Shape phase transition in neutron-rich even-even light nuclei with Z=20–28*

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Abstract: The E-Gamma Over Spin (E-GOS) analysis method is applied to the study of the shape phase transition of neutron-rich even-even light nuclei with Z=20–28. Some valuable results are gained through analysing E-GOS curves of Ca, Ti, Cr, Fe and Ni nuclei.

Key words: nuclear structure, shape phase transition, E-GOS curve, isospin effect

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1 Introduction

The structure of light nuclei is an important subject in the field of nuclear physics. The study of light nuclear structure not only deepens nuclear structure theory but also has great importance for astrophysics. Because of the complexity of nuclear structure, many theoretical models have been applied to the investigation of lighter nuclei [1–5].

The interacting boson model (IBM) is an algebraic model used to study nuclear collective motion. In the original version (IBM-1), only one kind of boson is considered, and it has been successful in describing various properties of medium and heavy even-even nuclei [6–11]. For lighter nuclei, the valence protons and neutrons fill the same major shells and isospin should be taken into account, so the IBM has been extended to the interacting boson model with isospin (IBM-3) [12]. In the IBM-3, three types of bosons including proton-proton (π), neutron-neutron (υ) and proton-neutron (δ) form the isospin T=1 triplet. In the lighter nuclei region where the protons and neutrons are in the same major shells, the IBM-3 can describe the low-energy levels of some nuclei well and explain their isospin and F-spin symmetry structure [3, 13–15]. The dynamical symmetry group for IBM-3 is $U(18)$, which starts with $U_{\rm sd}(6)\times U_{\rm c}(3)$ and must contain $SU_{\rm T}(2)$ and $O(3)$ as subgroups because the isospin and the angular momentum are good quantum numbers. The natural chains of IBM-3 group $U(18)$ are [16]:

$$U(18)\supset(U_{\rm c}(3)\supset SU_{\rm T}(2))\times(U_{\rm sd}(6)\supset U_{\rm d}(5)\supset O_{\rm d}(5)\supset O_{\rm d}(3)), \quad (1)$$

$$U(18)\supset(U_{\rm c}(3)\supset SU_{\rm T}(2))\times(U_{\rm sd}(6)\supset O_{\rm sd}(6)\supset O_{\rm d}(5)\supset O_{\rm d}(3)), \quad (2)$$

$$U(18)\supset(U_{\rm c}(3)\supset SU_{\rm T}(2))\times(U_{\rm sd}(6)\supset SU_{\rm sd}(3)\supset O_{\rm d}(3)). \quad (3)$$

The subgroups $U_{\rm d}(5)$, $O_{\rm sd}(6)$ and $SU_{\rm sd}(3)$ describe vibrational, γ-unstable and rotational nuclei respectively.

The shape phase transition is a major theme in the study of nuclear structure. Recently, the investigation of nuclear shape phase transition using the concept of dynamical symmetry has attracted great interest. The paper [17] presents a simple method for discerning the evolution from vibrational to rotational structure in nuclei as a function of spin, which is called E-Gamma Over Spin (E-GOS) curves. In this paper, this method will be applied to study the shape phase transition of neutron-rich even-even light nuclei with proton number Z=20–28.

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2 E-GOS Method

In the IBM-3 model, the theoretical yrast energy level for $U(5)$ limit is [17]:

$$E_I = \frac{I}{2}\hbar\omega, \tag{4}$$

for the $SU(3)$ limit, the theoretical yrast energy level is:

$$E_I = \frac{\hbar^2}{2J} I(I+1), \tag{5}$$

where J is the kinematic moment of inertia, and I is spin.

The gamma-ray decay energies for a perfect harmonic vibrator are given by

$$E_\gamma(I\to I-2) = \hbar\omega, \tag{6}$$

for an axially symmetric rotor:

$$E_\gamma(I\to I-2) = \frac{\hbar^2}{2J}(4I-2), \tag{7}$$

for $SO(6)$ limit, the gamma-ray decay energies are:

$$E_\gamma(I\to I-2) = [E(2^+)/4](I+2). \tag{8}$$

To study the shape and phase evolution, the ratio is introduced as in Ref. [17]:

$$R = \frac{E_\gamma(I\to I-2)}{I}. \tag{9}$$

When spin I approaches infinity, the R parameters for the vibrational, rotational and γ-unstable nuclei are

$$R_{\text{vibration}} \to 0, \tag{10}$$

$$R_{\text{rotation}} \to 4\left(\frac{\hbar^2}{2J}\right), \tag{11}$$

$$R_\gamma \to \frac{E(2^+)}{4}, \tag{12}$$

respectively.

