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Challenges in uncovering the origin of the proton's spin*

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Abstract One of the most fascinating challenges facing modern strong interaction physics is to understand the origin of the spin of the nucleon in terms of the spin and orbital angular momentum of the quarks and gluons. We review recent progress on this problem as well as some of the uncertainties associated with state of the art lattice QCD simulations. In particular, we explain the importance of the corrections associated with chiral extrapolation and finite volume corrections, especially for the term $B(0)$ extracted from the appropriate low moment of the deeply virtual Compton scattering amplitude.

Key words nucleon spin, QCD, lattice QCD, deeply-virtual Compton scattering, cloudy bag

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1 Introduction

Within simple, non-relativistic quark models the spin of the proton is carried entirely as the spin of the constituent quarks. Relativity, as represented by the MIT bag, for example, introduces a lower component with angular momentum one into the ground-state wave function and as a consequence replaces about 35% of the quark spin by quark orbital angular momentum. Thus, when the European Muon Collaboration measured the spin structure function of the proton accurately enough to extract the flavor singlet fraction of the proton spin arising from quark spin, the expectation was that it should be around 65%. Instead they found a value consistent with zero [1] and the so-called "proton spin crisis" was born.

It was very soon realized that through the quark-anti-quark box diagram, polarized gluons in the proton make an essentially point-like, non-partonic contribution to the proton spin structure function [2–4] going like $N_{\rm f}\alpha_{\rm s}(Q^2)\Delta G(Q^2)/2\pi$, with $N_{\rm f}$ the number of active flavors and ΔG the total helicity of the gluons, in a proton with positive helicity, at scale Q^2. Much of the experimental effort since the discovery of the spin crisis has gone into looking for the large polarized gluon content required to explain the measurement, namely $\Delta G(3\ {\rm GeV}^2) \sim 4.0$. The most recent values from RHIC and COMPASS are compatible with ΔG being very small on this scale, almost certainly less than 0.4 [5]. Even though the experimental values for the quark spin content have moved up significantly, to $\Sigma = 0.33 \pm 0.03 \pm 0.05$ [6, 7], it is now clear that the polarized glue is insufficient to explain the reduction from the expectation of 65% mentioned earlier.

As a result of these developments, work on the proton spin has taken two different paths. The first has been to redefine the proton spin problem as the challenge of how much of the proton's spin of 0.5 comes from gluon spin and orbital angular momentum and how much from the spin and orbital angular momentum of each quark flavor. One of the theoretical challenges here is find a widely agreed, physical definition of each of these quantities and there has been considerable debate on this issue [8, 9]. The second path has been to return [10] to alternative attempts to explain the the spin crisis [11, 12] in the light of the new, higher experimental value. Clearly these two paths ultimately have the same aim and a satisfactory answer to the first must eventually explain the second.

An important tool in both approaches is the capacity to combine information from lattice QCD with that from experiment in order to pin down the various contributions. In particular, the low moments of the energy momentum tensor

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$$\langle p'|T^{\mu,\nu}|p\rangle = \bar{u}(p')\big[A(\Delta^2)\gamma^{(\mu}\bar{P}^{\nu)} + B(\Delta^2)\bar{P}^{(\mu}\mathrm{i}\sigma^{\nu)\alpha}\Delta_\alpha/2M+\dots\big]u(p), \quad (1)$$

(with $\bar{P}=(p+p')/2$ and $\Delta=p-p'$), can be evaluated in lattice QCD and the combination $A(0)+B(0)$ for a particular quark flavor is indeed the total angular momentum carried by it [13, 14]. In a recent investigation of the successful explanation of the latest value of the proton spin sum by Myhrer and Thomas, it was shown that under QCD evolution the orbital angular momenta carried by up and down quarks actually cross over and contrary to expectations based on the values within the model (i.e. at the resolution scale approriate to a valence dominated quark model) agree at least qualitatively with the lattice QCD information [15]. This claim has recently been challenged [16] and we feel that it is worthwhile to examine the issues around that challenge in some detail as it goes to the heart of what we currently "know", as opposed to what we think we know. It is also fundamental when it comes to setting targets for future experimental programs in deeply virtual Compton scattering (DVCS). We argue that the current systematic errors on the quantity $B(0)$ extracted from lattice QCD are considerably larger than widely believed.

2 Explanation of the spin crisis

The explanation of the spin crisis by Myhrer and Thomas [10] (MT) involved two additional ingredients of any realistic modern description of hadron structure, namely the inclusion of the gluon exchange hyperfine interaction and the pion cloud required by chiral symmetry. Studies of the mass of the N and Δ in modern lattice QCD have established that there is no significant double counting in including both of these effects [17]. Individually they remove perhaps half of the difference between the modern spin sum rule and the relativistic quark model expectation of 65%. In combination they reduce the spin carried by the quarks to very close to the experimental value. Indeed, MT reported a value $\Sigma\in(0.35,0.40)$, which agrees very well with the experimental value.

