

# Study on the HOMs coupler design of spoke cavity

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**Abstract:** The superconducting spoke cavity has been proposed to accelerate the proton in the low energy section of high power proton linac for an Accelerator Driven Sub-critical System (ADS). Considering that the High Order Modes (HOMs) in the superconducting cavity have far reaching influence on power dissipation and beam stability, the analysis of HOMs of the spoke cavity is needed. In this paper, we put emphasis on the analysis of HOMs of the spoke cavity and the HOMs coupler design.

**Key words:** HOMs, spoke cavity, ADS

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## 1 Introduction

The superconducting spoke cavity has been proposed to accelerate the proton in the low energy section of high power proton linac for an Accelerator Driven Sub-critical System (ADS). According to some basic requirements of the ADS, we have designed a  $\beta=0.3$ ,  $f=352$  MHz spoke cavity [1]. For a superconducting cavity, the High Order Modes excited by bunches can seriously affect the subsequent charges passing through the cavity. If not sufficiently damped, they can lead to beam instabilities and beam loss. HOMs also increases the cryogenic losses due to the additional power dissipation in the cavity walls. Unless the modes are sufficiently damped, the additional refrigeration load can be expensive. In this paper, we put emphasis on the analysis of the HOMs of the spoke cavity and the HOMs coupler design.

## 2 High order modes

There are infinitely many resonant modes in a spoke cavity, but not every mode is harmful to the beam, therefore, it is necessary to identify the dangerous modes which have serious impact on the beam.

According to the theory of HOMs, the bunch-induced voltage is proportional to the shunt impedance ( $R_a/Q_0$ ). Therefore, the safest strategy

is to identify all the high  $R_a/Q_0$  modes [2].

$$\frac{R_a}{Q_0} = \frac{V_c^2}{\omega U}. \quad (1)$$

Here  $V_c$  is the accelerating voltage obtained by integrating the electric field along the axis.  $\omega$  is the mode frequency.  $U$  is the stored energy in the cavity. For some modes the electric field along the beam centre axis is almost zero, but they have stronger electric field along the line offset from the beam centre axis. So we integrate the electric field along the axis 2.5 cm away from the beam centre and get the voltage  $V_c^*$ , and the effective shunt impedance is defined as:

$$\frac{R_a^*}{Q_0} = \frac{V_c^{*2}}{\omega U}. \quad (2)$$

Considering of the danger to the beam and the dissipated power of the high order modes, we study the modes with frequency lower than  $3f_0$  ( $f_0$  is the frequency of the accelerating mode). Table 1 lists the result of 20 modes from ANSYS calculation [3].

M1 is the accelerating mode and the other 19 modes are high order modes. Among the 19 modes, M2, M4, M11, M12, M14 and M20 have relative high  $R_a/Q$  and  $R_a^*/Q$  and they can be excited easily. M5, M6, and M10 have a relatively high  $R_a^*/Q$ , and they can be excited by the transverse motion of the bunch. All of the high  $R/Q$  modes are harmful to the beam, so they must be damped by the HOM coupler.

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Table 1. The HOMs of the spoke cavity.

| modes | $f/\text{MHz}$ | $(R_a/Q)/\Omega$ | $(R_a^*/Q)/\Omega$ |
|-------|----------------|------------------|--------------------|
| M1    | 352            | 114.51           | 154.11             |
| M2    | 409.9          | 113.22           | 137.44             |
| M3    | 637.4          | 0.00845          | 0.00373            |
| M4    | 668.9          | 3.678            | 5.789              |
| M5    | 672.3          | 0.00811          | 13.378             |
| M6    | 716.3          | 0.00251          | 14.251             |
| M7    | 819.1          | 0.02846          | 0.00152            |
| M8    | 865.8          | 0.00046          | 0.00066            |
| M9    | 875.3          | 0.03394          | 0.00373            |
| M10   | 885.8          | 0.00028          | 0.651              |
| M11   | 997.6          | 3.818            | 1.87               |
| M12   | 1011.9         | 35.05            | 33.41              |
| M13   | 1029.4         | 0.04241          | 0.0362             |
| M14   | 1071.3         | 33.99            | 31.49              |
| M15   | 1083.7         | 0.011            | 0.00512            |
| M16   | 1107.2         | 0.0485           | 0.0547             |
| M17   | 1127.3         | 0.00447          | 0.00198            |
| M18   | 1131.9         | 0.0019           | 0.2345             |
| M19   | 1132.5         | 0.167            | 0.0535             |
| M20   | 1154           | 38.58            | 41.94              |