In order to schematically show the relation between the ratio R and the spin I, the E-GOS curves of vibrational, rotational and γ-unstable nuclei are plotted with the first excited state energy level at 1000 keV. Fig. 1 shows these theoretical limits.

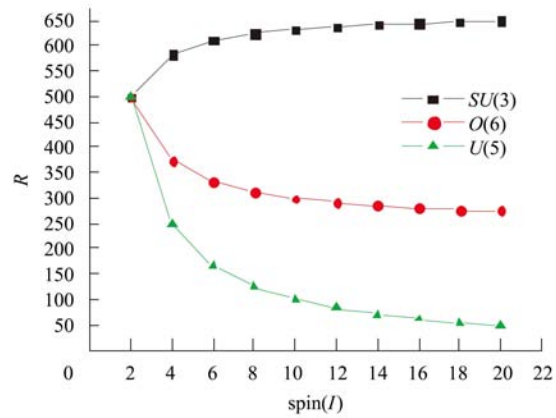

Fig. 1. E-GOS curves of vibrational, rotational and gamma-unstable nuclei.

3 Numerical results and discussion

When the E-GOS method is used to study the shape phase transition, the first excited yrast state experimental energy level is taken to plot the E-GOS curves. Moreover, in order to further investigate the

Table 1. The value E_4^+/E_2^+, β_2 and Q_0 of neutron-rich even-even light nuclei with Z=20–28.

nucleus	E_4^+/E_2^+	β_2	Q_0(b)	nucleus	E_4^+/E_2^+	β_2	Q_0(b)
^{40}Ca	1.3519	0.1201	0.3064	^{52}Fe	2.8092		
^{42}Ca	1.8058	0.2462	0.6487	^{54}Fe	1.8026	0.1984	0.8035
^{44}Ca	1.9732	0.2670	0.7257	^{56}Fe	2.4645	0.2498	1.0364
^{46}Ca	1.9123	0.1509	0.4225	^{58}Fe	2.5630	0.2607	1.1073
^{48}Ca	1.1754	0.0996	0.2868	^{60}Fe	2.5687		
^{44}Ti	2.2659	0.2579	0.7763	^{56}Ni	1.4530		
^{46}Ti	2.2598	0.3253	1.0017	^{58}Ni	1.6912	0.18	0.8234
^{48}Ti	2.3347	0.2685	0.8507	^{60}Ni	1.8806	0.2065	0.9663
^{50}Ti	1.7218	0.1671	0.5438	^{62}Ni	1.9915	0.1965	0.9398
^{52}Ti	2.2088			^{64}Ni	1.9388	0.1744	0.8519
^{48}Cr	2.4707	0.3676	1.2704	^{66}Ni	1.8750		
^{50}Cr	2.4023	0.2938	1.0433	^{68}Ni	1.4909		
^{52}Cr	1.6520	0.2229	0.8126				
^{54}Cr	2.1859	0.2523	0.9430				
^{56}Cr	2.6650						

shape and phase evolution, we list the value E_4/E_2 in Table 1. along with the quadrupole deformation parameter

$$\beta_2 = 4\pi\sqrt{B(E2:0_1^+ \to 2_1^+)}/(3eZR_0^2)$$

and the electric quadrupole moment Q_0 calculated using the relation

$$B(E2:0_1^+ \to 2_1^+) = \frac{5}{16\pi}e^2Q_0^2.$$

The experimental data are taken from Ref. [18]

The E-GOS curve in Fig. 2 describes the changing features of yrast energy levels for the even-even nuclei $^{40-48}$Ca, $^{44-52}$Ti, $^{48-56}$Cr, $^{52-60}$Fe and $^{56-68}$Ni (Z=20–28, N=20–40), and shows qualitatively the description of the shape phase transition of even-even nuclei with A=40–68.

As the E-GOS curve shows, the doubly magic nuclei ^{40}Ca, ^{48}Ca and ^{56}Ni have similar E-GOS curves at low spins ($I < 4\hbar$). Table 1 shows that their E_4/E_2=1.1754–1.4530, quadrupole deformation parameter β_2=0.0996–0.1201 and electric quadrupole moment Q_0=0.2868(b)–0.3064(b). The small values of E_4/E_2, β_2 and Q_0 indicate that these doubly magic nuclei have nearly spherical shape. From Fig. 2, the special feature of the E-GOS curves on ^{40}Ca, ^{48}Ca and ^{56}Ni results from a high-lying first excited state (2^+), which is a characterstic feature of spherical nuclei with a short-range (δ-like or pairing) residual interaction.

