CPC(HEP & NP), 2010, **34**(7): 953–957 **Chinese Physics C** Vol. 34, No. 7, Jul., 2010

Research on the prompt neutron multiplicity of fission fragments for ^{235}U(n,f)

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Abstract According to some experimental and evaluated data, the total excitation energy partitioning way between both of the fission fragments was given with a semi-empirical method. With the calculated energy partitioning way, the prompt neutron multiplicity as a function of fragment mass, $\bar{\nu}(A)$, for neutron-induced fission of ^{235}U at $E_{\rm n}$=0.0253 eV, 3 MeV, and 5 MeV was calculated. The results are checked with the total average prompt neutron multiplicities $\bar{\nu}$ and compared with the experimental and evaluated data.

Key words fission fragment, energy partition, neutron multiplicity

PACS 25.85.Ec, 24.10.Ca, 24.75.+i

1 Introduction

Substantial study on the mass, charge, kinetic energy, neutron and gamma release of the fission fragments has been carried out for a long time, and the prompt neutrons multiplicity as a function of fragment mass, $\bar{\nu}(A)$, is one of the most interesting topics in this regard. Prompt neutrons as the product of a fission reaction afford important information to understand the fission process quantitatively. In the practical application, an accurate $\bar{\nu}(A)$ can provide a possible way to deduce the pre-neutron (post-neutron) emission mass yields from the already known post-neutron (pre-neutron) emission mass yields.

For ^{235}U, the prompt neutrons multiplicity distribution $\bar{\nu}(A)$ is only experimentally known in the case of ^{235}U($n_{\rm th}$,f), and the theoretical calculation for $\bar{\nu}(A)$ is not so good. The main problem is the partitioning of total excitation energy ($E_{\rm TXE}$) between both light and heavy fission fragments, and this is a long-standing problem. In Ref. [1], the Monte Carlo approach was used to infer $\bar{\nu}(A)$ for neutron-induced fission of ^{235}U at $E_{\rm n}$=0.53 MeV with two different hypotheses for partitioning the $E_{\rm TXE}$, but the results are not good enough.

The main purpose of this paper is to simulate the possible energy partitioning in the light and heavy fragments using a semi-empirical method. Then, using this energy partitioning way, the prompt fission neutron multiplicity distributions of the n+^{235}U reaction at thermal, 3 MeV and 5 MeV neutron energies were investigated based on the statistical evaporation model. In this incident energy range, only one compound nucleus (^{236}U) undergoing fission is formed (the so-called first chance fission).

2 Theoretical approach

The key point about calculating $\bar{\nu}(A)$ is to simulate the emission of neutrons from each fission fragment, so how the $E_{\rm TXE}$ gets partitioned into the light and heavy fragments is needed to be known. According to Ref. [1], one semi-empirical method for partitioning the $E_{\rm TXE}$ is considered: the $E_{\rm TXE}$ is split according to the experimental values of the average total number of emitted neutrons [$\bar{\nu}_{\rm exp}(A)$], the average neutron kinetic energy [$\langle\varepsilon\rangle_{\rm exp}(A)$], and the total average energies removed by γ rays [$\bar{E}_{{\rm exp},\gamma}(A)$] as a function of fission fragment mass. In the present work, the initial excitation of each fragment is inferred by considering this semi-empirical method. This condition reads as follows:

$$E^*(A_{\rm L,H}) = E_{\rm TXE}\frac{\bar{\nu}_{\rm exp}(A_{\rm L,H})\langle\eta\rangle_{\rm L,H}+\bar{E}_{{\rm exp},\gamma}(A_{\rm L,H})}{\sum\limits_{\rm i=L,H}[\bar{\nu}_{\rm exp}(A_{\rm i})\langle\eta\rangle_{\rm i}+\bar{E}_{{\rm exp},\gamma}(A_{\rm i})]}, \tag{1}$$

