Investigation of the rescattering effect in D decay*

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Abstract: With two-body unitarity equations, we demonstrate the relation between the data of Dalitz analysis of a $D^+ \to K^-\pi^+\pi^+$ decay experiment and that of $K\pi$ scattering, and point out that there might be some underestimated ambiguity in the existing data sets, if the $I=1/2$ component of the $K\pi$ system is dominant in this decay process. It is suggested that the unitarity constraints should be built in to deal with the raw data to obtain an improved result from the Dalitz analysis.

Key words: D decay, πK scatterings, dispersion relation, unitarity

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1 Introduction

The controversial low lying scalar resonances have attracted much interest in phenomenological study for their deep relations with chiral symmetry breaking and non-perturbative aspects of QCD. On the experimental side, FermiLab [1, 2] and the BES collaboration [3–5] have found evidence for their existence. Theoretical studies based on some model-independent methods by analyzing the low energy $\pi\pi$ scattering phase shift can determine a second-sheet σ pole quite acurrately [6, 7]. The analysis of κ (or $K_0^*(800)$) has also been carried out as in the references listed in the Particle Data Group(PDG) Table [8]. However, most theoretical studies of κ pole are based on the old LASS experiment, in which the data points start from about 825 MeV due to the difficulty in analyzing the scattering process, but the pole position determined by them is usually lower and close to the $K\pi$ threshold. The lower data, especially those close to the threshold, are required to provide a solid foundation for those analyses and give more information about the resonance. The E791 group has made a pioneering model-independent partial wave analysis (MIPWA) to extract the S-wave component of the $K\pi$ system from the $D^+ \to K^-\pi^+\pi^+$ decay process [9], which provides a possibility of determining the $K\pi$ scattering phase close to the threshold. The FOCUS group also applied the MIPWA method to their higher statistic data [10]. Both of them have observed phase variations that clearly do not match the old measurements of the $I=1/2$ $K\pi$ scattering phase, which implies that the Watson's theorem could not apply to such weak decays with strong interactions of the final three particles. To find correct correspondence between the observations of the decay process and the scattering process, the discontinuity relations along the physical cuts, which represent the unitarity constraints of physical amplitudes, might be an unavoidable tool to bridge them, as suggested by Pennington [11].

2 The discontinuity relations in $D^+ \to K^-\pi^+\pi^+$

We consider the decay process $D^+(p) \to K^-(p_1) + \pi^+_a(p_2) + \pi^+_b(p_3)$, and define the Mandelstam variables as $s=(p_1+p_2)^2, t=(p_1+p_3)^2, u=(p_2+p_3)^2$, which satisfy $s+t+u=m_D^2+m_K^2+2m_\pi^2$. To make it clear, we label the final states $K^-\pi^+\pi^+$ as particle 1, 2 and 3, respectively. Go to the (12) c.m. frame, in which the initial state D^+ has three-momentum $\boldsymbol{q}$, as has the particle $\pi^+(p_3)$, while particle $K^-(p_1)$ and $\pi^+(p_2)$ have three-momentum $\boldsymbol{p}$ and -$\boldsymbol{p}$, respectively. The magnitudes of these momentum p and q satisfy

$$q^2=\lambda(s,m_D^2,m_\pi^2)/4s,\ p^2=\lambda(s,m_K^2,m_\pi^2)/4s, \quad (1)$$

where $\lambda(x,y,z)=x^2+y^2+z^2-2xy-2yz-2zx$. Due to the two identical pions, the full amplitude of the

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decay process is s↔t symmetric and reads

$$A(s,t)=A_{\rm s}(s,t)+A_{\rm t}(t,s)+A_{\rm u}(u,s), \qquad (2)$$

with $A_{\rm s}(s,t)=A_{\rm t}(s,t)$ and $A_{\rm u}(u,s)$ being s-t symmetric. The s-channel and t-channel amplitudes are conventionally partial-wave projected as $A_{\rm s}(s,t)=\sum_l a_l(s)P_l(\cos\theta_s)$, $A_t(t,s)=\sum_l b_l(t)P_l(\cos\theta_t)$, where $\theta_{\rm s}(=\theta_{13})$ is the angle between the three-momenta of $\rm K^-$ and $\pi_{\rm b}$ in (12) the rest system and $\theta_{\rm t}(=\theta_{12})$ is the angle between the three-momenta of $\rm K^-$ and π_a in (13) the rest system. a_l and b_l are the same functions with respect to different arguments in this process.

