. Phys. Lett., 2004, **21**: 1453
19 LEE T D. Particle Physics and Introductation to Field Theory. Beijing: Scientific Publishers, 1980. 232 (in Chinese)
20 Dolan L, Jackiw R. Phys. Rev. D, 1974, **9**: 3320
21 WANG D F, SONG H S, AN Y. Mod. Phys. Lett. A, 2002, **17**: 453
22 SONG H S, XU G N, AN Y. J Phys. A: Math. Gen., 1993, **26**: 4463
23 WANG D F, SONG H S. Chin. Phys. Lett., 2005, **22**: 2194