Ultra-high energy cosmic rays threshold in Randers-Finsler space*

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Abstract Kinematics in Finsler space is used to study the propagation of ultra high energy cosmic rays particles through the cosmic microwave background radiation. We find that the GZK threshold is lifted dramatically in Randers-Finsler space. A tiny deformation of spacetime from Minkowskian to Finslerian allows more ultra-high energy cosmic rays particles to arrive at the earth. It is suggested that the lower bound of particle mass is related with the negative second invariant speed in Randers-Finsler space.

Key words Randers-Finsler space, GZK, cut off

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1 Introduction

Decades ago, Greisen, Zatsepin and Kuz'min (GZK)[1] discussed the propagation of ultra-high energy cosmic rays (UHECR) particles through cosmic microwave background radiation (CMBR)[2]. Due to the photopion production process by CMBR, the UHECR particles will lose their energies drastically down to a theoretical threshold (about 5×10^{19} eV). That is to say, the UHECR particles whose energy beyond the threshold can not be observed[3]. This strong suppression is called the GZK cutoff. However hundreds of events with energies above 10^{19} eV and about 20 events above 10^{20} eV have been reported[4]. The latest UHECR data[5] identified with the GZK cutoff are 5 standard deviations. And the measured energy of the GZK cutoff is $(5\pm0.5\pm0.9)\times10^{19}$ eV. Also, the Pierre Auger collaboration[6] rejected the hypothesis that the cosmic rays spectrum continues with a constant slope above 4×10^{19} eV, with a significance of 6 standard deviations.

Although the latest experiment data appear to be consistent with the GZK cutoff, investigating the possible violation of the GZK cutoff is still of great theoretical interest. One should notice that the latest data[5, 6] are of 5 or 6 standard deviations. Small violations on the GZK cutoff can not be excluded.

The violation of the GZK cutoff strongly corresponds with the violation of Lorentz Invariance (LI)[7]. The violation of the LI and the Planck scale physics have long been suggested as possible solutions of the cosmic rays threshold anomalies[7]. LI is one of the foundations of the Standard Model of particle physics. Coleman and Glashow have set up a perturbative framework for investigating possible departures of local quantum field theory from LI[8, 9]. In a different approach, Cohen and Glashow suggested [10] that the exact symmetry group of nature may be isomorphic to a subgroup SIM(2) of the Poincare group. The mere observation of ultra-high energy cosmic rays and the analysis of neutrino data give an upper bound of 10^{-25} on the Lorentz violation[11].

In fact, Gibbons, Gomis and Pope[12] showed that the Finslerian line element $\mathrm{d}s = (\eta_{\mu\nu}\mathrm{d}x^{\mu}\mathrm{d}x^{\nu})^{(1-b)/2}(n_{\rho}\mathrm{d}x^{\rho})^{b}$ is invariant under the transformations of the group $\mathrm{DISIM}_b(2)$. The very special relativity is a Finsler geometry.

Recently, we proposed a gravitational field equation in Berwald-Finsler space[13]. The asymmetric term in the field equation violated LI naturally. A modified Newton's gravity is obtained as the weak field approximation of the Einstein's equation in Berwald-Finsler space[14]. The flat rotation curves of spiral galaxies can be deduced naturally without

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invoking dark matter in the framework of Finsler geometry.

In this letter, we use the kinematics in Randers-Finsler space to study the propagation of the UHECR particles through CMBR. We obtain a deformed GZK threshold for the UHECR particles interacting with soft photons, which depends on an intrinsic parameter of the Randers-Finsler space[15].

Denote by T_xM the tangent space at $x \in M$, and by TM the tangent bundle of M. Each element of TM has the form (x,y), where $x \in M$ and $y \in T_xM$. The natural projection $\pi: TM \to M$ is given by $\pi(x,y) \equiv x$. A Finsler structure[16] of M is a function

$$F: TM \to [0,\infty).$$

The Finsler structure F is regularity (F is C^∞ on the entire slit tangent bundle $TM\backslash 0$), positive homogeneity ($F(x,\lambda y) = \lambda F(x,y)$, for all $\lambda > 0$) and strong convexity (the $n\times n$ Hessian matrix $g_{ij} \equiv \frac{\partial^2}{\partial y^i \partial y^j}\left(\frac{1}{2}F^2\right)$ is positive-definite at every point of $TM\backslash 0$).

It is convenient to take $y \equiv \frac{\mathrm{d}x}{\mathrm{d}\tau}$ to be the intrinsic speed on Finsler space.