### 3 HOM coupler

Three types of HOM coupler have been developed. They are the waveguide coupler, coaxial coupler and beam tube coupler [4]. The waveguide coupler and beam tube coupler are usually installed on the beam tube. The highest frequency of the dangerous mode is 1.15 GHz, but the lowest cutoff frequency of the beam tube is 2.259 GHz. That means all the dangerous modes can not be guided out from the cavity. They are all trapped modes. On the other hand, the coaxial coupler is usually installed on the cavity and can support the TEM mode. All the HOMs can propagate in the coaxial coupler. Therefore the coaxial coupler is the best choice for the spoke cavity.

There are two methods for the coaxial coupler to pick up the HOMs power from the cavity. One is the antenna coupler, the other one is the loop coupler. In

this paper, we focus on the case of the antenna coupler. The geometry of this HOMs coupler is shown in Fig. 1. Fig. 2 is the equivalent circuit of the coupler.

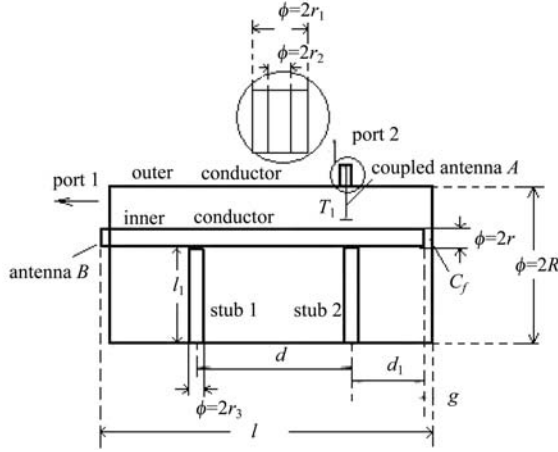


Fig. 1. The geometry of the antenna coupler.

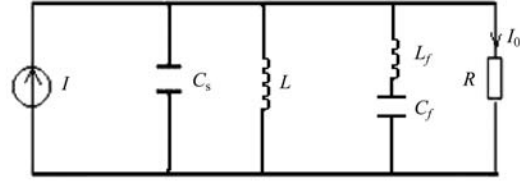


Fig. 2. The equivalent circuit of the coupler.

In Fig. 1, port 1 is connected to the cavity, port 2 is connected to an absorbing load  $R$ . Antenna  $B$  and its distributed capacitance  $C_s$  compose a current generator.  $I$  is the displacement current picked up by Antenna  $B$ . stub 1 and stub 2 are equivalent to an inductor, in parallel with the distributed capacitance. The two inductive stubs also support the inner conductor and provide cooling to the coupler antenna. On the right of the stub 2 is a coaxial resonator, which is equivalent to a series resonant filter in parallel with the load. The power of the HOMs is coupled into the load via the coupled antenna  $A$ .

The transfer function  $H(s)$  of the equivalent circuit is:

$$H(s) = \frac{L(1 - \omega^2 L_f C_f) \cdot s}{R[(1 - \omega^2 L C_s)(1 - \omega^2 L_f C_f) - \omega^2 L C_f] + L(1 - \omega^2 L_f C_f) \cdot s}. \quad (3)$$

In Eq. (3), if  $L_f C_f = 1/\omega^2$ ,  $H(s)$  is zero, which means that no power will pass through the load resistor  $R$ . As usual, we adjust the capacitive gap  $g$  to ensure the  $L_f C_f = 1/\omega_0^2$  ( $\omega_0$  is the frequency of the accelerating mode) to prevent the accelerating mode from being absorbed by the load. From Eq. (3), we can also find that  $H(s)$  of the HOM with the frequency  $\omega^2 = 1/(L C_s)$  has maximum modulus, which

denotes that this HOM can obtain maximum damp.

