

Derivation of a $1/r^2$ potential term for $Q\bar{Q}$ in a potential model

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Abstract We clarify the difference among potential models so far proposed to explain mass spectra of heavy-light mesons via transformations of the vacuum. Applying our idea to $Q\bar{Q}$ quarkonium, we obtain the extra term, $1/r^2$ with positive coefficient, other than non-relativistic potential terms expected for quarkonium.

Key words heavy quark effective theory, spectroscopy, symmetry

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1 Introduction

There are many articles to try to explain the heavy-light mesons since the discovery of the so-called D_{sJ} , D_{s0}^* (2317) and D_{s1}' (2460), by BaBar and CLEO. Except for our model [1, 2], other relativistic potential models fail to reproduce D_{sJ} particles. Accordingly most of the people think that these two particles are exceptional and should be explained by some other mechanism.

We would like to explain the difference of our relativistic potential model from others by considering the true vacuum of heavy-light system and its transformation. Then we will show another application of our method to $Q\bar{Q}$.

2 True vacuum and transformation

Let us introduce the notion of the true vacuum in heavy quark effective theory. Theory should be expanded around the true vacuum like the Higgs model. In HQET, it can be said that whether the model pays respect on heaviness of the heavy quark. In other word, one should expand the theory in terms of $1/m_Q$. If it is not, then we say it is not expanded around the

true vacuum.

We would like to introduce one more fact/notion that the transformation is different from the approximation. When one truncates the perturbation up to a certain order of a coupling constant, this is an approximation. In this case one may use the values of parameters the same as before truncation or closer values as before. One can instead use transformation on the model so that we obtain the true vacuum closer than before. In this case, values of parameters become quite different from those before transformation.

2.1 HQET in field theory (Georgi transformation)

Heavy-light system respects the heavy quark symmetry whose idea is to use $1/m_Q$ as an expansion parameter. In the language of the field theory, it is equivalent to have a propagator which does not propagate in the configuration but only in the time direction. This is given by

$$\psi_v = \exp(-imv \cdot x)\psi,$$

which depends on the heavy quark velocity [3]. This is a little bit modified so that at the lowest order it

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can give positive energy wave function. Here x and v are four-component coordinate and heavy quark classical velocity, respectively. The Lagrangian becomes after the transformation:

$$L = \bar{\psi}(i - m\boldsymbol{\partial})\psi = \bar{\psi}_v i v \cdot \partial \psi_v,$$

$$\frac{1 + \boldsymbol{\not{v}}}{2} \psi_v = \psi_v.$$

This is a unitary transformation so that the probability is conserved.

2.2 HQET in potential model

In order to construct the heavy quark effective theory in a potential model, we assume as in the field theory that the dominant wave function of a $Q\bar{q}$ bound state should include a positive-energy heavy quark state Q_+ . This is because at the lowest order a heavy quark becomes a static color source, a heavy quark becomes still, and hence it can be decomposed into positive and negative energy states. This is realized by adopting the Foldy-Tani-Wouthuysen transformation [4]. The FTW has a feature that it makes a free kinetic term diagonal.

$$Q^\dagger (i\vec{p} \cdot \vec{\alpha} + m_Q \beta) Q \rightarrow Q_{\text{FTW}}^\dagger \sqrt{m_Q^2 + p^2} \beta Q_{\text{FTW}}.$$

The explicit form of this transformation is given by

$$\psi = \exp(W \vec{p} \cdot \vec{\gamma}) \psi_{\text{FTW}}, \quad \tan W = p/(m_Q + E), \quad (1)$$

which is a unitary transformation. This gives, as expected,

$$m_Q(1 - \beta)\psi_{\text{FTW}} = 0 \quad \text{or} \quad \frac{1 + \beta}{2} \psi_{\text{FTW}} = \psi_{\text{FTW}}.$$

2.3 Lowest FTW transformation

Let us write down the lowest non-trivial FTW transformation that gives the second order terms in $1/m_Q$ for the effective Hamiltonian, which are the same as those we have used in 1997 [1].