Received 10 August 2009

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where $E_{\rm TXE}$ is the total excitation energy for the light and heavy pairs $A_{\rm L}$ and $A_{\rm H}$. L and H refer to the light and heavy fission fragment, $E^*(A_{\rm L,H})$ is the initial excitation energy of light or heavy system, $\langle\eta\rangle_{\rm L,H}$ is equal to the average energy removed per emitted neutron from light and heavy fragment

$$\langle\eta\rangle_{\rm L,H} = \langle\varepsilon\rangle_{\rm exp}(A_{\rm L,H}) + \frac{1}{2}E_{\rm B,2n}(Z_{\rm L,H}, A_{\rm L,H}), \qquad (2)$$

where $E_{\rm B,2n}(Z_{\rm L,H}, A_{\rm L,H})$ is the two neutron binding energy and $\langle\varepsilon\rangle_{\rm exp}(A_{\rm L,H})$ is the average neutron kinetic energy for neutron emission from a given fission fragment.

The total excitation energy $E_{\rm TXE}$ is given as follows:

$$E_{\rm TXE} = E_{\rm r}^* + E_{\rm B}(A_{\rm c}) + E_{\rm n} - E_{\rm TKE}, \qquad (3)$$

where $E_{\rm B}(A_{\rm c})$ is the binding energy of fission compound nucleus, and $E_{\rm n}$ is the kinetic energy of the neutron inducing fission. $E_{\rm TKE}$ is the total kinetic energy of the two fission fragments, and is taken from experimental data. $E_{\rm r}^*$ is the energy release in the fission process, which is given, in the case of binary fission, by the difference between the compound nucleus and the fission fragments (FF) masses:

$$E_{\rm r}^* = M(Z_{\rm c}, A_{\rm c}) - M(Z_{\rm L}, A_{\rm L}) - M(Z_{\rm H}, A_{\rm H}), \qquad (4)$$

where M is the mass excess in mega-electron-volts, c refers to the compound nucleus. In the present work, $E_{\rm B,2n}(Z_{\rm L,H}, A_{\rm L,H})$, $M(Z_{\rm L}, A_{\rm L})$ and $M(Z_{\rm H}, A_{\rm H})$ are replaced by their reduced values $E_{\rm B,2n}(A_{\rm L,H})$, $M(A_{\rm L})$ and $M(A_{\rm H})$, respectively, and as determined by weighting with the independent fission-fragment yields $Y_{\rm f}(A_{\rm L,H}, Z_{\rm L,H})$ of the same mass chain,

$$M(A_{\rm L,H}) = \frac{\sum\limits_Z M(A_{\rm L,H}, Z_{\rm L,H}) \times Y_{\rm f}(A_{\rm L,H}, Z_{\rm L,H})}{\sum\limits_Z Y_{\rm f}(A_{\rm L,H}, Z_{\rm L,H})}, \qquad (5)$$

$$E_{\rm B,2n}(A_{\rm L,H}) = \frac{\sum\limits_Z E_{\rm B,2n}(A_{\rm L,H}, Z_{\rm L,H}) \times Y_{\rm f}(A_{\rm L,H}, Z_{\rm L,H})}{\sum\limits_Z Y_{\rm f}(A_{\rm L,H}, Z_{\rm L,H})}. \qquad (6)$$

Now, the average prompt neutron multiplicity of each fission fragment A is calculated using the following relation:

$$\bar{\nu}(A_{\rm L,H}) = \frac{E^*(A_{\rm L,H}) - \bar{E}_\gamma(A_{\rm L,H})}{\langle\eta\rangle_{\rm L,H}} = \frac{E^*(A_{\rm L,H}) - \bar{E}_\gamma(A_{\rm L,H})}{\langle\varepsilon\rangle(A_{\rm L,H}) + \frac{1}{2}E_{\rm B,2n}(A_{\rm L,H})}. \qquad (7)$$

Actually, the experimental data $\bar{\nu}_{\rm exp}(A)$, $\langle\varepsilon\rangle_{\rm exp}(A)$ and $\bar{E}_{\rm exp,\gamma}(A)$ required in the above equation are available only for the thermal-neutron-induced fission of ^{235}U, and very few experimental data exist for other energy points. Therefore, in this work, the partition of $E_{\rm TXE}$ in $^{235}{\rm U}(n_{\rm th},{\rm f})$ system is first investigated with the above experimental data, and an energy partitioning way of $E_{\rm TXE}$ in the case of thermal neutron is obtained. Then using these results, the energy partitioning way of other energy points is deduced and the $\bar{\nu}(A)$ of n+^{235}U reaction at neutron energies of 0.0253 eV, 3 MeV and 5 MeV are investigated.