Table 1 shows that the singly magic nuclei $^{42-46}$Ca, $^{58-66}$Ni, ^{50}Ti, ^{52}Cr and ^{54}Fe have E_4/E_2=1.6912–1.9732, quadrupole deformation parameter β_2=0.1509–0.2670 and electric quadrupole moment Q_0=0.4225(b)–0.7257(b). The values E_4/E_2, β_2 and Q_0 of these singly magic nuclei are larger than those of doubly magic nuclei, which means that the shape of single magic nuclei obviously deviates from spherical at low spin.

From the E-GOS curve characteristics and the calculated shape parameters, these singly magic nuclei not only have a strong shell effect but also collective motion with increasing valence nucleons. Fig. 2 shows that ^{44}Ca, which has four valence neutrons, approaches the $U(5)$ limit when the spin $I < 6\hbar$, and the critical point of the phase transformation appears when $I = 6\hbar$.

The E-GOS curve shows various nuclear structure features for ^{44,46,48,52}Ti, $^{48-56}$Cr and ^{52,56,58,60}Fe whose valence protons and valence neutrons are in the same major shells. Table 1 shows that the quadrupole deformation parameter β_2=0.2498–0.3676 and electric quadrupole moment Q_0=1.0364(b)–1.2704(b) for the above nuclei, which show that they are deformed nuclei.

From Fig. 2, ^{44,46,48,52}Ti are $U(5)$-$SU(3)$ transitional nuclei and I=$8\hbar$ is the critical point for ^{46,48}Ti. For the above three $^{44-48}$Ti nuclei, the vibration characteristics gradually strengthen with increasing mass; this conclusion coincides with the calculations in Ref. [19] with the IBM-3. From the E-GOS curve, we find that $^{48-56}$Cr are $U(5)$-$SU(3)$ transitional nuclei, and the vibration characteristics strengthen with increasing neutron number when $I < 8\hbar$, which is the same as Ref. [14]. ^{52,56,58,60}Fe are also $U(5)$-$SU(3)$ transitional nuclei. The coexistence of interaction between homogeneous nuclei and inhomogeneous isotopes plays an important role in the structure and characteristics of light nuclei. As shown in Fig. 2, the characteristic of vibration is strengthened with increasing isospin three-component T_3=$(N$-$Z)/2$ (T_3=0, 2, 4) on isobars ^{48}Cr, ^{48}Ti, and ^{48}Ca. However, the E-GOS curves show more complicated phenomena with the increasing isospin three-component on isobars ^{52}Fe, ^{52}Cr, ^{52}Ti and ^{56}Ni, ^{56}Fe, ^{56}Cr.

From Fig. 2, ^{68}Ni with 28 protons and 40 neutrons is similar in E-GOS curve to the doubly magic nuclei ^{40,48}Ca and ^{56}Ni when $I < 4\hbar$. So we infer that N=40 may be a magic number for neutrons and ^{68}Ni may be a doubly magic nucleus. In order to show this, Fig. 3 gives the relationship curves between the energy levels of the first and second excited states with the number of neutrons. Based on these curves, it is seen that the doubly magic nuclei ^{40,48}Ca and ^{56}Ni have higher exciting energies on the first and second excited states. In comparison, ^{68}Ni has also high first and second exciting energetic levels, so it should also be a doubly magic nucleus. In fact, the possible doubly magic nucleus ^{68}Ni has also been studied by many other groups either theoretically or experimentally, with similar conclusions [20, 21].