If the input signal is  $I = A \sin(\omega t)$ , then the response of the circuit is  $I_o(s) = H(s)I(s)$ . Using the inverse laplace transform theory, we got the output signal in Time Domain:

$$I_o(t) = \frac{-Aab\omega}{b^2 + \omega^2 a^2} e^{-\frac{b}{a}t} + \frac{2Aab\omega}{b^2 + \omega^2 a^2} \cos(\omega t). \quad (4)$$

Here  $a = L(1 - \omega^2 L_f C_f)$ ,  $b = R(1 - \omega^2 L C_s)(1 - \omega^2 L_f C_f) -$

$\omega^2 LC_f R$ . The first term in Eq. (4) decays exponentially and could be neglected. The next term represents the absorbance by the resistor. The scatter parameters of the circuit with normalized impedance at all ports are  $S=20\log(I_o/I)$ . Choosing the  $1/\omega_0^2 = L_f C_f$  and  $1/\omega_1^2 = LC_s$ , ( $\omega_1=409.9$  MHz is the frequency of the mode M2), Fig. 3 shows the  $S$  parameters curve with frequency from 300 MHz to 1.2 GHz. At the point that  $\omega=352$  MHz, the curve has a minimum value, which means that the accelerating mode has been effectively rejected. At the point that  $\omega=409.9$  MHz, the curve has a maximum value, which means that the mode M2 has been effectively damped. Another minimum value takes place at the point that  $\omega=600$  MHz. We should avoid the minimum value taking place at the point nearby the dangerous modes. From the 700 MHz to 1.2 GHz, in which most of the dangerous modes concentrate, the average value of the  $S$  parameter is more than  $-10$  dB.

To sum up, given the properly geometric parameters, this type of coupler can have an excellent performance to attenuate the HOMs.

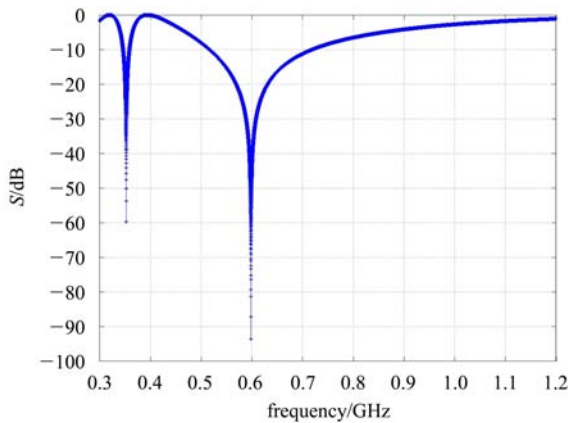


Fig. 3. The  $S$  parameters curve of the equivalent circuit of the coupler.

The challenges in designing coaxial couplers are twofold: (a) to maximize the coupling to the fields over the broad frequency band of the dangerous HOMs, and (b) to provide excellent rejection for the accelerating mode. Based on the above discussion, during the 3D high frequency simulation, we adjust the parameters of the coupler, such as  $g$ ,  $l$ ,  $l_1$ ,  $d$ , etc to improve the performance of the coupler.

#### 4 HOM coupler design

When transmitting the weak signal, in order to obtain the lowest attenuation, the radius ratio ( $R/r$ )

of the coaxial cable's outer conductor to inner conductor should be 3.6. On the other hand, in the coaxial resonator, only the TEM mode should exist, this requires highest frequency ( $f_{\max}$ ) of the dangerous mode to be lower than the coaxial resonator's lowest cutoff frequency [5].

$$f_{\max} < \frac{c}{\pi(R+r)}. \quad (5)$$

The  $f_{\max}$  of the dangerous mode is 1.154 GHz, so we can get  $R=0.0647$  m, and  $r=0.0179$  m.