There has long been a body of work on multiparticle interactions [12], particularly by Ascoli and collaborators on 3-pion final states dating from the discovery of a_1 particle and its possible structures [13]. Much more recently, Caprini has shown that one can deduce a unitarity relation with rescattering [14], implicit in these studies. Neglecting those contributions from three-particle and more intermediate states, the discontinuity of the decay amplitude with respect to the variable s reads

$$\frac{1}{2{\rm i}}\{A(s+{\rm i}\epsilon,t)-A(s-{\rm i}\epsilon,t)\} = \frac{1}{8\pi^2}\int\frac{{\rm d}\boldsymbol{k}_1}{2\omega_1}\frac{{\rm d}\boldsymbol{k}_2}{2\omega_2}\delta^4(P)T^*(s,t')A(s,t''), \qquad (3)$$

where $P=p_1+p_2-k_1-k_2$, $t'=(p_1-k_1)^2$ and $t''=(k_1+p_3)^2$, respectively. Assuming the pole dominance of the amplitude and using standard partial-wave expansion techniques, the imaginary part of each partial wave amplitude is obtained as:

$$\begin{aligned}&\frac{1}{2{\rm i}}[a_l(s+{\rm i}\epsilon)-a_l(s-{\rm i}\epsilon)]\\ &=\rho(s)(t_l(s))^*\left\{a_l(s)+\frac{2l+1}{2}\int{\rm d}z_s(s,t')P_l(z_s(s,t'))\left[A_t(t',s)+A_u(u',s)\right]\right\}\\ &=\rho(s)(t_l(s))^*\left\{a_l(s)+\frac{2l+1}{2}\int{\rm d}z_s(s,t')P_l(z_s(s,t'))\left[\sum_{l'}P_{l'}(z_t(t',s))b_{l'}(t')+\sum_{l'}P_{l'}(z_u(u',s))c_{l'}(u')\right]\right\}. \qquad (4)\end{aligned}$$

To make the formula less complicated, the isospin indices are omitted and this point is addressed later. If the integral part of the r.h.s. of Eq. (4) is neglected, one immediately obtains the well-known Watson's theorem: the phase of $a_l(s)$ is equal to the phase shift $\delta_l(s)$. However, these two phases are different when the rescattering effects are taken into account. As in the process we are interested in here, the u-channel part can be omitted, because $\pi_a^+\ \pi_b^+$ coupling is not expected to be significant, as verified by the experiments. Thus, with the isospin index denoted, Eq. (4) turns out to be

$$\begin{aligned}{\rm Im}a_l^I(s) &= \frac{1}{2{\rm i}}[a_l^I(s+{\rm i}\epsilon)-a_l^I(s-{\rm i}\epsilon)]\\ &=\rho(s)(t_l^I(s))^*\{a_l^I(s)+h_l^I(s)\}, \qquad (5)\end{aligned}$$

where

$$h_l^I(s)=\frac{2l+1}{2}\int{\rm d}z_s(s,t')P_l(z_s(s,t'))A_t^{\{I\}}(t',s). \qquad (6)$$

It is worth emphasizing that the superscript I of $A_t^{\{I\}}$ here is still the total isospin of K and π_a but not K and π_b. The recoupling coefficients $C_{II'}$ to ensure $A_t^{\{I\}}(t,s)=\sum_{I'}C_{II'}A_t^I(t,s)$ can be obtained through the standard angular-momentum coupling procedures. It means that the $I=1/2$ and $I=3/2$ parts will be entangled with each other in principle, and the data needed for applying such an ambitious analysis are precise.

