CPC(HEP & NP), 2010, **34**(9): 1331–1334 Chinese Physics C Vol. 34, No. 9, Sep., 2010

Quark-clustering in cold quark matter*

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Abstract Quarks are proposed to be grouped together to make quark-clusters due to the strong interaction in cold quark matter at a few nuclear densities, because a weakly coupling treatment of the interaction between quarks there would be inadequate. Cold quark matter is then conjectured to be in solid state (i.e., forming a crystal structure) if the inter-cluster potential is deep enough to localize clusters in lattice. Such a solid state of cold quark matter would be very necessary for us to understand different manifestations of pulsar-like compact stars, and could not be ruled by first principles.

Key words quark matter, neutron stars, pulsars, nuclear matter, quantum chromodynamics

PACS 21.65.Qr, 97.60.Jd, 97.60.Gb

1 Introduction to the physics of matter at supra-nuclear density

On one hand, pulsar-like compact stars are unique laboratories for the physics of cold matter at supranuclear densities, which certainly relates to the fundamental color interaction between quarks. However, one of the challenges for physicists today is to understand the strong interaction at low energy scales. Although QCD (quantum chromo-dynamics) is believed to be the underlying theory for describing the elementary strong interaction and has been tested well at high energy scale, the non-perturbative nature of QCD at low energy scale makes it difficult for us to deal with particle systems under strong interaction, especially the case of cold matter at a few nuclear densities.

On the other hand, one of the challenges for astrophysicist is to understand the nature of pulsar-like compact stars, although it has been more than 40 years since the discovery of the first pulsar. Of particular interest is whether the density in such compact stars could be high enough to result in unconfined quarks (quark matter). Stars composed of deconfined quarks (and possible gluons) as the dominant degrees of freedom are called quark stars (or strange stars because of the existence of strange quarks), in contrast with normal neutron stars whose dominant degrees of freedom are hadrons, and there are possible observational evidences that pulsar-like stars could be quark stars [1]. It depends on the state of cold quark matter at supra-nuclear densities to model quark stars, which is unfortunately not certain due to the non-perturbative nature of QCD.

Therefore, the study of cold quark matter opens an important window to connect three active fields: particle physics, condensed matter physics, and astrophysics. The combination of these fields could be helpful and necessary for us to explore the nature of both strong interaction and pulsar-like compact stars.

2 Cold quark matter: color superconducting v.s. quark clustering

QGP (quark-gluon plasma) is a state predicted theoretically in QCD, when the temperature is extremely high ($\gtrsim 200$ MeV). QGP can be also call hot quark matter, and could be produced in the experiments of relativistic heavy ion collisions. Cold quark matter, on the other hand, could only be produced at extremely high chemical potential, and it could

Received 19 January 2010

* Supported by NSFC (10778611, 10973002), National Basic Research Program of China (2009CB824800) and LCWR (LHXZ200602)

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only exist in rare astrophysical conditions, the pulsar-like compact stars. In principle, we could explore the properties of cold quark matter by astrophysical observations, sine there are possible evidences that pulsar-like stars are actually quark stars.

At first glance, the ground state of extremely dense quark matter seems that of an ideal Fermi gas. Nevertheless, it has also been found that the highly degenerate Fermi surface is unstable against the formation of quark Cooper pairs, which condense near the Fermi surface due to the existence of color attractive channels between the quarks. In fact, a BCS-like color superconductivity, similar to electric superconductivity, has been formulated within perturbative QCD (pQCD) at ultra-high baryon densities. It has further been argued, based on QCD-like effective models, that color superconductivity could also occur even at the more realistic baryon densities of pulsar-like compact stars. Therefore a color superconductivity state [2] is currently focused on in studying cold quark matter.

However, is there any other possibility about the state of cold quark matter? What kind of cold quark matter can we expected from observations of compact stars rather than from first principles? We will discuss another conjectured state of cold quark matter here, i.e., the state of quark-clusters.

