Renormalizability and nonrenormalizable interactions*

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Abstract Arguments are provided which show that extension of renormalizability in quantum field theory is possible. By an appropriate choice of effective Lagrangian, a dressed Feynman propagator is obtained. In this scheme, higher order Feynman diagrams become self-convergent and nonrenormalizable interactions become renormalizable. As an example, the vacuum fluctuation effects on ρ meson mass for the vector-tensor coupling model is discussed. It is found that the result can agree with the experimental value when coupling constant is adjusted.

Key words renormalizability, dressed scheme, vacuum fluctuation, effective mass

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1 Introduction

It is now generally believed that renormalizability is not a fundamental requirement of quantum field theory. In fact, the widely acknowledged effective field theory[1—3] contains nonrenormalizable interactions. It has been especially emphasized by Weinberg[1] that renormalizability is unnecessary for the following main reasons: (1) it places a too stringent restriction on the possible types of renormalizable interactions and (2) as regards the cancellation of ultraviolet divergences(UV), nonrenormalizable theories are actually also renormalizable, if all of the possible interactions allowed by symmetries are included in the Lagrangian, because then there will be enough counterterms to cancel every UV divergence. However, it is still desirable to find means to broaden the extent of renormalizability, since for a renormalizable interaction only a finite number of counterterms in the Lagrangian is needed for the elimination of infinities, while an infinite number is necessary, if it is a nonrenormalizable interaction. Hereafter we shall always understand renormalizability in the above restrictive sense specified by "finite number". We would like to show that such an extension of renormalizability is indeed possible if an effective Lagrangian were chosen properly.

2 Theory

Consider the following Fermion Lagrangian density, for instance:

$$\begin{aligned}\mathcal{L}_f = & -\overline{\psi} N \cosh[a(\gamma_\mu \partial_\mu)^2]\gamma_\mu \partial_\mu \psi - \overline{\psi} M \psi = \\ & -\overline{\psi}(\gamma_\mu \partial_\mu + M)\psi - \overline{\psi}(N\cosh[a(\gamma_\mu \partial_\mu)^2]-1)\times \\ & \gamma_\mu \partial_\mu \psi = \mathcal{L}_f^0 + \mathcal{L}' \end{aligned} \quad (1)$$

where ψ is the fermion field, N a constant for normalization and a a parameter far lesser than 1. Clearly, compared to the free lagrangian density $\mathcal{L}_f^0$, $\mathcal{L}_f$ has a remainder term $\mathcal{L}'$, which may be considered as a self-interaction part of the fermion field. The Dyson-Schwinger equation for the fermion propagator $G(k)$ reads

$$G(k) = G^0(k) + G^0(k)\Sigma(k)G(k) \quad (2)$$

where the superscript "0" indicates a zeroth order approximation and $G^0(k) = -[\gamma_\mu k_\mu - \mathrm{i}M]^{-1}$, $k_\mu \equiv (\boldsymbol{k}, \mathrm{i}k_0)$. The fermion self-energy can be written as $\Sigma(k) = \Sigma_\mathrm{d}(k) + \Sigma_\mathrm{r}(k)$, where $\Sigma_\mathrm{d}(k) = (N\cosh(ak^2) - 1)\gamma_\mu k_\mu$ is the contribution of self-interaction Lagrangian $\mathcal{L}'$, while $\Sigma_\mathrm{r}(k)$ is what contributed by the interaction lagrangian $\mathcal{L}_I$. We may introduce a dressed propagator $G_\mathrm{d}(k)$ and rewrite Eq.(2) in the

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$$G_{\rm d}(k)=G^0(k)+G^0(k)\Sigma_{\rm d}(k)G_{\rm d}(k)= -[N\cosh(ak^2)\gamma_\mu k_\mu-{\rm i}M]^{-1}, \quad (3)$$

$$G(k)=G_{\rm d}(k)+G_{\rm d}(k)\Sigma_{\rm r}(k)G(k). \quad (4)$$

Set $N=[\cosh(aM_{\rm N}^2)+2aM_N^2\sinh(aM_{\rm N}^2)]^{-1}$ and $M=N\cosh(aM_{\rm N}^2)M_{\rm N}$, where M_N is the free nucleon mass. It is seen that the pole of the dressed propagator $G_d(k)$ is $\gamma_\mu k_\mu={\rm i}M_{\rm N}$ and the residue at the pole equals to 1. Eq. (3) shows if $a=0$, then $G_{\rm d}(k)=G^0(k)$; if $a\neq 0$, $\cosh(ak^2)$ makes $G_{\rm d}(k)$ have multiple poles in entire complex plane. However, in fact, there is only a physical pole on the real axis and the others will drive to infinity in complex plane as a decreases. According to Eq. (4) it is not difficult to see that a perturbation series can also be expanded in terms of $G_{\rm d}(k)$ (dressed scheme, DS) instead of $G^0(k)$ (ordinary scheme, OS). Clearly, the same remark also applies to boson propagators, a dressed boson propagator $\Delta_{\rm d}(k)$ also may be introduced, i.e.