The well studied $\Delta I=1/2$ rule in the decay $\rm K\to\pi\pi$ teaches us that the $\Delta I=1/2$ components dominate over $\Delta I=3/2$ transitions and such kinds of dominance are common to both kaon and hyperon decays. This is in keeping with the quark line picture that an s-quark changing to a u-quark is dominated by a weak process. In D decay to Kππ, the c-quark changes into an s-quark by emitting an off-shell $\rm W^+$ boson that materializes as a π^+. Thus, it might be a good approximation to suppose that the $I=1/2$ part dominates the Kπ system.

Eq. (5) represents two constraints with its real part and imaginary part, respectively. The imaginary-part constraint (referred as IMC later) has a concise form,

$$\delta_l\equiv{\rm Arg}[t_l^I]={\rm Arg}[a_l^I+h_l^I]\pm n\pi,\quad n=0,1,2\cdots \qquad (7)$$

and the real-part (as REC) has a more complicated form,

$$\delta_l=\frac{1}{2}\left(\Psi_l+{\rm ArcSin}\left[\frac{2{\rm Im}a_l}{|g_l|}-{\rm Sin}[\Psi_l]\right]\right), \qquad (8)$$

where $g_l=a_l+h_l$ and $\Psi_l={\rm Arg}[a_l+h_l]$. IMC is more convenient for practical applications, but care must be taken to obtain the correct n value by re-inputting it into Eq. (5). REC does not work well while the argument of inverse Sine function is out of range $[-1,1]$, when the representation of $A(s,t)$ violates unitarity.

3 Numerical analysis

If the MIPWA fit to the $D^+ \to K^-\pi^+\pi^+$ has only a constant global phase ambiguity, one can, when assuming the S-wave component is dominated by I=1/2 part, simply apply the unitarity formula to compute the Kπ I=1/2 scattering phase shift of all the partial waves. The total amplitude can be obtained with polynomial representations of S-wave phase and magnitude and the isobar-model descriptions of the reference waves, P- and D- waves, whose parameters are from the experimental fit. With the analytic formula of $A(s,t)$, the re-scattering integral, $h_l(s)$, can also be explicitly written down to find the scattering phase with a global phase shift. Herein, we suppose that $h_0(s)$ does not produce an extra large error, so the reproduced scattering phases have the same errors as $\mathrm{Arg}[a_0(s)]$. The two-body unitarity is not embedded in experimental analysis, so we should not expect that IMC and REC can be satisfied simultaneously. The results based on either of them compared with LASS $I=1/2$ data [15] are shown in Fig. 1.

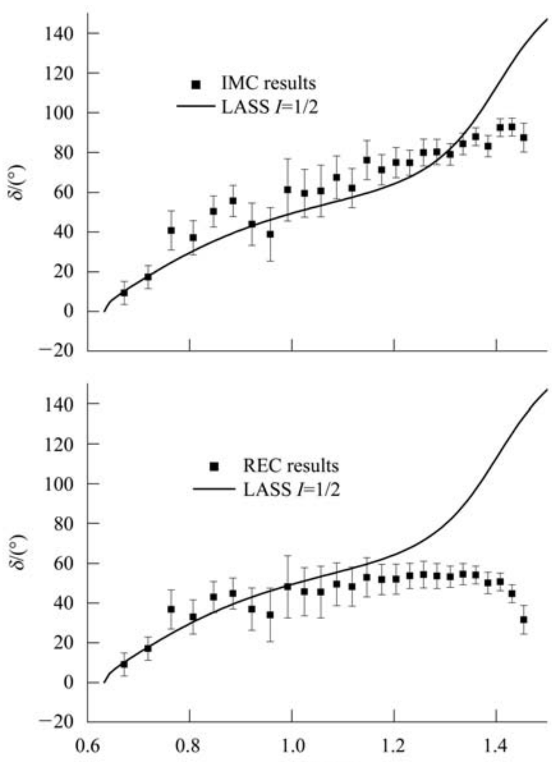

Fig. 1. The reproduced Kπ I=1/2 scattering phase from E791 data compared with the LASS data.

