# Beta decay of nuclide $^{56}$Fe, $^{56}$Co, $^{56}$Ni, $^{56}$Mn, $^{56}$Cr and $^{56}$V due to strong electron screening in stellar interiors*

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**Abstract** According to a new electron screening theory, we discuss the beta decay rates of nuclide $^{56}$Fe, $^{56}$Co, $^{56}$Ni, $^{56}$Mn, $^{56}$Cr and $^{56}$V with and without strong electron screening (SES). The results show that SES has only a slight effect on the beta decay rates for $\rho/\mu_e < 10^8$ g/cm$^3$. However the beta decay rates would be influenced greatly for $\rho/\mu_e \geqslant 10^8$ g/cm$^3$. Due to SES, the maximum values of the $C$-factor (in %) on beta decay rates of $^{56}$Fe, $^{56}$Co, $^{56}$Ni, $^{56}$Mn, $^{56}$Cr and $^{56}$V is of the order of 95.03%, 35.02%, 98.05%, 80.33%, 98.30% and 98.71% at $T_9 = 4.0$ and 98.83%, 98.89%, 99.65%, 10.32%, 4.10% and 40.21% at $T_9 = 7.0$, respectively.

**Key words** beta decay, stellar interiors, strong electron screening

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## 1 Introduction

According to the stellar evolution theory, stars with masses larger than $8M_\odot$, after burning for millions of years, will collapse when the nuclear reactions in the core stop. The matter of the core then consists mainly of $^{56}$Fe. This collapse proceeds very fast and stops in the central region when its density goes beyond the nuclear matter density with a strong shock starting to travel outward. This shock wave reaches the outer mantle in a few seconds and supplies the explosion energy of a few times $10^{51}$ ergs. The supernova explosion will then proceed because of the unstable nucleic burning and iron nucleic collapse.

Some researches show that beta decay and electron capture play an important role in the process of supernovae explosions and are the resource and factory for producing a large quantity of neutrinos. Fuller, Fowler and Newman (FFN) [1] discussed in their pioneering work the beta decay assuming a single G-T resonance. Using an average strength function, the beta decay in presupernovae stellar cores was investigated by Kar and Ray [2]. By using FFN's technique and considering the influence of a quenching factor, Aufderheide [3] also investigated the beta decay of a great deal of iron group nuclei. But their discussions have lost sight of the influence of strong electron screening (SES) on beta decay.

As is well known, SES has attracted a strong interest of nuclear astrophysicists due to its consequences for the calculation of reaction rates at energies around the Gamow peak in low-energy astrophysical relevant reactions. In particular, it is extremely important to calculate accurately the screening corrections on the stellar weak-interaction rates such as the beta decay and electron capture in the stages of presupernova stellar evolution and nucleosynthesis. This issue has been studied by some authors, such as Gutierrez et al, [4] Bravo and Garcia-Senz [5], Luo and Peng [6, 7], Liu and Luo [8–10], and Iton et al. [11].

According to a new screening potential theory [11], based on the $p$-$f$ shell model, we investigate the beta decay rates of some typical iron group nuclide by the method of Aufderheide [3]. The G-T resonance transition contribution has always been a very interesting and challenging issue, especially for beta decay and electron capture. Thus in this paper, we pay attention to the influence of the G-T resonance transition contribution, which includes the range of low energy (ground state) to higher energies (excited state).

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## 2 The beta decay rates in stellar interiors