$$\Delta_{\rm d}(k)=-{\rm i}[N_2\cosh(bk^2)k^2+m^2-{\rm i}\varepsilon]^{-1}. \quad (5)$$

In the following, we shall consider the pseudovector π-N interaction (PVI) $\mathcal{L}_{pv}={\rm i}f\overline{\psi}\gamma_\mu\gamma_5\boldsymbol{\tau}\cdot(\partial_\mu\boldsymbol{\phi})\psi$ and the tensor ρ-N coupling interaction (TC) $\mathcal{L}_{\rho T}=\frac{f_\rho}{4M_N}\overline{\psi}\Sigma_{\mu\nu}\boldsymbol{\tau}\cdot\psi\boldsymbol{F}_{\mu\nu}$, where $\Sigma_{\mu\nu}=\frac{1}{2{\rm i}}[\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu]$ and $F_{\mu\nu}=\partial_\mu\boldsymbol{A}_\nu-\partial_\nu\boldsymbol{A}_\mu$. Following the argument given in Ref. [1], one finds easily that the superficial degree of divergence of Feynman diagram $d_{\rm F}$ can be written as

$$d_{\rm F}=4-\Sigma_\kappa E_\kappa(2-p_\kappa)-N_{\rm i}r_{\rm i}-N_{\rm i}', \quad (6\text{-}1)$$

$$r_{\rm i}=4-1-\Sigma_\kappa 2(2-p_\kappa), \quad (6\text{-}2)$$

where E_κ is the number of external lines of field κ, $N_{\rm i}$ the number of vertices of interaction i in the Feynman diagram and $N_{\rm i}'$ the number of vertices which are connected with external lines, where p_κ denotes the power of the propagator $\Delta_\kappa(k)$ or $G(k)$ of field κ, i.e. $\Delta_\kappa(k)\sim k^{-2p_\kappa}$. In OS, $G^0(k)\sim k^{-1}$ and $\Delta^0(k)\sim k^{-2}$, according to Eq.(6) $r_{\rm pv}=r_{\rm \rho T}=-1$, this says that $d_{\rm F}$ grows with $N_{\rm i}$, thus as is wellknown, PVI and TC are nonrenormalizable.

Now let us study DS. The one loop approximation to the nucleon self-energy (see Fig.1(a))for PVI reads

$$\Sigma(k)=3f^2\int\frac{{\rm d}^4q}{(2\pi)^4}\gamma_\mu q_\mu\gamma_5G(k-q)\gamma_\nu q_\nu\gamma_5\Delta_\pi({\rm q}), \quad (7)$$

while that to the ρ meson self-energy (see Fig.1(c))for TC is given by

$$\Pi_{\mu\nu}(k)=-\int\frac{{\rm d}^4q}{(2\pi)^4}{\rm Tr}[\Gamma_\mu G(q)\widetilde{\Gamma}_\nu G(k+q)], \quad (8)$$

where $\Gamma_\mu=-\frac{{\rm i}f_\rho k_\alpha}{2M_{\rm N}}(\gamma_\alpha\gamma_\mu-\delta_{\alpha\mu})\tau_{\rm i}, \widetilde{\Gamma}_\nu=\frac{{\rm i}f_\rho k_\rho}{2M_{\rm N}}(\gamma_\rho\gamma_\nu-\delta_{\rho\nu})\tau_{\rm j}$. Substituting Eqs. (4) and (5) into Eq. (7) and (8), one finds that both $\Sigma(k)$ and $\Pi_{\rho t}(k)$ are convergent. Note that not all of Feynman diagrams are convergent in DS(for instance, tadpole diagrams(see Fig. 1(b)), because there is only one propagator involved in the Feynman integral, thus according to the power of k^2 to judge convergent character in the Minkowski space is incredible). However, even in this case, $d_{\rm F}$ doesn't grow with the number of vertices of interaction. Thus, in the restrictive sense, extension of renormalizability is realizable.

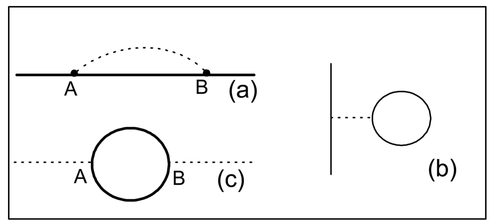

Fig. 1. Fermion self-energy (a), (b) and meson self-energy (c).