3 Excitation energy partitioning at thermal energy

The quantities entering the calculation are based on the experimental data or evaluated data and do not depend on the kind of theoretical model. $E_{\rm TKE,exp}(A_{\rm L}, A_{\rm H})$, $\bar{\nu}_{\rm exp}(A)$, $\langle\varepsilon\rangle_{\rm exp}(A)$ and $\bar{E}_{\rm exp,\gamma}(A)$ are used as a way of partitioning the $E_{\rm TXE}$ between the light and heavy fragments. In this work, they are replaced by $E_{\rm TKE,eval}(A)$, $\bar{\nu}_{\rm eval}(A)$, $\langle\varepsilon\rangle_{\rm eval}(A)$ and $\bar{E}_{\rm eval,\gamma}(A)$ respectively, which are determined by fitting the experimental data. The results are shown with lines in Fig. 1.

With the data of Fig. 1 and Eqs. (1–7), the total excitation energy partitioning way at thermal neutron induced n+^{235}U reaction is investigated, and the result is given by the solid curve in Fig. 2. This is the first time for giving an energy partitioning way based on the experimental data, not on any theoretical hypotheses. The dashed line represents the energy partitioning way, proportional to the mass of the fragments, presented in the Los Alamos model [6]. It is obvious that the heavy fragment has a relatively higher excitation energy in terms of the dashed line. The heavier fragment, which receives a larger portion of the excitation energy in the fission system, evaporates absolutely more neutrons than the lighter fragment. This energy partitioning way is used in the TALYS code [7], therefore, the calculated $\bar{\nu}(A)$ are discrepant from the experimental data.

According to the temperature-dependent multinode fission model [7], the shell effect results in the asymmetric fission and the liquid drop results in the symmetric fission. The probability of symmetric and asymmetric fission depends on the excitation energy. At a very low temperature (at very low excitation energy), asymmetric fission mode is dominant, and the energy partitioning way is presented by the solid line (in Fig. 2). With an increase in temperature (excitation energy), the asymmetric fission mode melts