Let's begin with some of order-of-magnitude estimates. For a typical pulsar, the mass is about $1.4\mathrm{M}_\odot$ and the radius is about 10 km, then the average density mass density is $\rho \simeq 2.4\rho_0$, where ρ_0 is the nuclear matter density. Let us see some properties of quark matter with mass density $\rho \sim 3\rho_0$. Strange quark matter is most reliable cold quark matter, which is composed of u, d and s quarks, so their number densities are $n_\mathrm{u} = n_\mathrm{d} \sim n_\mathrm{s} \approx 0.48\ \mathrm{fm}^{-3}$, and the total quark number density is $n_\mathrm{q} \approx 1.4\ \mathrm{fm}^{-3}$, with the distance between quarks $l_\mathrm{q} = n_\mathrm{q}^{-1/3} \approx 0.9$ fm. The de Broglie wavelength is $\lambda_\mathrm{q} = h/\sqrt{3m_\mathrm{q}kT} \approx 4\times10^3\ \mathrm{fm}\ (m_\mathrm{q}/300\ \mathrm{MeV})^{-\frac{1}{2}}(T/10^6\mathrm{K})^{-\frac{1}{2}}$. Therefore, for $m_\mathrm{q} \simeq 300$ MeV and the temperature $T \simeq 10^6$ K (the typical temperature inside pulsars), $\lambda_\mathrm{q} \gg l_\mathrm{q}$, which means that the quantum Fermi-Dirac statistics should be applied. Turning off the interaction between quark, then we can get the degenerate quark chemical potential at zero temperature. For the non-relativistic case, $\mu_\mathrm{s}^\mathrm{NR} \sim \mu_\mathrm{u}^\mathrm{NR} = \mu_\mathrm{d}^\mathrm{NR} = \hbar^2(3\pi^2 n_\mathrm{u})^{\frac{2}{3}}/2m_\mathrm{q} \approx 380\ \mathrm{MeV} \gg T$, and for the extremely relativistic case, $\mu_\mathrm{s}^\mathrm{ER} \sim \mu_\mathrm{u}^\mathrm{ER} = \mu_\mathrm{d}^\mathrm{ER} = \hbar c(3\pi^2 n_\mathrm{u})^{\frac{1}{3}} \approx 480\ \mathrm{MeV} \gg T$.

However, the strong interaction should play an important role. The QCD coupling constant decreases as the energy scale μ increases, and in pQCD it can be written approximately as

$$\alpha_\mathrm{s} \approx \frac{4\pi}{\left(11 - \frac{2}{3}n_\mathrm{f}\right)\ln\frac{\mu^2}{\Lambda^2}}, \tag{1}$$

where n_f is the number of quark flavors, $\Lambda \sim 200$–300 MeV is the renormalization parameter, and μ is of the order of chemical potential. Therefore, for $n_\mathrm{f} = 3$, we can get $\alpha_\mathrm{s} \simeq (0.8$ to $3)$ for $\rho \sim 3\rho_0$ although pQCD would not be applicable in this case. We can also see that even if $\rho \sim 10^6\rho_0$, then $\alpha_\mathrm{s} \sim 0.15$, which indicates that the interaction is still much stronger than the electromagnetic interaction (with coupling constant $\alpha_\mathrm{em} \simeq 1/137$). In a word, one could possibly have $\alpha_\mathrm{s} > 1$ in the realistic cold quark matter, and this strong interaction could render the quark grouped into clusters, rather than making quarks condensate in momentum space to form color super-conductivity.

If Q_α-like particles are created in cold quark matter [3], the distance between neighbored clusters is about 2 fm if the mass density $\rho = 3\rho_0$. From the Heinsenberg's uncertainty relation, the quarks inside one cluster with length scale l have typically the kinetic energy of $\sim \hbar^2/(m_\mathrm{q}l^2)$. They are bound by color interaction with energy of $\sim \alpha_\mathrm{s}\hbar c/l$. The length scale l can be estimated by equaling the kinetic energy and color interaction energy, then $l \sim \hbar c/(\alpha_\mathrm{s} m_\mathrm{q} c^2) \sim 1\ \mathrm{fm}/\alpha_\mathrm{s}$, and the interaction energy $\sim 300\alpha_\mathrm{s}^2$ MeV. Then quarks are bound tightly inside one quark-cluster, and because the length scale of one cluster ($l < 1$ fm) is much shorter than the inter-cluster distance (~ 2 fm) in this case, quark-clusters can be considered as classical particles in cold quark matter.

