Level statistics for the even-even Yb isotopes*

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Abstract The level statistics of the even-even Yb isotopes are studied by using the energy levels calculated by the projected shell model. The spectrum of intrinsic states and band energies are also studied to discuss the generation of chaoticity. The energy dependence of the chaoticity is investigated, and a chaos to order transition is found.

Key words chaos, level statistics, Yb isotopes, projected shell model

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1 Introduction

In nuclear physics there are many calculations on level statistics during the last two decades. However, the generation of the energy levels in nuclei may become a problem. Level statistics needs long sequences that is pure and successive. Levels found in experiments can hardly satisfy these conditions, so we may turn to nuclear models. The models that are widely used are shell model[1, 2], particle-rotor model[3, 4], cranked shell model[3, 4] and projected shell model[5]. The shell model is used for light and spherical nuclei, and it was shown in Ref. [2] that the level statistics agree well with the prediction of GOE. However, for heavy and deformed nuclei most of the energy level calculations were done in a single j shell, so the results can not describe the dynamics of the actual nuclear system. In this paper we will deal with the realistic case of the Yb isotopes which are heavy and deformed nuclei, with three major shells taken into consideration. The energy levels are obtained from the projected shell model.

The paper is organized as follows. In section 2 we will give a brief introduction to level statistics. Our results and some discussions are given in section 3. Finally in section 4 there is a short summary.

2 Theoretical framework

The framework of the projected shell model is given in Ref. [6]. Here we only give some introduction to level statistics.

As mentioned above, first we must choose long sequence with levels of the same spin and parity. Then there is an important step called unfolding[7]. For the sequence we can define the cumulative function $N(E)$, which gives the number of levels with energy less than or equal to E and is thus a staircase function. It can be divided into two parts: $N(E)=N_{\rm av}(E)+N_{\rm fl}(E)$ where $N_{\rm av}(E)$ gives the smooth part and $N_{\rm fl}(E)$ the fluctuating part. The aim of unfolding is to scale the smooth part into a uniform one, i.e. to map each $E_{\rm i}$ into an $X_{\rm i}$ so that $N(X)$ is a linear function with the slope equal to 1. To do this we need to get the average level density $\rho(E)$, then there is

$$N(E_{\rm i})=\int_{-\infty}^{E_{\rm i}}\rho(E){\rm d}E. \qquad (1)$$

Usually $\rho(E)$ is generated by fitting the numerical level densities to some fixed form with some free parameters. The unfolding method we used comes from Ref. [8].

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After unfolding we can calculate the quantities defined in the random matrices theory. The most important one is the nearest- neighbor spacing distribution (NNS) $P(s)$. It gives the probability of finding two neighboring unfolded levels with spacing s. For regular systems there is

$$P(s)=\exp(-s), \tag{2}$$

while for fully chaotic system with time-reversal symmetry it is

$$P(s)=\frac{\pi}{2}s\exp(-\frac{\pi}{4}s^2), \tag{3}$$

which is also called the GOE (Gaussian orthogonal ensemble) distribution. For systems that are neither fully chaotic nor fully regular, there is Berry-Robnik distribution[9]:

$$P(q,s)=(1-q)^2\exp(-(1-q)s)\mathrm{erfc}(\frac{\sqrt{\pi}}{2}qs)+ (2q(1-q)+\frac{\pi}{2}q^3s)\exp(-(1-q)s-\frac{\pi}{4}q^2s^2), \tag{4}$$

where q is decided according to the numerical $P(s)$ to get a best fitting. If the system is fully chaotic, q=1, while for fully regular systems q=0. Thus q can be treated as a measure to the degree of chaoticity of the system. It is called the Berry-Robnik parameter.