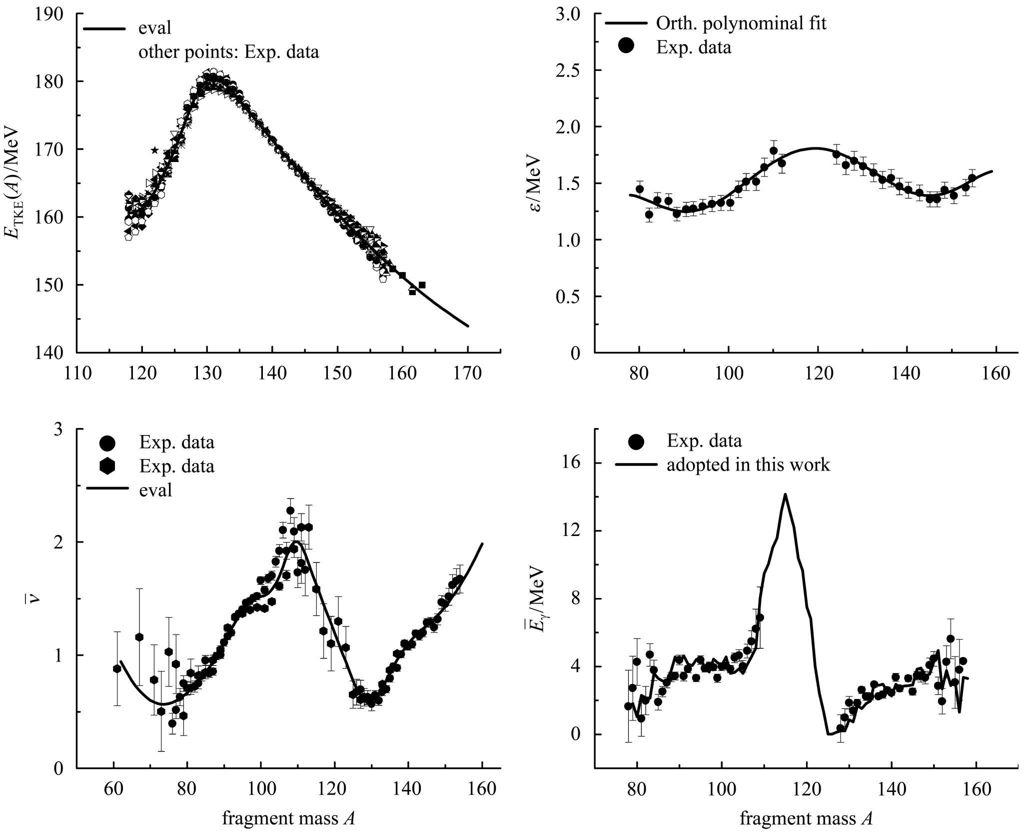

Fig. 1. The data used to partition the $E_{\rm TXE}$ at thermal neutron induced n+^{235}U reaction. The points are experimental data and the lines are evaluated data. $E_{\rm TKE,exp}(A_{\rm L}, A_{\rm H})$ is taken from Ref. [2], $\langle\varepsilon\rangle_{\rm exp}(A)$ is taken from Ref. [3], $\bar{\nu}_{\rm exp}(A)$ is taken from Ref. [4] and $\bar{E}_{\rm exp,\gamma}(A)$ is taken from Ref. [5].

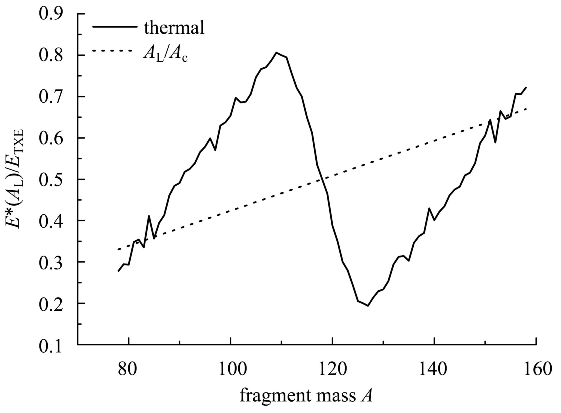

Fig. 2. The energy partitioning way at thermal neutron energy.

together and then disappears for high excitation energies(about 50 MeV), i.e., only symmetric fission mode exists, then no shell effects should be taken into account. In this case, the energy partitioning way can be described by the way of Los Alamos model. If the excitation energy is between the two cases mentioned above, both shell effect and liquid drop should contribute to the energy partition.

With the above two energy partitioning ways and the systematic parameters of fission fragment mass distribution of n+^{235}U fission system [8], in which the weight of symmetric and asymmetric fission as incident energy function is given, the energy partitioning ways at 3.0 MeV and 5.0 MeV are deduced, and shown in Fig. 3. For the sake of comparison, the energy partitioning ways at 14.0 MeV and 20.0 MeV are also deduced. From Fig. 3, one can see that the energy partitioning ways tend to the $A_{\rm L}/A_{\rm c}$ line with the increasing of the neutron incident energy.