In the low-energy region, both of the reproduced Kπ I=1/2 scattering phase sets are in agreement with the reanalyzed results of the old LASS data [15]. The global phase shifts to make their threshold values reasonable are 85° for REC and 110° for IMC. They are determined by making the reproduced data close to the threshold coincide with the ChPT prediction. The reproduced P-wave scattering phase is not so good as the S-wave, and the two results from IMC and REC have a much greater difference. Recalling that the Breit-Wigner representation used in the experiment does not describe the phase accurately (see Fig. 6 shown in Ref. [9]), these results are at an acceptable level. However, the reproduced D-wave result has an obvious slope, which might imply that there is an uncounted s,t-dependent phase ambiguity of the total amplitude. Another reason why the results of P- and D- waves appear to be less satisfactory is that their contributions are suppressed by the Legendre function factors so there is a much larger uncertainty. However, the results reproduced from the FOCUS experiment data, as shown in Fig. 2, are obviously different from the LASS data.

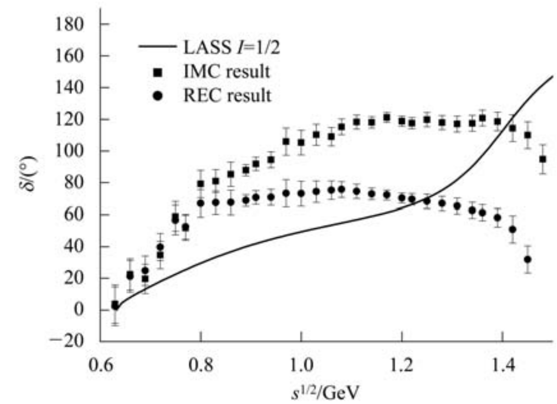

Fig. 2. The reproduced I=1/2 S-wave phases from the FOCUS data.

Remember that in the Dalitz analysis there might exist an ambiguity of the total amplitude represented by an energy-dependent global phase factor, since the observed density is in proportion to $|A(s,t)|^2 = |A_s(s,t)+A_t(t,s)|^2$. One of the sufficient conditions to get the same Dalitz plot as shown in $D^+ \to K^-\pi^+\pi^+$ experiments is $A(s,t) \to A(s,t)e^{i\Phi(s,t)}$ with $\Phi(s,t)$ being the same function when exchanging s and t[1]. This kind of ambiguity comes from the non-accuracy of the models used in the reference waves or even the commonly used Breit-Wigner representation, which is only an approximation of the dynamical structure of the physical reality. The difference between them can be simulated by such a phase factor, with more general characters of the amplitude. We can simply

1) We are grateful to Prof. M. Pennington for pointing this out.

parameterize this possible phase factor in a polynomial form as $\Phi(s,t) = \phi(s) + \phi(t)$, and $\phi(s) = \sum_n c_n(s-s_0)^n$ with $s_0 = (m_K + m_\pi)^2$. There is at least a constant global phase shift existing in the experiment data, since the experimental phases are defined with respect to the phase of $K^*(892)$ resonance.

The possible phase ambiguity is determined by finding the best matching of the reproduced $I=1/2$ scattering phases with the regenerated LASS $I=1/2$ phases in accordance with the ChPT [15]. Introducing the phase ambiguity means that the physical decay amplitude is not $A(s,t)$ from MIPWA but $A(s,t)e^{i\Phi(s,t)}$, and the physical partial wave amplitudes are redefined as

$$a'_l(s) = \int_{-1}^{+1} \frac{2l+1}{2} A(s,t) e^{i\Phi(s,t)} P_l(\cos\theta_s) d\cos\theta_s. \quad (9)$$

To keep more information from the experiments, the integrations of re-scattering are computed by the interpolation method. However, the deficiency of this method makes REC constraints non-executable in the vicinity of the integration limits, so only IMC is applied here. The best fits for the E791 and FOCUS data are almost at the same qualitative level, as shown in Fig. 3.