$$\psi = \begin{pmatrix} 1 & -\frac{\vec{\sigma}\vec{p}}{2m_Q} \\ \frac{\vec{\sigma}\vec{p}}{2m_Q} & 1 \end{pmatrix} \psi_{\text{LFTW}}, \quad \tan W = \frac{p}{2m_Q}, \quad (2)$$

which gives

$$U_{\text{LFTW}}(-\vec{p})(\vec{\alpha} \cdot \vec{p} + \beta m_Q) U_{\text{LFTW}}^{-1}(\vec{p}) =$$

$$\beta \left(m_Q + \frac{p^2}{2m_Q} \right) - \frac{p^2}{m_Q^2} \vec{\alpha} \cdot \vec{p} + O((1/m_Q)^3).$$

Compared with Eq. (1), the transformation given by Eq. (2) becomes very simple four by four matrix.

2.4 Free fermion field transformation

The former consideration naturally extends to another transformation which is adopted by many people. That is, many people use, at the lowest order in $1/m_Q$, the positive-energy heavy quark field is contained in the Hamiltonian. This transformation is given by,

$$\psi = \begin{pmatrix} 1 & 0 \\ \frac{\vec{\sigma}\vec{p}}{2m_Q} & 0 \end{pmatrix} \psi_{\text{FFF}} =$$

$$\begin{pmatrix} 1 & -\frac{\vec{\sigma}\vec{p}}{2m_Q} \\ \frac{\vec{\sigma}\vec{p}}{2m_Q} & 1 \end{pmatrix} \frac{1 + \beta}{2} \psi_{\text{FFF}}. \quad (3)$$

This is as can be seen not a unitary transformation because of a positive-energy projection operator, $(1 + \beta)/2$. This transformation is intentionally or not intentionally adopted by, e.g., Refs. [5–7], etc. This transformation only uses a positive energy component of the heavy quark so that negative energy contribution is neglected in calculations.

2.5 Summary of transformations

We give in Table 1 which paper adopts which transformation, whether it respects HQET (true vacuum) or not, values of D_{sJ} .

Table 1. Comparison of transformations.

Ref.	TF	TV	D_{sJ}
[8]	No	No	2.48, 2.55
[2]	FTW	Yes	2.325, 2.467
[5]	FFF	Yes	2.525, 2.593
[6]	FFF	Yes	2.38, 2.51
[7]	FFF	Yes	2.463, 2.535

In Table 1, Ref. means reference number, TF which transformation is used, TV true vacuum is used or not, and D_{sJ} their mass values. The observed mass values of D_{sJ} are given by 2.317, 2.460 GeV.

3 Application to $Q\bar{Q}$

In this section, we apply the Foldy-Tani-Wouthuysen transformation on both two heavy quarks of quarkonium $Q\bar{Q}$.

To obtain the eigenvalue equation in the first order of $1/m_Q$, we introduce the Foldy-Tani-Wouthuysen transformation for both of the heavy quarks Q_1 and \bar{Q}_2 in the Hamiltonian. To make the eigenvalue equation compact and convenient, we apply the charge conjugate transformation on the anti-heavy quark Q_2 so that one can multiply the anti-heavy quark

(\bar{Q}_2) operators from right with the wave function and heavy quark (Q_1) operators from left. These operators are unitary and are explicitly given by

$$U_{\text{FTW}i} \equiv \exp(W_i \vec{\gamma}_{Q_i} \cdot \vec{t}_i), U_c \equiv U_{\bar{Q}_{2c}} = i\gamma_{\bar{Q}_2}^0 \gamma_{\bar{Q}_2}^2, \quad (4)$$

where

$$\tan W = \frac{p}{m+E}, \quad \vec{t} = \frac{\vec{p}}{p}, \quad (p = |\vec{p}|). \quad (5)$$

Using these definitions, the transformation is given by

$$(U_{\text{total}} H U_{\text{total}}^{-1}) \otimes (U_{\text{total}} \psi) = E U_{\text{total}} \psi,$$

$$U_{\text{total}} = U_c U_{\text{FTW}2} U_{\text{FTW}1}. \quad (6)$$

Expanding $U_{\text{total}} H U_{\text{total}}^{-1}$, $U_{\text{total}} \psi$, and E in $1/m_1$ and $1/m_2$, we obtain a series of equations in each order of $1/m_i$.