3 To understand observations in solid quark star models

Quark-clusters in cold quark matter would crystallize to form classical solid, if the inter-cluster potential is deep enough. Various astrophysical observations could be understood in terms of solid quark star models.

High-mass pulsars. The solid quark star models are much different from the conventional models (e.g., MIT bag model) in which quarks are relativistic particles. For a relativistic system, the pressure is proportional to the energy density, so it cannot have stiff equation of state. In a solid quark star, on the other hand, quarks are grouped in clusters and these clusters are non-relativistic particles, which means that it could have stiffer equation of state. A stiffer equation of state could have very important astrophysical implications because it can lead to a higher maximum mass. Some recent observations have indicated massive ($\sim 2\mathrm{M}_\odot$) pulsars [4], which could be explained naturally in solid quark star models. One of the solid quark star models is that the inter-cluster potential could be described by the Lennard-Jones form just like the case in a system of inert gas [5]

$$u(r)=4U_0\left[\left(\frac{r_0}{r}\right)^{12}-\left(\frac{r_0}{r}\right)^{6}\right], \qquad (2)$$

where U_0 is the depth of the potential well, r_0 is the range of the inter-cluster force. In this model, the equation of state can be very stiff, because at a small inter-cluster distance, there is a very strong repulsion. Under some reasonable values of parameters, a quark star with Lennard-Jones matter could be very massive ($> 2\mathrm{M}_\odot$). The mass-radius curves and mass-central density curves (the central density only includes the rest mass energy density) are shown in Fig. 1. The mass of a star increases and reaches the maximum value as the increases of the central density, and after that, the star will become gravitationally instable.

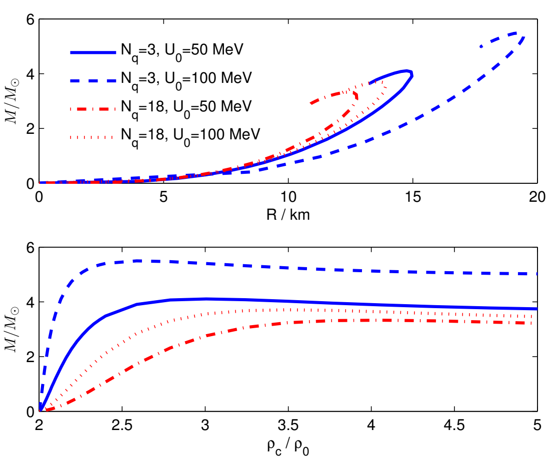

Fig. 1. The mass-radius and mass-central density (rest-mass energy density) curves, for a given surface density $\rho_s = 2\rho_0$. N_q is the number of quarks in one quark-cluster, and U_0 is the depth of the inter-cluster potential.

Energy releases during star-quakes. A solid stellar object would inevitably result in star-quakes when strain energy develops to a critical value, and a huge of gravitational and elastic energies would then be released. Actually there are two types of stress force inside solid stars, and two factors could result in the development of stress energy in a solid star. (i) As a quark star cools (even spinning constantly), the changing state of matter may cause the development of anisotropic pressure distributed inside solid matter. (ii) A uniform fluid star would keep approximately its eccentricity of Maclaurin spheres, and the eccentricity decreases as a star spins down. However, for a solid star, the shear stress would prevent the eccentricity of the star from decreasing during spin-down. In this case, even the state of matter does not change, and stress energy could still develop as the solid star spins down.

During an accretion process, the solid star could additionally support the accreted matter against gravity by these forces, unless the forces become so strong that a star-quake occurs. This is the so-called AIQ (Accretion-Induced star-Quake) mechanism proposed previously [6], which might be responsible to the bursts (even the supergiant flares) and glitches observed in soft γ-ray repeaters/anomalous X-ray pulsars [7]. The gravitational energy releases during star-quakes for the polytropic quark star model have been calculated [8]. The results for polytropic index $n = 1$ are shown in Fig. 2. Three supergiant flares from soft γ-ray repeaters have been observed, with released photon energy being order of $\sim 10^{47}$ ergs. Our numerical results imply that for all the parameters we chosen, the released energy could be as high as the observed.