The beta decay rates for the $k$th nucleus $(Z,\ A)$ in thermal equilibrium at temperature $T$ are given by a sum over the initial parent states $i$ and the final daughter states $j$ [3],

$$\lambda_k^{bd(0)}(\rho,T,Y_{\rm e}) = \ln 2\sum_i \frac{(2J_i+1)\exp[-E_i/k_{\rm B}T]}{G(Z,A,T)} \times\sum_j \frac{\xi(\rho,T,Y_{\rm e},Q_{ij})}{ft_{ij}}, \quad (1)$$

where $J_i$ and $E_i$ are the spin and excitation energy of the parent states, $k_{\rm B}$ is the Boltzmann factor, $G(Z,A,T)$ is the nuclear partition function and $ft_{ij}$ is the comparative half-life connecting states $i$ and $j$, $Q_{ij} = Q_{00} + E_i - E_j$ is the nuclear energy difference between the states of $i$ and $j$, respectively. $Q_{00} = M_{\rm p}c^2 - M_{\rm d}c^2$, $M_{\rm p}$ and $M_{\rm d}$ are the masses of the parent nuclear and the daughter nucleus, respectively, and $E_i$, $E_j$ are the excitation energies of the $i$ th and $j$ th nuclear state. The beta decay phase space integral $\xi(\rho,T,Y_{\rm e},Q_{ij})$ is written as

$$\xi(\rho,T,Y_{\rm e},Q_{ij}) = \frac{c^3}{(m_{\rm e}c^2)}\int_0^{\sqrt{Q_{ij}^2-m_{\rm e}^2c^4}} {\rm d}pp^2(Q_{ij}-\varepsilon_{\rm n})^2 \times\frac{F(Z+1,\varepsilon_{\rm n})}{1+\exp[(U_{\rm F}-\varepsilon_{\rm n})/k_{\rm B}T]}, \quad (2)$$

where $p$, $m_{\rm e}$, $U_{\rm F}$ and $\varepsilon_{\rm n}$ are the electron momentum, mass, chemical potential and energy, respectively. $F(Z+1,\ \varepsilon_{\rm n})$ is the coulomb wave correction, which is the ratio of the square of the electron wave function distorted by the coulomb scattering potential to the wave function of the free electron. The electron chemical potential is found by inverting the expression for the lepton number density,

$$n_{\rm e} = \frac{1}{2\pi^2\lambda_{\rm e}^3}\sum_{n=0}^{\infty} q_{n0}\int_0^{\infty}(f_{-\rm e}-f_{+\rm e}){\rm d}p_z, \quad (3)$$

where $\lambda_{\rm e}$ is the Compton wavelength, $q_{\rm n0} = 2-\delta_{\rm n0}$ is the electron degenerate number,

$$f_{-\rm e} = \left[1+\exp\left(\frac{\varepsilon_{\rm n}-U_{\rm F}-1}{kT}\right)\right]^{-1}$$

and

$$f_{+\rm e} = \left[1+\exp\left(\frac{\varepsilon_{\rm n}+U_{\rm F}+1}{kT}\right)\right]^{-1}$$

are the electron, positron distribution functions, respectively.

According to Ref. [3], the sum over the parent in the total beta decay rate can be broken up the sum into two parts, one for the low energy region near the ground state and the other for the resonance region dominated by G-T resonance transition. Thus it becomes

$$\lambda_k^{bd(0)}(\rho,T,Y_{\rm e}) = \lambda_0+\lambda_{\rm GT}, \quad (4)$$

$$\lambda_0 = \ln 2\frac{(2J_0+1)}{G(Z,A,T)}\exp(-E_{\rm peak}/k_{\rm B}T) \times\frac{\xi(\rho,T,Y_{\rm e},E_{\rm peak}+Q_{00})}{ft_{\rm eff}}, \quad (5)$$

$$\lambda_{\rm GT} = \ln 2\exp(-E_{\rm BGTR(0)}/k_{\rm B}T)\frac{G(Z+1,A,T)}{G(Z,A,T)} \times\frac{\xi(\rho,T,Y_{\rm e},E_{\rm BGTR(0)}+Q_{00})}{ft_{0\to{\rm BGTR(0)}}}, \quad (6)$$

$$ft_{\rm eff} = 6.06\times 10^4,\ ft_{0\to{\rm BGTR(0)}} = \frac{10^{-3.596}}{|M_{\rm BGTR(0)}|^2}, \quad (7)$$

where $J_0$ is the initial spin of the parent state, $E_{\rm BGTR(0)}$ is the energy difference between the orbit that the new neutron occupies in the G-T resonance and the ground state.