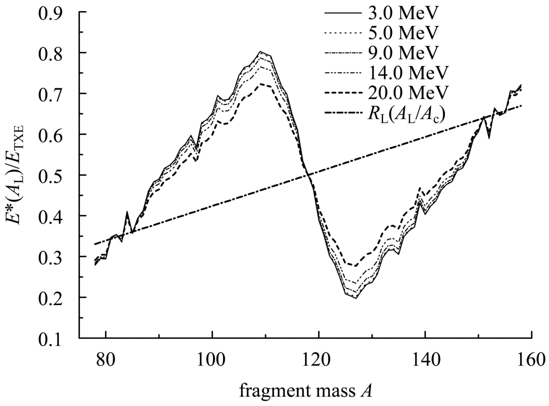

Fig. 3. The energy partitioning way at different neutron energies.

4 Calculations of $\bar{\nu}(A)$

With the calculated energy partitioning ways and Eqs. (2–7), the prompt neutron multiplicity distribution $\bar{\nu}(A)$ for neutron induced n+^{235}U reaction at thermal energy, $E_{\rm n}=3.0$ MeV and 5.0 MeV are calculated. For a thermal neutron case, the $E_{\rm TKE,eval}(A)$, $\langle\varepsilon\rangle_{\rm eval}(A)$ and $\bar{E}_{\rm eval,\gamma}(A)$ determined by fitting the experimental data are used to calculate the $\bar{\nu}(A)$. But no experimental data of $E_{\rm TKE}(A)$, $\langle\varepsilon\rangle(A)$ and $\bar{E}_{\gamma}(A)$ are available for n+^{235}U reaction at 3.0 MeV and 5.0 MeV. Therefore, some empirical approaches are used.

According to Ref. [9], an empirical express of the average total γ emission energy for the n+^{235}U system is as follows:

$$\langle E_{\gamma}^{\rm tot}\rangle=(6.6\pm0.03)+(0.0777\pm0.004)E_{\rm n}\ ({\rm MeV}),\quad(8)$$

where $E_{\rm n}$ is the incident neutron energy. Therefore, a linear relation with $\langle E_{\gamma}^{\rm tot}(E_{\rm n})\rangle/\langle E_{\gamma}^{\rm tot}({\rm thermal})\rangle$ is used. For 3.0 MeV neutron,

$$\bar{E}_{\gamma,3.0}(A)=\frac{\langle E_{\gamma}^{\rm tot}(3.0)\rangle}{\langle E_{\gamma}^{\rm tot}({\rm thermal})\rangle}\times\bar{E}_{\gamma,{\rm thermal}}(A)=$$
$$1.0353\times\bar{E}_{\gamma,{\rm thermal}}(A),\quad(9)$$

and for 5 MeV, $\bar{E}_{\gamma,5.0}(A)$=1.0588×$\bar{E}_{\gamma,{\rm thermal}}(A)$.

For the average neutron kinetic energy (at $E_{\rm n}$ energy) as a function of the mass number of the FF, $\langle\varepsilon\rangle_{E_{\rm n}}(A)$, according to the statistical theory [10], the average energy of Maxwell evaporation spectrum $\langle\varepsilon\rangle=\dfrac{3}{2}T_{\rm M}$. $T_{\rm M}$ is the nuclear temperature and is proportional to the excitation energy of fission fragment $\sqrt{E^{*}}$. So a relation of the $\langle\varepsilon\rangle_{\rm exp,thermal}(A)$ and $\langle\varepsilon\rangle_{E_{\rm n}}(A)$ is given, namely for a given initial fission fragment,

$$\langle\varepsilon\rangle_{E_{\rm n}}(A)=\sqrt{\frac{E_{E_{\rm n}}^{*}}{E_{\rm thermal}^{*}}}\times\langle\varepsilon\rangle_{\rm exp,thermal}(A),\qquad(10)$$

where $E_{\rm thermal}^{*}$ is the excitation energy of the fission fragment gained from the $E_{\rm TXE}$ at thermal neutron, and $E_{E_{\rm n}}^{*}$ is the excitation energy of the fission fragment gained from the $E_{\rm TXE}$ at En energy point. $\langle\varepsilon\rangle_{\rm exp,thermal}(A)$ is the experimental values of the thermal neutron case.