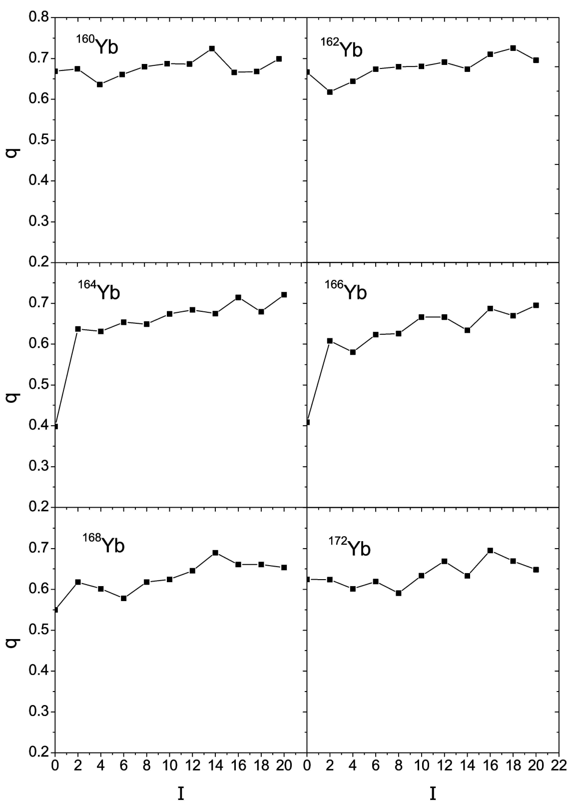

Fig. 1. The values of q for different Yb isotopes as a function of spins.

3 Results

We calculated the level statistics for the even-even Yb isotopes ^{160}Yb to ^{168}Yb and ^{172}Yb. First we take the whole spectrum into account to get a general idea of the level statistics of Yb isotopes. The results are shown in Fig.1. Generally speaking, the behavior of the levels of Yb isotopes has significant deviations from the prediction of GOE, but it is also different from Poisson, which indicates that there is chaotic motion in these atomic nuclear systems. However, the behavior is different in different isotopes. In Fig.1 we can find that values of q show a decrease as the mass number increases from 160 to 172. In Fig.2 we give the histogram of $P(s)$ for ^{162}Yb and the $P(s)$ plot for the GOE and Poisson distribution. It is shown that the nearest neighbor distribution is intermediate between the GOE and Poisson limit, which is consistent with the result of the Berry-Robnik parameter.

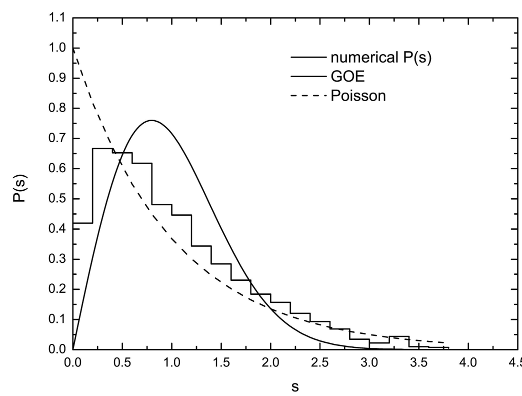

Fig. 2. $P(s)$ plot for ^{162}Yb, I=12. The histogram is the numerical results of $P(s)$. The dash line shows Poisson distribution while the solid line shows GOE distribution.

Then, it is interesting to investigate where the chaoticity in the spectrum comes from. We have done calculations to give some hints to this problem. Taking ^{162}Yb as an example, we calculated the level statistics of its intrinsic states—the sequence of two quasi particle excitation energies. The intrinsic states are the eigen states of the "Nilsson+BCS" Hamiltonian and have parity and K (projection of the angular momentum on the intrinsic z axis) good quantum numbers. We do statistics on the sequence formed by levels of the same quantum numbers. The value of q for the K=2 two-qp states (with positive parity) is 0.5011, and for the K=6 sequence it is 0.5687, while as shown in Fig.1 for the eigen value sequence of ^{162}Yb they are always larger than 0.6. This concludes that the intrinsic states are relatively regular, compared with the eigen value sequence.