The influence of the G-T resonance transition contribution, which includes the range of lower energy from the ground state and higher energy from the excited states, has been discussed in, detail by FFN [1]. The total G-T matrix element $|M_{\rm BGTR(0)}|^2$ between the initial parent state $|\psi_{\rm i}^{\rm P}\rangle$ and the final daughter state $|\psi_{\rm f}^{\rm D}\rangle$ has the form

$$|M_{\rm BGTR(0)}|^2 = \left|\left\langle\psi_{\rm f}^{\rm D}\right|\sum_{\rm n}\sigma_{\rm n}(\tau_{\pm})_n\left|\psi_{\rm i}^{\rm P}\right\rangle\right|^2. \quad (8)$$

In the condition of the temperature $T$ (in units of MeV), ranging from roughly 0.3 MeV to 0.8 MeV, and the electron chemical potentials, ranging from 0.5 MeV up to at least 8 MeV, $E_{\rm peak}$ is approximated by

$$E_{\rm peak} = 5T-Q_{00}+U_{\rm F}+\delta(T,U_{\rm F}), \quad (9)$$

$$\delta(T,U_{\rm F}) = -0.6604+0.9429T-0.02119U_{\rm F} -0.9432TU_{\rm F}-0.0009524U_{\rm F}^2 +0.06224TU_{\rm F}^2. \quad (10)$$

If $E_{\rm peak}$ is negative, the transitions from ground state to ground state are computed.

Itoh et al. [11] calculated the screening potential which was caused by relativistic degenerate electrons with the linear response theory. This condition, under which the electrons are strongly degenerate, is expressed as $kT \ll E_{\rm c} \ll E_0$, where $E_0$ is the no-resonant reaction effective energy and $E_{\rm c}$ is the coulomb potential energy per nucleon. The screening energy per-electron is expressed as

$$D=7.525\times10^{-3}Z\left(\frac{10Z\rho_7}{A}\right)^{1/3}J\left(r_{\rm s},R\right){\rm MeV}, \quad (11)$$

where $J(r_{\rm s},\ R)$, $r_{\rm s}$, $R$ can be found in Ref. [11]. The formula (11) is valid for $10^{-5}\leqslant r_{\rm s}\leqslant10^{-1}$, $0\leqslant R\leqslant50$, which are usually fulfilled in the presupernova environment.

The electron is strongly screened and the screening energy is high enough not to be neglected in a high density plasma and will decrease from $\varepsilon$ to $\varepsilon'=\varepsilon-D$ in the decay reaction due to electron screening. On the other hand, the screening decreases to some extent the number of electrons whose energy is higher than the threshold energy of beta decay. The threshold energy increases from $\varepsilon_{\rm s}$ to $\varepsilon_{\rm s}=\varepsilon_0+D$. Thus the phase space integral with screening is given by

$$\xi^{\rm SES}\left(\rho,T,Y_{\rm e},Q_{ij}\right) = \frac{c^3}{(m_{\rm e}c^2)}\int_1^{Q_{ij}+D}{\rm d}\varepsilon_{\rm n}\varepsilon_{\rm n}p(Q_{ij}+D-\varepsilon_{\rm n})^2\frac{F\left(Z+1,\varepsilon_{\rm n}-D\right)}{1+\exp\left[\left(U_{\rm F}-\varepsilon_{\rm n}\right)/k_{\rm B}T\right]} \quad (12)$$

and therefore the beta decay rate in SES becomes

$$\lambda_k^{bd({\rm SES})}\left(\rho,T,Y_{\rm e}\right)=\lambda_0^{\rm SES}+\lambda_{\rm GT}^{\rm SES}, \quad (13)$$

In order to compare the contribution of the beta decay rate with and without SES, we define the following quantity,

$$C=\left(\lambda_k^{bd({\rm SES})}-\lambda_k^{bd(0)}\right)/\lambda_k^{bd({\rm SES})}. \quad (14)$$