According to the experimental data [2], it is found that the total FF kinetic energy $E_{\rm TKE}$ almost does not change with the increase of the incident neutron energy in n+^{235}U reaction, and this is reasonable, because $E_{\rm TKE}$ is the result of Coulomb repulsion. Therefore, in this work, $E_{\rm TKE}$ is assumed independent of the neutron incident energy.

With Eqs. (3–10) and the data of Fig. 2 and Fig. 3, the prompt fission neutron multiplicity distributions $\bar{\nu}(A)$ for n+^{235}U reaction at $E_{\rm n}=0.0253$ eV, 3.0 MeV and 5.0 MeV are calculated with statistical evaporation theory. The calculated results of thermal neutron (open circle) are shown in Fig. 4, and are in good agreement with the experimental data (solid circle and solid triangle) of Nishio and E.E.Maslin [4] within uncertainties except for some fluctuations between the mass number A=100 and 125, where the uncertainties of the experimental data are also large.

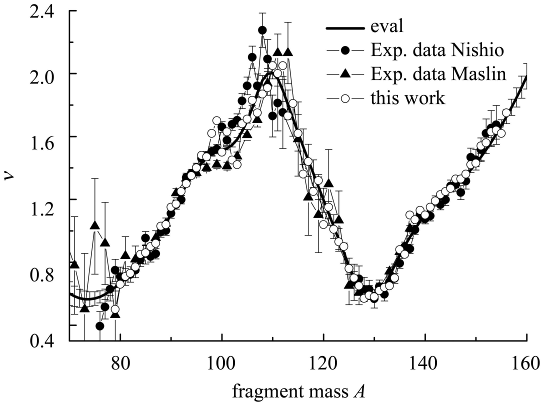

Fig. 4. The average neutron multiplicity $\bar{\nu}$ as a function of the FF mass for n(0.0253 eV)+^{235}U reaction.

In order to check the calculation results, the average prompt neutron number $\bar{\nu}$ from $\bar{\nu}(A)$ distribution with the chain yields of the thermal-neutron-induced fission of ^{235}U is calculated. The chain yields are taken from ENDF/B-VII. The result is $\bar{\nu}$=2.368189, and is in good agreement with the experimental data taken from EXFOR [11].

The calculated results of $\bar{\nu}(A)$ at $E_{\rm n}$=3.0 MeV and 5.0 MeV are presented in Fig. 5. Since no experimental data are available for $\bar{\nu}(A)$ at these energies, these results can not be checked directly, but it can be checked indirectly by the calculation of $\bar{\nu}$. This value can be deduced from the chain yields and the $\bar{\nu}(A)$ distribution calculated in this work.

Evaluated data exist for the chain yield of the n+^{235}U system in ENDF/B-VII library, but only for thermal neutron, 0.5 MeV and 14.0 MeV. The chain yields for the n+^{235}U system at 3.0 MeV and 5.0 MeV are given by linear interpolation between 0.5 MeV and 14.0 MeV. The calculation results of the total average prompt neutron number from $\bar{\nu}(A)$ are $\bar{\nu}$=2.7541 for 3.0 MeV and $\bar{\nu}$=3.0412 for 5.0 MeV, and compared with the evaluated values [12] (line) and experimental values [11] (open symbol) in Fig. 6. The results show the present calculation well reproduces the average prompt fission neutron multiplicity $\bar{\nu}$ for the first chance fission in the n+^{235}U system.

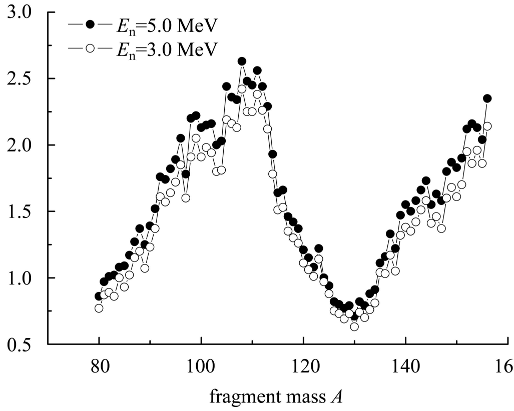

Fig. 5. The average neutron multiplicity $\bar{\nu}$ as a function of the FF mass for n(E_n)+^{235}U reaction.