Very recently Bass and Thomas [18] re-examined this in a slightly more sophisticated treatment in which g_A^3 was required to agree with experiment. This raised the range a little, to 0.42 ± 0.07 – still in agreement with the experimental data but with a little room for a small residual contribution from the axial anomaly. (A value of $\Delta G(3\ \mathrm{GeV}^2)\sim 0.4$ yields a reduction in Σ of order 0.06.) More interesting in that work was the corresponding evaluation of g_A^8, which is required in order to extract the quark spin content from the integral of g_1^p. These authors found that $SU(3)$ symmetry was broken at the level of 20%, yielding a value of $g_A^8=0.46\pm0.05$. This in turn raises the experimental value of $g_A^{(0)}|_{\mathrm{inv}}$ to $0.36\pm0.03\pm0.05$.

Within the MT model it is possible to decompose the proton spin into the contributions from the spin and orbital angular momentum of the quarks and anti-quarks. The result is that $L^{\mathrm{u,d}}\sim(0.25,0.06)$, where each term includes the quark and anti-quark contributions. These values differ dramatically from the typical values coming out of lattice calculations [19], namely $L^{\mathrm{u,d}}\sim(-0.21,0.22)$ – where, in deriving these values from $J^{\mathrm{u,d}}$ we used $\Delta u=+0.84$ and $\Delta d=-0.44$. This rather dramatic difference, can be understood, at least qualitatively, in terms of the evolution from a scale consistent with a valence dominated quark model (well below 1 GeV^2) to the scale of the lattice QCD calculations (of order 4 GeV^2) [15].

The issue raised very recently by Wakamatsu is the extent to which the agreement between the evolved values of $L^{\mathrm{u,d}}$ within the MT model and the values extracted from experimental data and lattice QCD is or is not satisfactory. This is entirely a matter of how one assesses the errors in the latter and it is that we consider next.

3 Error assessment

For the time being it is not possible to calculate the flavor singlet contributions to the angular momentum in lattice QCD because the disconnected terms have proven too difficult to distinguish from the noise in the calculation. (We note that for the electric and magnetic form factors of the proton, as well as for the first moment of the parton distributions, there has recently been significant progress [20, 21] but while this promises well for the future it adds nothing to the present discussion.) For the present, there is no believable error estimate for the singlet combinations from this source and therefore we do not discuss this further. Clearly, in these circumstances the value for $J^{\mathrm{u+d}}$ obtained in Ref. [15], namely 0.30 at a scale of 4 GeV^2, is in perfectly acceptable agreement with that found by Hägler et al., namely $(0.25,0.29)$.

The really interesting case is the non-singlet term involving $L^{\mathrm{u-d}}$, where there is no disconnected term and the lattice calculation therefore has a claim to reliability. This case is also fascinating from the physics point of view because, as observed by Wakamatsu and his collaborators [22], the widely used chiral quark soliton model yields a very different value from other models. In particular, there is a very large contribution from the non-linear pion field in the model which leads to a large negative value for $L^{\rm u-d}$, namely -0.33 (at the model scale). In this case, the QCD evolution curves for $L^{\rm u}$ and $L^{\rm d}$ do not cross.

To evaluate $L^{\rm u-d}$ one needs to determine $A^{\rm u-d}$, $B^{\rm u-d}$ and $\Delta u-\Delta d$:

1) The first is very well determined by measurements of the parton distribution functions, yielding $A^{\rm u-d}=0.155\pm0.005$.

2) The only way to determine $B(0)$ is through DVCS or lattice QCD. For the former the only determination, using data from Hermes and JLab, is at an uncomfortably low value of Q^2 and totally dependent on the model used. This leaves lattice QCD for the moment, where the measurement of $B(0)$ in general is very complex. As we see from Eq. (1), the B-term is proportional to $\varDelta$ and hence vanishes when tends to zero. Thus, in addition to the usual lattice requirements that one extrapolate to $a=0$, $L=\infty$ and the physical quark masses (i.e. the continuum limit in an infinite box at the physical pion and kaon masses), to obtain $B(0)$ one must also calculate at finite momentum transfer and extrapolate to $\varDelta^2=0$. In practice, the extrapolations in the lattice spacing, a, and volume, L^3, have not yet been done. However, the extrapolation in m_π and $\varDelta^2$ has been done using a low order chiral fit, linear in $\varDelta^2$ up to 1.2 GeV^2. It is not yet possible to assess the systematic errors associated with this procedure but a re-analysis by Wang and Thomas [23] using a finite range regulator yielded similar results to those found by Hägler et al.