At the beginning of designing the HOMs coupler, we must find the appropriate  $g$  and  $d_1$  to ensure that the coaxial resonate filter can filter out the accelerating mode. This procedure can be done via simulating a coaxial cavity, as in Fig. 4. The electric field mostly concentrates in the capacitive gap. A tiny change of the gap  $g$  will lead to a significant frequency variation of the coaxial cavity.

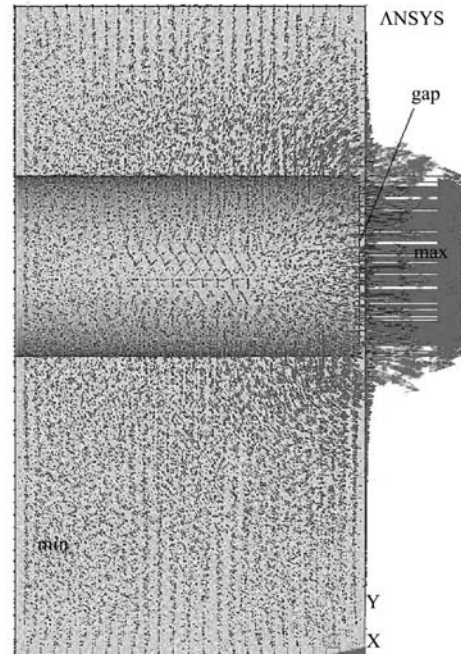


Fig. 4. Electric field of the coaxial resonate filter, max represents the maximum electric field.

Keeping the frequency of the coaxial cavity to be 352 MHz, Table 2 shows the relation between  $g$  and  $d_1$ . The capacitive gap  $g$  increases with the increase of  $d_1$ .

Table 2. The relation between capacitive gap  $g$  and the inductive length  $d_1$ , under resonate frequency=352 MHz.

|          | $f_0=352$ MHz |      |       |      |
|----------|---------------|------|-------|------|
| $d_1$ /m | 0.05          | 0.06 | 0.07  | 0.08 |
| $g$ /mm  | 0.735         | 0.92 | 1.132 | 1.14 |

The initial structure parameters of the HOM coupler are listed in Table 3. In order to simplify the design, without considering the effect of the field pattern, we connect the coupler to a coaxial transmission line, as shown in Fig. 5. Power is fed into the coaxial line from port 1. Both port 2 and port 3 are connected to a matched load.

Table 3. The geometric parameters of the coupler.

|         |          |
|---------|----------|
| $R/m$   | 0.0647   |
| $r/m$   | 0.0179   |
| $r_1/m$ | 0.0072   |
| $r_2/m$ | 0.002    |
| $r_3/m$ | 0.01     |
| $g/m$   | 0.001132 |
| $d/m$   | 0.06     |
| $d_1/m$ | 0.07     |
| $l_1/m$ | 0.04671  |
| $L/m$   | 0.171132 |

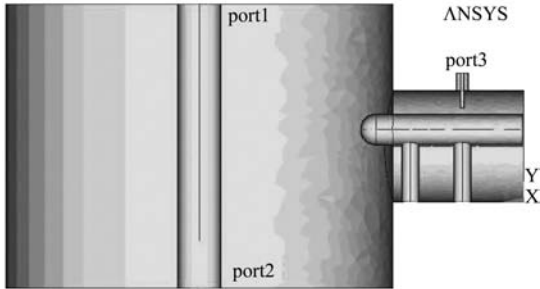


Fig. 5. Simulating the coupler on a coaxial line.

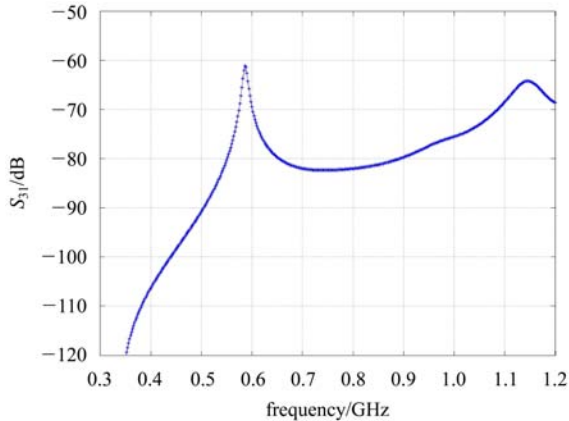


Fig. 6. The  $S$  parameters curve.