In the case of $Q_1 = Q_2$, i.e., $m_1 = m_2 = m$, we obtain the following non-trivial eigenvalue equation in the lowest order of $1/m$.

$$\frac{p^2}{m} \psi_{0+1}^{s+,+} + \left[S(r) + V(r) + \frac{9V^2}{4m} \right] \psi_{0+1}^{s+,+} = E_{0+1} \psi_{0+1}^{s+,+}, \quad (7)$$

$$\frac{p^2}{m} \psi_{0+1}^{v+,+} + \left[S(r) + V(r) + \frac{V^2}{4m} \right] \psi_{0+1}^{v+,+} = E_{0+1} \psi_{0+1}^{v+,+}, \quad (8)$$

where the subscript “0+1” means that the non-trivial equation is obtained by adding 0-th and 1st order equations in $1/m$ and $E_{0+1} = E_0 + E_1$. The superscripts s and v mean that an i -th order wave function is decomposed as follows:

$$\psi_i = \begin{pmatrix} \psi_i^{+,-} & \psi_i^{+,+} \\ \psi_i^{-,-} & \psi_i^{-,+} \end{pmatrix},$$

$$\psi_i^{a,b} = \psi_i^{\text{sa},b} + (\vec{n} \cdot \vec{\sigma}) \psi_i^{\text{va},b}, \quad (9)$$

with $\vec{n} = \vec{r}/r$. Here we have denoted without confusion the transformed wave function, $(U_{\text{total}} \psi)_i$, as ψ_i . For $a = \pm$, they correspond to positive and negative energy components of a heavy quark and for $b = \pm$, they correspond to those of an anti-heavy quark. As one can see from Eqs. (7, 8) and $V^2 \propto 1/r^2$, the dominant potential near the origin is given by $1/r^2$ with a positive coefficient in the first order of $1/m$. Hence one should include this term when solving Eqs. (7, 8) at the lowest non-trivial order differential equation. Contrary to the lattice results [9], we have no problem to solve these equations because of positive coefficients for $V^2 \propto 1/r^2$ terms. When they are negative, one faces a critical value for this coefficient below which one can not obtain a solution [9, 10].

References

- 1 Matsuki T, Morii T. Phys. Rev. D, 1997, **56**: 5646–5667
- 2 Matsuki T, Morii T, Sudoh K. Prog. Theor. Phys., 2005, **117**: 1007–1012
- 3 Georgi H. Phys. Lett. B, 1990, **240**: 447–450
- 4 Foldy L L, Wouthuysen S A. Phys. Rev., 1950, **78**: 29–36; Tani S. Prog. Theor. Phys., 1951, **6**: 267–285
- 5 Morishita J, Kawaguchi M, Morii T. Phys. Rev. D, 1988, **37**: 159–178
- 6 ZENG J, Van Orden J W, Roberts W. Phys. Rev. D, 1993, **52**: 5229–5241; Di Piero M, Eichten E. Phys. Rev. D, 2001, **64**: 114004
- 7 Ebert D, Galkin V O, Faustov R N. Phys. Rev. D, 1998, **57**: 5663–5669
- 8 Godfrey S, Kokoski R. Phys. Rev. D, 1991, **43**: 1679–1687
- 9 Koma Y, Koma M, Wittig H. Phys. Rev. Lett., 2006, **97**: 122003
- 10 Landau L D, Lifshitz E M. Quantum Mechanics. Elsevier Science Ltd, 2003