2 GZK threshold in Randers-Finsler space

In 1941, G Randers[17] studied a very interesting class of Finsler manifolds. The Randers metric is a Finsler structure F on TM with the form

$$F(x,y) \equiv \sqrt{\eta_{ij}\frac{\mathrm{d}x^i}{\mathrm{d}\tau}\frac{\mathrm{d}x^j}{\mathrm{d}\tau}} + \frac{\eta_{ij}\kappa^i}{2m}\frac{\mathrm{d}x^j}{\mathrm{d}\tau}. \quad (1)$$

The action of a free moving particle on Randers space is given as

$$I = \int_s^r \mathcal{L}\mathrm{d}\tau = m\int_s^r F\left(\frac{\mathrm{d}x}{\mathrm{d}\tau}\right)\mathrm{d}\tau. \quad (2)$$

Define the canonical momentum p_i as

$$p_i = m\frac{\partial F}{\partial\left(\frac{\mathrm{d}x^i}{\mathrm{d}\tau}\right)}. \quad (3)$$

Using Euler's theorem on homogeneous functions, we can write the mass–shell condition as

$$\mathcal{M}(p) = g^{ij}p_ip_j = m^2 . \quad (4)$$

The modified dispersion relation in Randers spaces is of the form

$$m^2 = \eta^{ij}p_ip_j - \eta^{ij}\kappa_i(\mu, M_p)p_j + O(\kappa^2), \quad (5)$$

where we have used the notation

$$\eta_{ij} = \mathrm{diag}\{1,-1,-1,-1\}, \quad (6)$$

$$\kappa_i = \kappa\{1,-1,-1,-1\}, \quad (7)$$

and η^{ij} is the inverse matrix of η_{ij}. Here κ can be regarded as a measurement of LI violation. We consider the head-on collision between a soft photon of energy ϵ, momentum q and a high energy particle m_1 of energy E_1, momentum p_1, which leads to the production of two particles m_2, m_3 with energies E_2, E_3 and momentums p_2, p_3, respectively. By making use of the energy and momentum conservation law and the modified dispersion Eq. (5), we obtain the deformed GZK threshold in Randers-Finsler space

$$E_{\mathrm{th}} = \frac{(m_2+m_3)^2 - m_1^2}{4(\epsilon - \kappa/2)}. \quad (8)$$

Taking roughly the energy of soft photon to be 10^{-3} eV, we give a plot for the dependence of the threshold $E_{\mathrm{th}}^{\mathrm{N}}$ on the deformation parameter κ in Fig. 1.

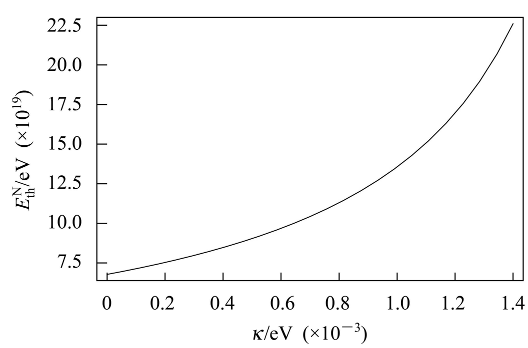

Fig. 1

We can see clearly that a tiny deformation of spacetime (κ with the order of the CMBR) can provide sufficient correction to the primary predicted threshold for the propagation of UHECR particles through the CMBR[1]. If the nature of our universe is Finslerian, more UHECR particles should be detected than Greisen, Zatsepin and Kuzmin expected.

Another invariant speed in Randers-Finsler space is expressed as[15]

$$C_2 = \frac{\kappa^2 - 4m^2}{\kappa^2 + 4m^2}. \quad (9)$$

From the above discussion, we know that the deformation parameter κ may be the same order as CMBR. So far as we know, there is no observational evidence for the existence of the second invariant speed C_2. Thus, we suppose that C_2 is negative or C_2 is beyond the speed of light. The negative condition of the invariant speed C_2 deduces that $m \geqslant \kappa/2$. This gives particle mass a lower bound for a massive particle.

The condition under which C_2 is beyond the speed of light deduces that the mass of the particle is negative. In such a case, C_2 may correspond to the speed of the Goldstone boson.

3 Conclusion

Recently, there has been renewed interest in experimental tests of LI and CPT symmetry. Kostelecky[18] has tabulated experimental results for LI and CPT violation in the minimal Standard-Model Extension. Our result would not violate the minimal Standard-Model Extension, since κ can be eliminated by a redefinition of the energy and momentum. κ is very small, the minor change in energy and momentum can be neglected except for soft photon.

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