Sweeping the frequency from 300 MHz to 1.2 GHz, we obtained the sweeping frequency curve of the scattering parameter from port 1 to port 3, as shown in Fig. 6. The scattering parameter  $S_{31}$  of the accelerating mode is near  $-120$  dB, almost no attenuation for the working mode. For the HOMs, the attenuation is far more than the working mode, but less than  $-60$  dB. The power of the HOMs extracted from the

coaxial line is very weak. In order to improve the attenuation of HOMs, we adjust the stub's length  $l_1$  and change the angle between Antenna  $A$  and the coupler during the simulation. The result indicates that when the inner conductor deviates from the center axis and the angle that the Antenna  $A$  makes with the inner conductor is  $30^\circ$ , the attenuation of HOMs can be improved obviously [6]. More discussions can be found in Ref. [6]. Another method to enhance the ability of attenuating HOMs is to dig a hole in the inner conductor and install a ball at the end of the Antenna  $A$ , as shown in Fig. 7.

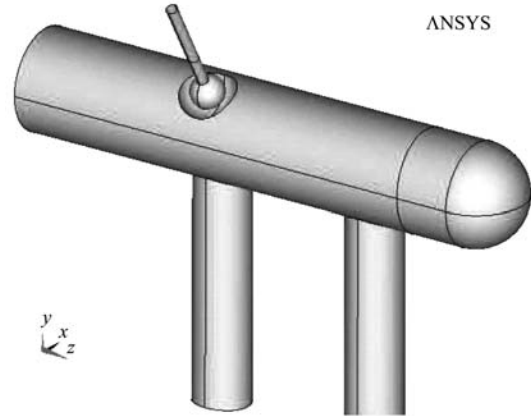


Fig. 7. Improvement of the Antenna  $A$ .

Figure 8 shows the new sweeping frequency curve after taking these methods.

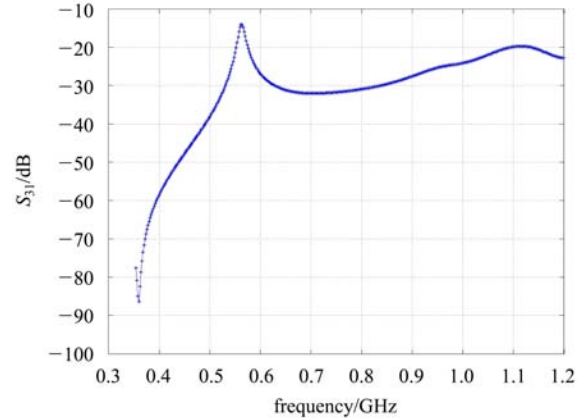


Fig. 8. The  $S$  parameters curve of the improved coupler.

Obviously, the attenuation for all modes is improved about 40 dB. For HOMs, except the mode with frequency 409.9 MHz, the averaged attenuation reaches up to  $-25$  dB. The coupler has effectively damped these HOMs. We can also see that the coupler still provides excellent rejection for the accelerating mode whose attenuation is only  $-80$  dB.

Finally, we must connect this coupler to the spoke cavity to examine its performance, as shown in Fig. 9.

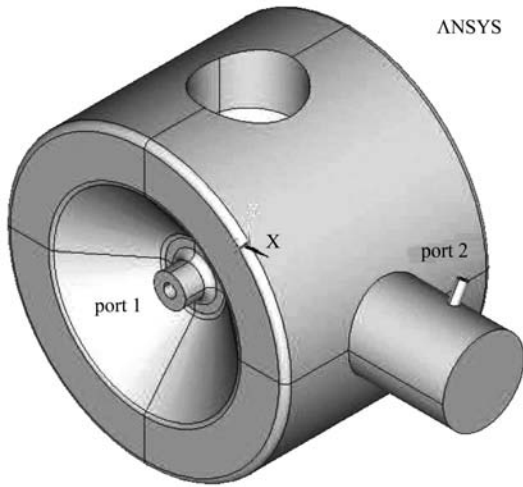


Fig. 9. Simulating the coupler on a spoke cavity.