The really challenging problem associated with $B(0)$ is rather less obvious. In fact, it contains the information on the spin content of the proton and hence, implicitly, $g_{\rm A}^3$, or $\Delta u-\Delta d$. The extremely well known problems of calculating $g_{\rm A}^3$ on the lattice are still there in the calculation of $B(0)$ but hidden below the veneer of all the other challenges such as extrapolating in m_π and $\varDelta^2$. We discuss the issues with $\Delta u-\Delta d$ next.

3) The anomalous behaviour of $g_{\rm A}$ in the chiral limit, where the gradient of the pion field appearing in the axial current makes a surface contribution, has been known for many years. Studies within chiral quark models have shown that this can lead to anomalous behaviour in $g_{\rm A}$ on small volumes at low m_π [24]. For example, a careful study by the RBC-UKQCD collaboration showed that even on a lattice of side 2.7 fm and at $m_\pi^2\sim0.12$ GeV^2, $g_{\rm A}$ was only 1.08 ± 0.05 and decreasing rapidly as m_π decreased [25]. They confirmed that this was primarily an effect of the lattice volume, concluding that "to keep FVE's [finite volume errors] at or below 1%, then for $m_\pi=330$ MeV, spatial sizes of 3.4–4.1 fm are necessary".

The LHPC simulations of $A(0)$ and $B(0)$ were based on a lattice of size 2.5 fm, which is certainly not large enough to compute $g_{\rm A}^3$ accurately. Indeed, the RBC-UKQCD results would suggest that the corresponding value of $g_{\rm A}^3$ at the physical pion mass could be below 1. We stress that this is *implicit* in the calculation. Without a separate calculation of $g_{\rm A}^3$ using the same chiral extrapolation on the same configurations one can only guess at the values of Δu and Δd that need to be subtracted from $J^{\rm u}$ and $J^{\rm d}$ in order to deduce $L^{\rm u}$ and $L^{\rm d}$. (All this is separate from the issue of the uncertainty introduced by the need to extrapolate in $\varDelta^2$.) What is absolutely clear is that subtracting (one half of) the physical value of $g_{\rm A}^3$ from $J^{\rm u-d}$ to obtain $L^{\rm u-d}$ almost certainly introduces a very large error.

We stress that these remarks are intended to enhance our understanding of this very important problem and as guidance for future work and by no means as a criticism of the superb effort that has gone into the lattice determination of the low moments of the energy-momentum tensor.

As just an illustration of the potential effect of the finite volume corrections to the lattice QCD simulations of $B(0)$, we suppose that the value of Δu–Δd, at the physical pion mass, implicit in the work of Hägler et al., is 0.9, rather than the experimental value of 1.27. We stress that this value is conservatively high with respect to the value suggested for this lattice size in the RBC-UKQCD work. Using $J^{\rm u-d}=0.22$ one would then derive $L^{\rm u-d}=-0.23$, rather than the value -0.42 suggested in Ref. [25] and used by Wakamatsu. If the spin were more effected by finite volume corrections than $g_{\rm A}^3$, a reasonable assumption that nevertheless needs more study, $L^{\rm u-d}=-0.23$ would be a much better estimate of the physical value at 4 GeV^2. This is also, perhaps coincidentally, in very good agreement with the value (at 4 GeV^2) derived within the MT model.

4 Conclusion

This is a very exciting time to be working in hadronic physics. We can reasonably expect to resolve the very fundamental question of the origin of the proton spin within the next 5–10 years. This will come as a result of advances in lattice QCD as well as in experimental physics – with the 12 GeV upgrade at JLab allowing the systematic study of DVCS for the first time. The results of this work will provide deep new insights into how QCD works.

However, for the time being we "see as through a glass darkly". We have a very satisfactory explanation of the spin crisis in terms of the effect of gluon exchange and chiral symmetry, with firm predictions that, as a consequence, a large fraction of the proton spin is carried as orbital angular momentum by the quarks and anti-quarks. Nevertheless, when it comes to testing this explanation, there are potentially large systematic errors associated with the interpretation of the state of the art lattice simulations, especially for $L^{\mathrm{u-d}}$. That, in turn, has a severe impact on the possible conclusions one can draw about the distribution of spin and angular momentum on the quarks. It will also be very important to eventually test whether or not the rather unusual role of the sea in the chiral quark soliton model, which clearly distinguishes it from other models, is supported by lattice QCD and experimental data.

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