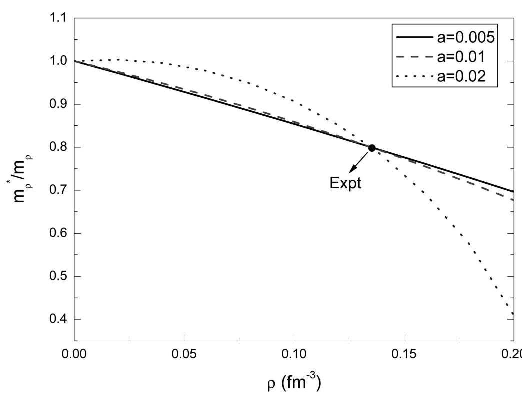

Fig.2. effective mass of ρ meson vs density for parameter $a=0.005, 0.01, 0.02$.

3 Example

Yeas ago, the experiments[4] of TAGX collaboration have shown that, when the density of the nucleon medium equals to $0.7\rho_0$, the mass of neutral ρ-meson reduces to 610 MeV, where ρ_0 is the saturation density. This is a quantitative result of the deduction of ρ-meson mass in a dense medium. However, as was shown by Refs. [5,6], the value of m^*_ρ calulated by the vacuum polarization diagrams of ρ-N vector-tensor coupling interactions(VTC) increases with density in conflict with the experiments qualitatively. To reconcile the discrepancy, Shiomi and Hatsuda[6] pointed out that one must take the vacuum fluctuation effects into account. Now let us examine vacuum fluctuation effects on the ρ-meson for VTC with interaction $\mathcal{L}_{\rho VT} = ig_\rho\overline{\psi}\gamma_\mu\boldsymbol{\tau}\psi\boldsymbol{A}_\mu + \mathcal{L}_{\rho T}$(see above). First of all, we will calculate the self-energy of ρ meson in medium under one loop approximation. As they are long and space consuming, we only present the essential steps here, which are necessary for our calculations. In dense medium the propagator of nucleon reads

$$G(k) = 2i\pi(\gamma_\mu k^*_\mu + iM^*)\delta[k^{*2} + M^{*2}]\theta(k_F - |k|) - \frac{\gamma_\mu k^*_\mu + iM^*}{k^{*2} + M^{*2} - i\varepsilon}, \quad (9)$$

where $k^*_\mu = N\cosh(ak^2)\gamma_\mu k_\mu, M^* = N\cosh(aM^{*2}_N)M^*_N$ and M^*_N is the effective nucleon mass in medium. The VTC vertices of interaction are

$$\Gamma_\mu = [g_\rho\gamma_\mu - \frac{f_\rho}{2M_N}ik_\alpha(\gamma_\alpha\gamma_\mu - \delta_{\alpha\mu})]\tau_i,$$

$$\tilde{\Gamma}_\nu = [g_\rho\gamma_\nu + \frac{f_\rho}{2M_N}ik_\alpha(\gamma_\alpha\gamma_\nu - \delta_{\alpha\nu})]\tau_j. \quad (10)$$

Substituting Eqs. (9) and (10)into Eq. (8), it can be calculated numerically. The effective mass of ρ meson m^*_ρ is defined as the pole of the propagator $D^{\mu\nu}_\rho(q)$ in the limit $\boldsymbol{q} \to 0$. We find

$$m^{*2}_\rho = \frac{N_2\cosh(\beta m_\rho{}^2)m_\rho{}^2 + i\Pi_\rho(q_0^2 = m^2_\rho, q \to 0)}{N_2\cosh(\beta m^{*2}_\rho)},$$

where $\Pi_\rho(q) = \frac{1}{3}\Pi_{\mu\mu}(q)$[7]. In the calculations, we have chosen the parameters as $M_N = 939$ MeV, $m_\rho = 768$ MeV, $\beta = 0.01$ and $N_2 = [\cosh(\beta m^2_\rho) + \beta m^2_\rho \sinh(\beta m^2_\rho)]^{-1}$, following the same reason as for the determination of N. M^*_N is adopted from the Bonn potential model, which is more reasonable, because it takes account of more intermediate mesons, especially the ρ meson. When we choose $g_\rho = 2.72$ and $f_\rho = 3.7g_\rho$ for weak tensor coupling[8], we find the self-energy of ρ meson is negative, which is popular. However, the absolute value is so large that m^{*2}_ρ becomes negative too. The value being large can be understood because the self-energy turns to infinity when $a \to 0$ and when a is small, the self-energy becomes very large inevitably. In order to fit experimental value $m^*_\rho/m_\rho = 0.8$ for $\rho/\rho_0 = 0.7$, we need adjust g_ρ and f_ρ. For a equal to 0.005, 0.01, 0.02 respectively, g_ρ and f_ρ should decrease 34.48, 14.62, 4.42 times correspondingly. The result is shown in Fig.2, where solid, dash and dot line refer to $a = 0.005, a = 0.01$ and $a = 0.02$, respectively. We see that effective mass of ρ meson decreases with density, this is consistent with the result in Refs.[5,6,9].

In summary, even in the restrictive sense, extension of renormalizability is possible if an effective Lagrangian is chosen properly. our calculation of effective mass of ρ meson shows that the coupling constant can be adjusted to obtain proper result in this scheme.

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