Figure 3 also shows a comparison of the possible phase ambiguities, $\phi(s)$, of E791 and FOCUS data. One will find that both of them might have such an ambiguity that varies much quickly at the region near the $K\eta'$ threshold, but in low-energy region the phase variation from E791 data is more steady than that from the FOCUS data.

As demonstrated in Eq. (9), the decay amplitudes of all partial waves are redefined while including a phase factor of $e^{i\phi(s,t)}$. The renewed partial wave decay amplitudes are severely changed. Fig. 4 shows the comparison of the absolute values between the renewed partial waves and the original MIPWA representations from E791. An expert might decide that this kind of P-wave representation is no longer what is commonly used in analysis. Even though the peak corresponding to $K^*(892)$ is almost unchanged, it presents another structure at the low energy region close to the $K\pi$ threshold. We would like to point out that this result just gives evidence of the drawback of the MIPWA method. This method is an almost model independent method to determine the S-wave amplitude, but it is partially model-dependent in applying the Breit-Wigner representation to

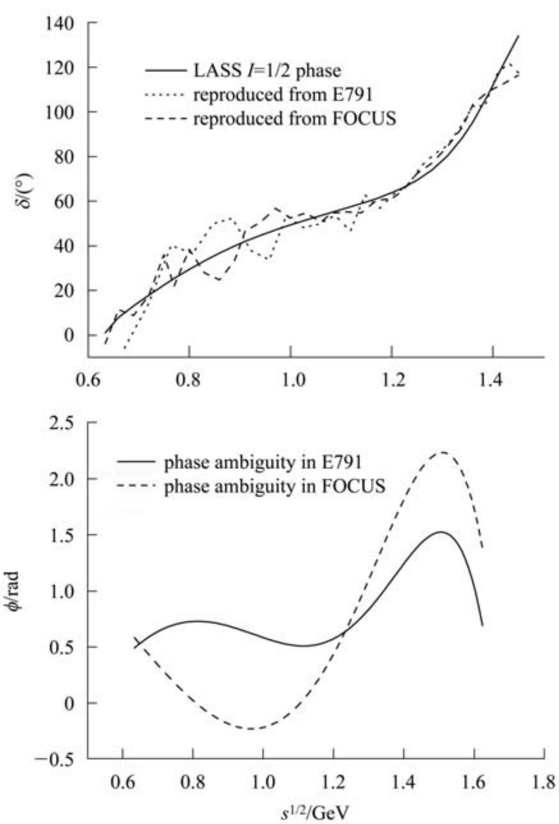

Fig. 3. The possible phase ambiguity of E791 and FOCUS results at a similar qualitative level of fit. Upper: the fit results with interpolation. Lower: the possible phase ambiguity.