## 3 Some numerical results and discussion

Figures 1 and 2 show the beta decay rates of $^{56}$Fe, $^{56}$Co, $^{56}$Ni, $^{56}$Mn, $^{56}$Cr and $^{56}$V as a function of the density at a temperature of $T_9$=4.0, 7.0 with and without SES ($T_9$ is the temperature in units of $10^9$ K). Only a slight effect on the beta decay rates can be seen for $\rho/\mu_{\rm e}<10^8$ g/cm$^3$. However, the influence increases for $\rho/\mu_{\rm e}\geqslant10^8$ g/cm$^3$, as can be seen in both figures. For instance, in Fig. 1, for $\rho/\mu_{\rm e}\geqslant10^8$ g/cm$^3$, the NEL rates of $^{56}$Fe, $^{56}$Co, $^{56}$Ni decrease by almost more than 5 orders of magnitude. However, they decrease by no more than 4 orders of magnitude for $^{56}$Mn, $^{56}$Cr and $^{56}$V.

A brief glance at Figs. 1 and 2 reveals that at the relative lower and the higher temperature, the effect on the beta decay rates is different due to the G-T transitions occurring at different densities. The higher the density, the larger is the influence on beta decay. This is so because the electron energy is so low at the lower temperature that the G-T transitions may not dominate. On the other hand, the Fermi energy of the electron gas increase, with its density, which may affect the G-T transition process largely and therefore also the beta decay.

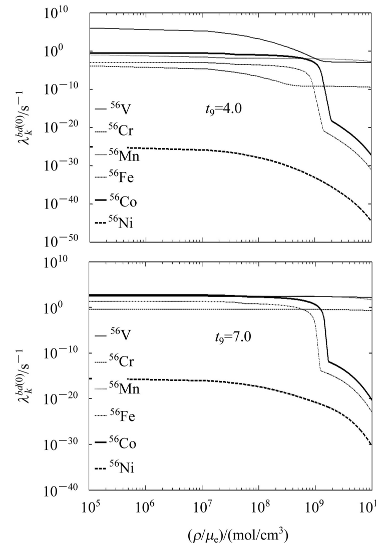


Fig. 1. The beta decay rates of the nuclides $^{56}$Fe, $^{56}$Co, $^{56}$Ni, $^{56}$Mn, $^{56}$Cr and $^{56}$V as a function of the density at a temperature of $T_9=4.0$ and 7.0 without SES.

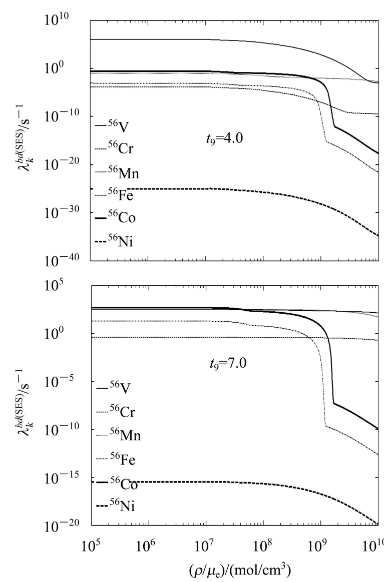


Fig. 2. The beta decay rates of the nuclides $^{56}$Fe, $^{56}$Co, $^{56}$Ni, $^{56}$Mn, $^{56}$Cr and $^{56}$V as a function of the density at a temperature of $T_9=4.0$ and 7.0 with SES.