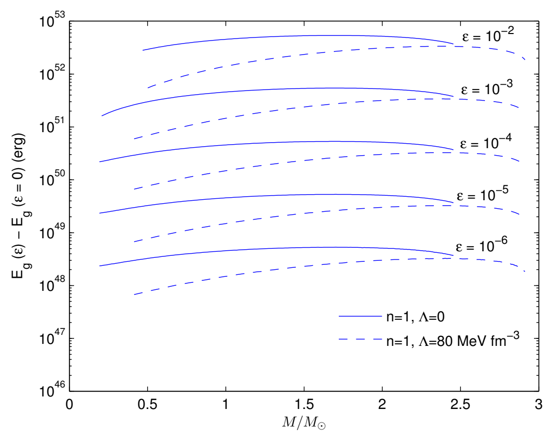

Fig. 2. The gravitational energy difference between stars with and without anisotropic pressures, which may be released during sequential star quakes. Λ is the difference of vacuum energy between inside and outside of a quark star, and ε denotes the deviation of tangential $P_\perp$ from the radial pressure P_r, $\varepsilon = (P_\perp - P_r)/P_r$.

Pulsar glitches. Pulsar glitches, which are the occasional disruptions of regular pulses, provide us with rich source of information concerning the properties of pulsars. Glitches can happen during the star-quakes of solid quark stars, but cannot be explained in terms of conventional quark star models in which quark stars are considered to be in liquid state. In terms of normal neutron stars, the leading glitch model involves the angular momentum transfer in the crust from the superfluid to the normal component; however, the observations of long period precession for pulsars are inconsistent with this glitch model, and in addition, it can hardly explain the recent discovered slow glitches. A solid pulsar with crust could solve these problems, and both normal and slow glitches can then be modeled [9, 10].

Others. It is worth mentioning that some observations of pulsars favor bare strange stars (i.e., a quark star without crust) over normal neutron stars, although they do not depend on the specific state of matter inside quark stars. Clear drifting sub-pulses suggest that both positively and negatively charged particles should be strongly bound on the surface of a pulsar, which could be explained naturally if the pulsar is a bare strange star. Another example is the observational properties of dead pulsars that do not have radio emission because the potential drop in the open field line region is lower than a critical value: one would expect that both thermal X-ray emission and atomic lines from atmospheres of neutron stars should have been discovered; however, no clear atomic features has been found, and such featureless spectrum could be a probe for identifying bare strange stars. Moreover, a one-dimensional supernova calculation shows that the lepton-dominated fireball supported by a bare quark surface do play a significant role in the explosion dynamics under a photon-driven scenario [11]. To distinguish bare strange stars and neutron stars is important, which is in fact the first step to demonstrate the existence of solid quark stars. A quark star should be in solid state in order to explain observational properties of pulsar-like compact stars, as we have discussed in this section.

4 Conclusions and discussions

Realistic cold quark matter is suggested in a solid state, where quark-clustering occurs, and solid quark stars could also be very necessary to understand various observations. To explore the real QCD phases of cold quark matter, it should be necessary to combine different research fields, such as the Lattice QCD, the QCD-based effective models (e.g., NJL model and DSE), and the phenomenological models proposed from astrophysical side.

In this paper, the quark stars are supposed to be normal or classical solid, in which the quark-clusters are in periodic lattices and the barrier penetration is negligible. However, another possibility of quantum or super solid can still not be ruled out in effective QCD models. The BCS-type quark pairing was proposed to form at a Fermi surface of cold quark matter, and the shear moduli of the rigid crystalline color super-conducting quark matter could be much larger than those of neutron star crusts [12], and this might provide explanations to pulsar glitches too.

Cold quark matter could be in normal solid state if interaction of the lattices (quark-clusters) is so strong that each lattice can only vibrate deep inside the potential well. In contrast, quantum solid state could exist when the interaction of lattices is relatively weak and the potential wells are not deep enough, so that the quantum penetration is significant. It could be interesting to observationally distinguish between normal solid and quantum solid state in the future.

We would like to acknowledge useful discussions at our pulsar group of PKU.

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