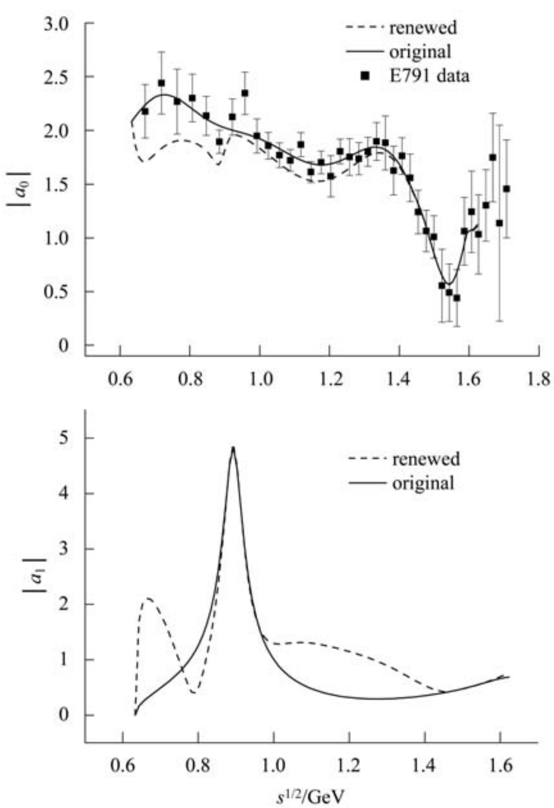

Fig. 4. Comparison of the renewed modulus of S- and P-wave decay amplitudes. The data are from E791.

describe the "reference" waves. The Dalitz plot, measuring $|A(s,t)|^2=|\mathcal{S}+\mathcal{P}+\mathcal{D}+\mathcal{S}'+\mathcal{P}'+\mathcal{D}'|^2$, might fail to separate the partial waves very accurately when the "reference" waves are not in an accurate description, if no other constraint is included. That means that there is a mis-allocation in the experimental results of the different partial waves because the searching procedures for the maximum likelihood choose another acceptable result in several ambiguities. Let us have a close look at the data of E791 in Fig. 4. The cusp at about 900 GeV in the magnitude of S-wave amplitude happens, perhaps, because the MIPWA cannot completely separate the origin of $K^*(892)$ effect. The lower peak close to the threshold in the renewed P-wave is mainly induced from the S-wave component by the integration

$$\frac{3}{2}\int_{-1}^{+1} a_0(s)\mathrm{e}^{\mathrm{i}\Phi(s,t)}P_1(\cos\theta_\mathrm{s})\mathrm{d}\cos\theta_\mathrm{s}.$$

Its effect on the Dalitz plot are highly suppressed by the $\cos\theta_\mathrm{s}$ factor in the partial wave integration, so the P-wave result is easily distorted by the fit procedure to satisfy the measured values of another region in the Dalitz plot. It might be considered as the contribution from low-lying resonances in the S-wave, perhaps κ resonance, but MIPWA assigns it to the P-wave.

4 Summary

In conclusion, the MIPWA analysis of $D^+\to K^-\pi^+\pi^+$ provides an experimental possibility of measuring the low-energy $K\pi$ scattering phase shifts close to the threshold. There is clear evidence that the experimental results might still have some underestimated ambiguities and there might be a sizable uncertainty in separating the contributions from different partial waves. A better analysis of the same data with constraints from two-body unitarity is suggested. Our conclusion is based on the assumption that the $I=1/2$ part of the $K\pi$ system is dominant over $I=3/2$ part, which is reasonably similar to the $\Delta I=1/2$ rule in Kaon decay. However, a more complete analysis taking into account both of their contributions will help to clarify this problem.

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References

1 Aitala E et al. Phys. Rev. Lett., 2001, **86**:770
2 Aitala E et al. Phys. Rev. Lett., 2002, **89**:121801
3 Ablikim M et al. Phys. Lett. B, 2004, **598**:149
4 Ablikim M et al. Phys. Lett. B, 2006, **633**:681
5 Ablikim M et al. Phys. Lett. B, 2007, **645**:19
6 Caprini I, Colangelo G, Leutwyler H. Phys. Rev. Lett., 2006, **96**:132001
7 ZHOU Z Y et al. JHEP, 2005, **02**:043
8 Amsler C et al. Phys. Lett. B, 2008, **667**:1
9 Aitala E et al. Phys. Rev. D, 2006, **73**:032004
10 Link J et al. Phys. Lett. B, 2009, **681**:14
11 Pennington M. Int. J. Mod. Phys. A, 2006, **21**:5503
12 Aitchison I. J. Phys. G, 1977, **3**:121
13 Ascoli G, Wyld H. Phys. Rev. D, 1975, **12**:43
14 Caprini I. Phys. Lett. B, 2006, **638**:468
15 Link J et al. Phys. Lett. B, 2007, **653**:1