Figure 3 shows the factor $C$ for $^{56}$Fe, $^{56}$Co, $^{56}$Ni, $^{56}$Mn, $^{56}$Cr and $^{56}$V as a function of the density at a temperature of $T_9=4.0$ and 7.0 for the case with SES. One can see that SES has only a slight effect on the beta decay rates for $^{56}$Fe and $^{56}$Co at the temperature of $T_9 = 4.0$ and 7.0, and for the nuclides $^{56}$Mn, $^{56}$Cr, $^{56}$V at the temperature of $T_9 = 7.0$ for $\rho/\mu_e < 10^8$ g/cm$^3$. However, the beta decay rates of the great majorty of the nuclides would be highly influenced for $\rho/\mu_e \geqslant 10^8$ g/cm$^3$. On the other hand, the maximum values of the $C$-fator (in %) on beta decay rates for $^{56}$Fe, $^{56}$Co, $^{56}$Ni, $^{56}$Mn, $^{56}$Cr and $^{56}$V due to SES are of the order of 95.03%, 35.02%, 98.05%, 80.33%, 98.30% and 98.71% at $T_9 = 4.0$ and are of the order of 98.83%, 98.89%, 99.65%, 10.32%, 4.10% and 40.21% at $T_9 = 7.0$, correspondingly. This is due to the fact that the $Q$-values of $^{56}$Fe, $^{56}$Co and $^{56}$Ni are negative and these beta decay reactions are endothermic. It is the other way round for $^{56}$Mn, $^{56}$Cr and $^{56}$V, which have positive $Q$-values and the beta decay reactions are exothermic. On the other hand, for larger atomic numbers (such as the nuclides $^{56}$Fe, $^{56}$Co, $^{56}$Ni), the SES will increase and lead to a greater influence on beta decay.

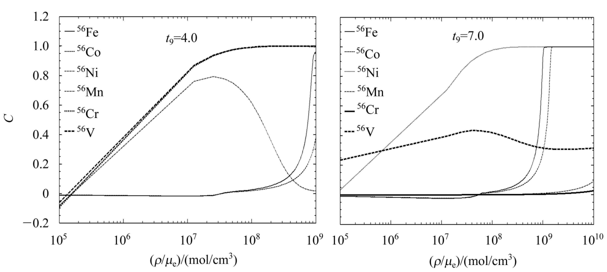


Fig. 3. The factor $C$ for the nuclides $^{56}$Fe, $^{56}$Co, $^{56}$Ni, $^{56}$Mn, $^{56}$Cr and $^{56}$V as a function of the density at temperatures of $T_9 = 4.0$ and 7.0.

From Figs. 1–3 one can conclude that at the same temperature, SES has a different effect on beta decay rates at different densities. The lower the density, the smaller the influence on NEL is at the same temperature. This is because the electron Fermi energy and electron screening potential are so low at the lower density (such as a density of $\rho/\mu_e < 10^8$ g/cm$^3$) that SES affects the beta decay reaction very little. On the other hand, at low density and high temperature the electron Fermi energy is so small and the electron average energy is high enough that the influence on beta decay will increase strongly. With increasing of the electron number density the electron Fermi energy becomes so high at high density that the electrons degenerate very easily and the electron screening potential will increase strongly.

As is well known, the motion of the electron will be hindered due to the coulomb repulsion of the screening cloud when it travels away from the atomic nucleus. Thus the electron will be blocked from moving out due to SES. Especially at the range of lower energy, it is very difficult for the electron to move through the screening cloud, hence the SES has a large effect on lower energy emission electrons. On the other hand, SES which is equal to that atomic nucleus decreases the coulomb repulsion of emission electron. It may increase the energy of emission electron, so the beta decay rates will be aggrandized greatly due to SES.

## 4 Conclusion

We discuss beta decay rates with and without SES of $^{56}$Fe, $^{56}$Co, $^{56}$Ni, $^{56}$Mn, $^{56}$Cr and $^{56}$V. The results show that SES has only a slight effect on the beta decay rates for $\rho/\mu_e < 10^8$ g/cm$^3$. However, the beta decay rates of most nuclei would be influenced greatly for $\rho/\mu_e \geqslant 10^8$ g/cm$^3$.

What are the role and status played by SES in stars, such as white dwarfs, neutron stars and some magnetars? How does SES affect the beta decay? How does SES influence the electron gas under the stellar conditions of high density and temperature? These problems of SES have always been very interesting and challenging subjects in stellar interiors. Thus the conclusion obtained in this work may significantly help further research in nuclear astrophysics, especially research into presupernova stellar evolution and nucleosynthesis.


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