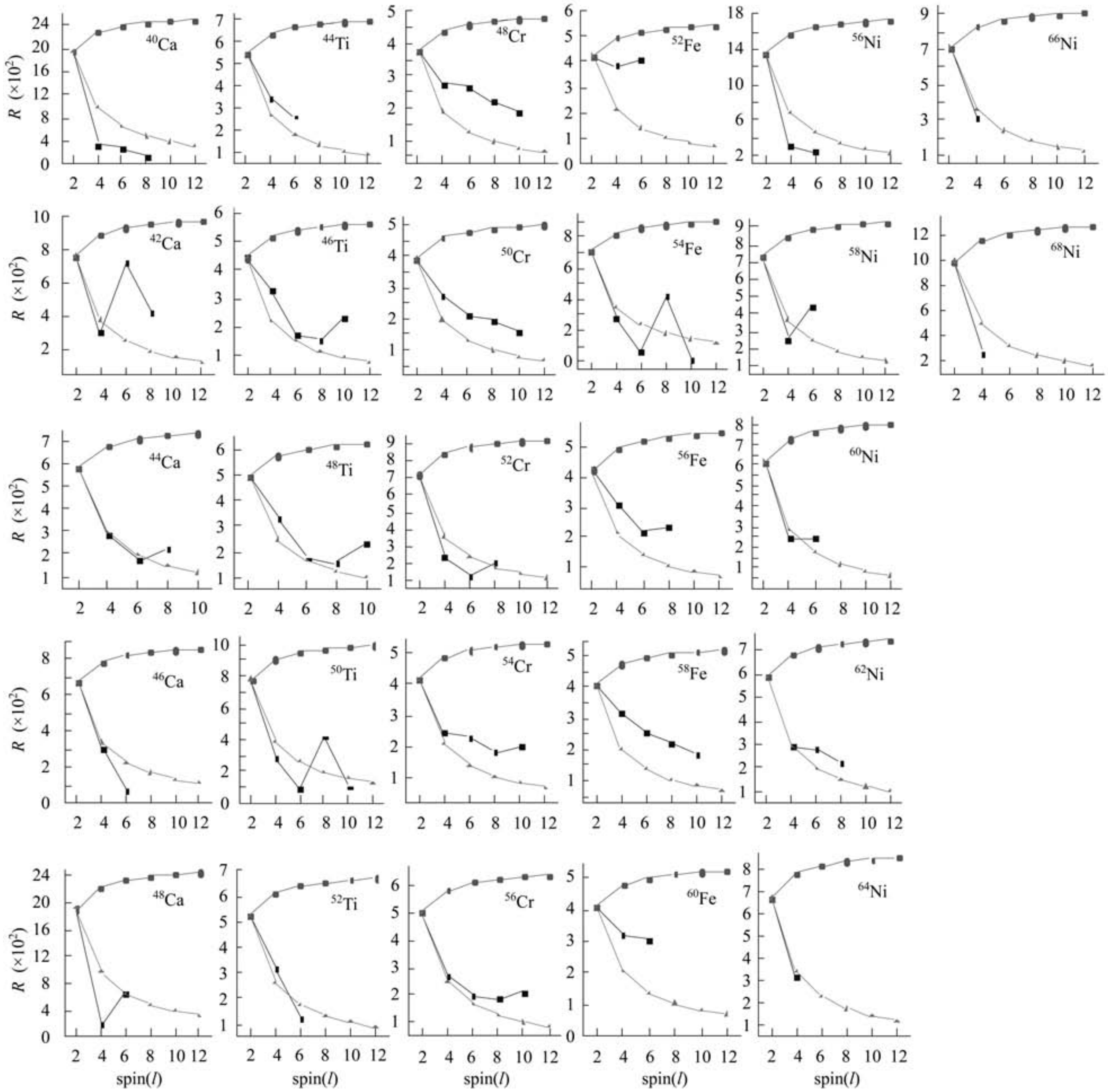

Fig. 2. E-GOS curves of neutron-rich even-even light nuclei with Z=20–28.

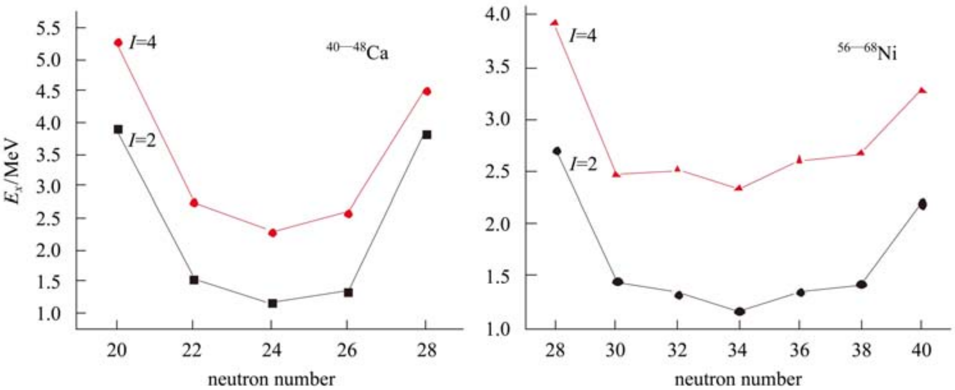

Fig. 3. Relationship curves between the energy levels of the first and second excited states with number of neutrons.

4 Conclusions

The E-Gamma Over Spin (E-GOS) analysis method is applied to study the shape phase transition of neutron-rich even-even light nuclei with Z=20–28. It is found that doubly magic nuclei have similar E-GOS curves at low spins and singly magic nuclei have characteristics of collective motion when the number of valence nucleons increases and approaches mid-shell. The isospin effect plays an important role in the shape phase transition of neutron-rich even-even light nuclei. It is also shown that ^{68}Ni is a doubly magic nucleus.

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