Power is fed into the spoke cavity from port 1. Port 2 is connected to a matched load. When designing a HOMs coupler, the efficiency  $\eta$  of the cavity

must be considered, which is defined as:

$$\eta = \frac{\beta}{1 + \beta}. \quad (6)$$

Here  $\beta$  is coupling factor of cavity and  $\beta = Q_0/Q_e = S_{21}^2/(1 - S_{11}^2 - S_{21}^2)$ , according to Eq. (5), if we want to increase  $\eta$ , we must increase  $\beta$ .

Changing the angle between the coupler and the spoke, we can find that when the coupler makes a  $45^\circ$  angle with the spoke, the coupler has a maximum attenuation for HOMs, and a minimum attenuation for the accelerating mode. Table 4 lists the sweeping results of these special modes. For the accelerating mode and two HOMs at  $f=409.9$  MHz and  $f=716.3$  MHz,  $\eta$  is close to zero. The coupler almost can't abstract their power out from the cavity. Compared with the three modes, other HOMs have relatively higher  $\eta$ , which means that these modes got effective attenuation.

Table 4. The sweeping results of HOMs (when calculating  $Q_1$  and  $Q_e$ ,  $Q_0$  is  $1 \times 10^9$ ).

| $f$ /MHz | $S_{11}$ /dB | $S_{21}$ /dB | $\beta$                | $\eta$                  | $Q_1$                | $Q_e$                   |
|----------|--------------|--------------|------------------------|-------------------------|----------------------|-------------------------|
| 352      | -0.53516     | -47.9128     | $1.404 \times 10^{-3}$ | $1.4036 \times 10^{-4}$ | $9.9986 \times 10^8$ | $7.1234 \times 10^{12}$ |
| 409.9    | -0.51891     | -51.5945     | $6.151 \times 10^{-5}$ | $6.1506 \times 10^{-5}$ | $9.9994 \times 10^8$ | $1.6258 \times 10^{13}$ |
| 668.9    | -0.36129     | -26.6661     | 0.0277                 | 0.02699                 | $9.7301 \times 10^8$ | $3.6054 \times 10^{10}$ |
| 672.3    | -0.63062     | -12.9129     | 0.6084                 | 0.37825                 | $6.2175 \times 10^8$ | $1.6437 \times 10^9$    |
| 716.3    | -0.32467     | -45.3211     | $4.164 \times 10^{-4}$ | $4.1625 \times 10^{-4}$ | $9.9958 \times 10^8$ | $2.4014 \times 10^{12}$ |
| 997.6    | -0.18974     | -21.5880     | 0.1937                 | 0.16227                 | $8.3773 \times 10^8$ | $5.1624 \times 10^9$    |
| 1011.9   | -0.66961     | -9.88719     | 2.5495                 | 0.71827                 | $2.8173 \times 10^8$ | $3.9224 \times 10^8$    |
| 1071.3   | -0.16352     | -24.8896     | 0.0962                 | 0.08778                 | $9.1222 \times 10^8$ | $1.0393 \times 10^{10}$ |
| 1154     | -0.11012     | -23.9159     | 0.1936                 | 0.16222                 | $8.3778 \times 10^8$ | $5.1645 \times 10^9$    |

## 5 Conclusion

In this paper, we have analyzed the High Order Modes in the spoke cavity, and found out the dangerous HOMs. Contraposed with these modes, a coaxial HOM coupler has been designed. The simulation

result confirms that most of the danger modes have been damped effectively, except two modes whose frequencies are 409.9 MHz and is 716.3 MHz. The coupler has a relatively satisfactory performance. As for how to damp the two special HOMs, future study is needed.

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