The chaoticity in the "intrinsic" states can be easily understood as that the "Nilsson+BCS" Hamiltonian has broken the rotational symmetry. But it is not enough to explain the chaoticity shown in the eigen value sequence. According to the framework of the projected shell model, two things happen after we get the two-qp states: they are projected onto states with good spin quantum numbers; and the "total" Hamiltonian is diagonalized in the space formed by the projected two-qp basis. Among these two steps, which one is responsible for the "extra" chaoticity shown in the eigen value sequence?

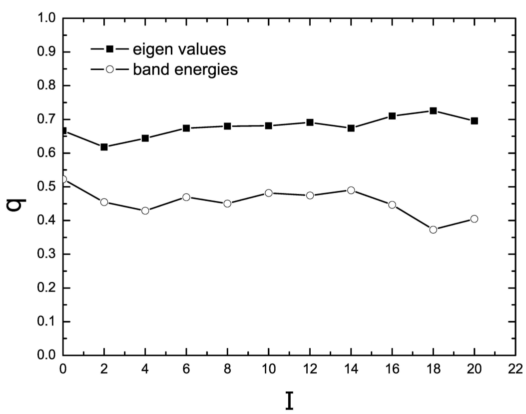

Fig. 3. The values of q for eigen value sequence and band energy sequence of ^{162}Yb as functions of spins.

To answer this question we do statistics on the sequence formed by the diagonal elements of the "total" Hamiltonian matrix. The diagonal elements are the expectation energies of the projected two-qp basis (also called "band energies" in the projected shell model). The results for ^{162}Yb are given in Fig. 3. It can be seen that these sequence are not more chaotic than the intrinsic states, in fact they are even more regular. Thus we may conclude that the projection step doesn't contribute to the chaoticity. The "extra" chaoticity is caused by the two-body interactions in the Hamiltonian, or the non-diagonal elements of the Hamiltonian matrices. Because of them different projected states (or different bands) are mixed in the diagonalization process, and this mixture tends to increase chaos.

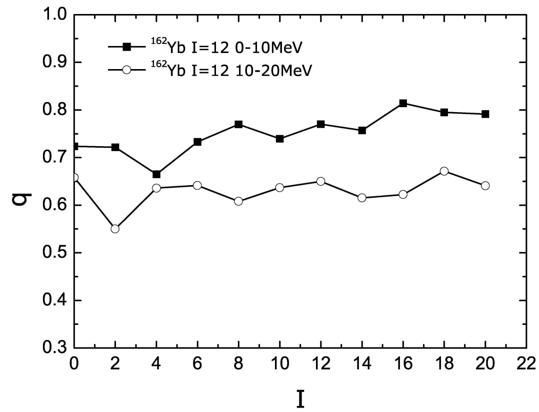

Fig. 4. The values of q for 0-10 MeV and 10-20 MeV above yrast as functions of spins.

There are other interesting results in the level statistics of the Yb isotopes. One of them is the energy dependence of the chaoticity. We do statistics on levels of the 0—10 MeV region and the 10—20 MeV region respectively. The results for ^{162}Yb, I=12 are shown in Fig. 4. The values of q of the 0—10 MeV (above yrast) region is obviously larger than that of the 10—20 MeV region, for nearly all the isotopes and all the spins that are considered.

4 Summary

We have done level statistics of the even-even Yb isotopes using the sequence obtained from the projected shell model. The results show that the NNS is different from the prediction of not only the GOE distribution but also the Poisson distribution. The chaoticity show a decrease as the mass number increases, and a chaos to order transition is found as the excitation energy increase. The level statistics of the intrinsic two-quasi particle states and the expectation energy of the projected basis are also studied to discuss the generation of the chaoticity. The mixing of different bands caused by the two-body interaction of the Hamiltonian tends to increase the chaoticity.

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