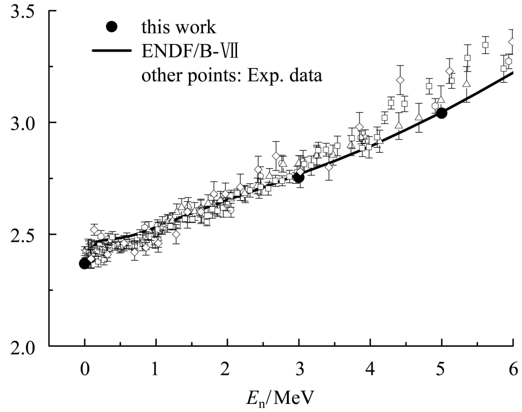

Fig. 6. The calculated results (solid circle) and comparisons with the experimental data (taken from EXFOR [11]) and the evaluated data (taken from ENDF/B-VII [12] with solid line).

In the E_n range above 5.0 MeV, where more than one fission chance is involved, the calculation will become more complex. More studies for higher incident neutron energies are in progress, and it is useful to gain the knowledge of the post fission behavior.

5 Conclusion remarks

1) With available experimental data and a semi-empirical method, the total excitation energy partitioning ways between both of the two fission fragments are given for the first time in n+^{235}U system, although we do not know yet how to explain them theoretically at present. It should be studied further.

2) There are two possible assumptions about the γ emission mechanism. One is the energy released by γ emission which is only in the case that the excitation energy is lower than the neutron binding energy, namely the neutron could not emitted again. In this way, the total γ emission energy $\langle E_\gamma^{\text{tot}} \rangle$ is independent of the neutron incident energy, and the other one is that there exists γ competition, namely γ is also emitted in the course of neutron emission, in this way, the $\langle E_\gamma^{\text{tot}} \rangle$ is dependent of the neutron incident energy. The latter one is adopted in this work (see Eq. (10)). From the present calculation results, it can be concluded that this assumption is reasonable for the first chance fission in n+^{235}U reaction. But for higher energy, where the (n, nf) and (n, 2nf) channels will open, the reasonability of this assumption needs to be studied.

3) There are some fluctuations of $\bar{\nu}(A)$ around A=100, which mainly come from the reduced mass and neutron binding energy of the fission fragments, affected by the pair and shell effect.

References

1 Lemaire S, Talou P, Kawano T et al. Phys. Rev. C, 2005, **72**: 024601

2 www.nds.org/exfor/, Experimental Nuclear Reaction Data (EXFOR). Database (2009), Entry Numbers: 40017, 40235, 21834

3 Nishio K, Nakagome Y, Yamamoto H, Kimura I. Nucl. Rhys. A, 1998, **632**: 540

4 www-nds.iaea.org/exfor/, Experimental Nuclear Reaction Data (EXFOR) Database (2009), entry nuber 22464, 21095

5 Pleasonton F, Ferguson R L, Schmitt H. Phys. Rev. C, 1972, **6**: 1023

6 Madland D C, Nix J R. Nucl. Sci. Eng., 1982, **81**: 213

7 Duijvestijn M C, Koning A J. Phys. Rev. C, 2001, **64**: 014607

8 LIU Ting-Jin, SUN Zheng-Jun, SHU Neng-Chuan. Chin. Phys. C, 2008, **32**: 556–560

9 Madland D G. Nucl. Rhys. A, 2006, **772**: 113–137

10 LU Xi-Ting et al. Nuclaer Physics. Beijing: Atomic Energy Press, 2000. 273–322

11 www-nds.iaea.org/exfor/, Experimental Nuclear Reaction Data (EXFOR) Database (2009), entry nubers: 12326, 14051, 21252, 40132, 40871, 41378

12 www.nndc.bnl.org/endf/, Evaluated Nuclear Data File (ENDF) (2009), target=^{235}U, projectile=n, MT=456, MF=1.