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# An explanation of the $\Delta_{D_{35}}(1930)$ as a $\rho\Delta$ bound state*

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**Abstract** Constituent quark models based on two-body potentials systematically overpredict the mass of $\Delta_{D_{35}}(1930)$. A possible solution to this problem comes out from the application of a schematic hybrid model, containing three-quark as well as meson-baryon components, to the light-quark baryon spectrum. The $\Delta_{D_{35}}(1930)$ and its partners $\Delta_{D_{33}}(1940)$ and $\Delta_{S_{31}}(1900)$ are found to contain a significant $\rho\Delta$ component. Then, through the use of the hidden gauge formalism, it is shown that these resonances can be dynamically generated from the $\rho$-$\Delta$ interaction. In particular $\Delta_{D_{35}}(1930)$ can be interpreted as being essentially a $\rho\Delta$ bound state. This interpretation suggests that the inclusion of $\rho\Delta$ as an effective inelastic channel in data analyses could improve the extraction and identification of the resonance.

**Key words** Baryon spectra, dynamically generated resonances, quark model

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## 1 Introduction

The nature of the $\Delta_{D_{35}}(1930)$, a three-star resonance in the Particle Data Group (PDG) review[1] with mass $M$ = 1900—2020 MeV and width $\Gamma$ = 220—500 MeV, has been a matter of controversy since its discovery, from a partial wave analysis of elastic and charge exchange $\pi$p scattering data, in 1976[2]. Existing $SU(6)\times O(3)$ models in that time based on the three-quark (3q) harmonic oscillator did not predict a $(56,1^-)$ multiplet in the $N=3$ band, to which the resonance was assigned, in the 1900 MeV region. Instead a mass 160 MeV higher than the experimental value was obtained (see Ref. [3]). This was interpreted as possible evidence for an hybrid state involving gluonic or nonvalence quark degrees of freedom. This interpretation was questioned a few years later[4] through a revision of the role played by anharmonic perturbations. Since then more refined three-quark models for baryons, based on two-body interactions, which we shall generically denote as $3q^{2b}$, have been developed[5—7]. From them a quite precise description of the well established ground states of baryons from $J=1/2$ to $J=7/2$ or more has been attained (for us ground states refer to the lowest energy N, $\Delta$... states of any $J^P$). In all these models the $\Delta_{D_{35}}(1930)$ (ground state of $\Delta(5/2^-)$) is out of the systematics: the predicted mass is significantly above (80—230 MeV) the PDG average mass.

On the other hand there is nowadays compelling evidence of baryon resonances containing more than 3q components. A paradigmatic case is the $\Lambda(1405)$ which requires (3q models overpredict its mass by about 150 MeV) the consideration of a N$\overline{\text{K}}$ component for its explanation[8]. Here we show that the $\Delta_{D_{35}}(1930)$ may be a similar case requiring a $\rho\Delta$ component to give account of its properties. For this purpose we first revise in section 2 the experimental status of $\Delta_{D_{35}}(1930)$ and examine in section 3 how its mass is systematically overpredicted in quark models based on two-body potentials. The relevant role that $S$-wave meson-baryon components, with thresholds close above the PDG mass averages, may be playing for the determination of the masses of some light-quark resonances, including $\Delta_{D_{35}}(1930)$ and its partners $\Delta_{D_{33}}(1940)$ and $\Delta_{S_{31}}(1900)$, is analyzed in section 4. There a schematic model containing both 3q and meson-baryon (mB) interacting components is considered. The resulting dominance of the $\rho\Delta$ component for $\Delta_{D_{35}}(1930)$, $\Delta_{D_{33}}(1940)$ and $\Delta_{S_{31}}(1900)$

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drives us to consider the possibility that these resonances may be dynamically generated from the ρΔ interaction at the hadronic level. In section 5 we analyze this possibility by making use of the hidden gauge formalism to manage the vector meson-baryon interaction. As a result the three resonances are generated as ρΔ bound states. In particular the absence of $3q$ configurations close in energy for the $\Delta_{D_{35}}(1930)$ allows us to interpret this resonance as a pure ρΔ bound state. In section 6 we go back to data analyses to emphasize the relevance of the inclusion of ρΔ as an effective inelastic channel in order to properly extract and identify the $\Delta_{D_{35}}(1930)$. Finally in section 7 we summarize our main results and conclusions.

## 2 Experimental status

The PDG Breit-Wigner mass ($M$) and width ($\Gamma$) of $\Delta_{D_{35}}(1930)(***)$ are obtained as averages from the masses and widths extracted from several phenomenological models. These models differ in the fitted set of data and in their input for the couple channel problem. To be more precise and for later purpose we shall centre on three of these models, identified in the PDG review as ARNDT, MANLEY and CUTKOSKY, respectively, corresponding to the analyses carried out by the Carnegie-Melon-Berkeley (CMB), Kent State University (KSU) and Virginia Politechnical Institute-George Washington University (VPI-GWU) groups. So CMB [9] fits $\pi N \to \pi N$ data and considers, apart from πN and a non-resonant (NR) ππN channel, a set of effective inelastic channels: πΔ, ρN, ηN, σN, ωN, $\pi N^*$, ρΔ. As for KSU[10] it fits $\pi N \to \pi N$ and $\pi N \to \pi NN$ data using πN, (NR) ππN, πΔ, ρN, $\rho_3 N$, σN, $\pi N^*$ and adding specific inelastic channels for some partial waves: ηN for $S_{11}$, ρΔ for $D_{35}$,...On the other hand VPI-GWU[11] fits $\pi N \to \pi N$ and $\pi N \to \eta N$ data including πN, ηN, πΔ, ρN.

It should be pointed out that most analyses (including CMB and KSU) extract a $\Delta_{D_{35}}$ resonance with a mass close to 1930 MeV. A notable exception is the VPI-GWU model with a mass of 2233±53 MeV and a large width of $773 \pm 187$ MeV. We shall come back to this discrepancy later.

## 3 Mass overprediction from $3q^{2b}$ models

The high $\Delta_{D_{35}}$ mass predicted by $3q^{2b}$ models can be easily understood from the application of the Isgur-Karl quark-quark potential[5] to baryons. This reads

$$V_{ij} = br_{ij} - \frac{2}{3}\frac{\alpha_s}{r_{ij}} + V_{ij}^{\text{hyp}},$$

where $r_{ij}$ is the distance between quarks $i$ and $j$, $b$ is the effective parameter for confinement, $\alpha_s$ the effective chromoelectric quark-quark coupling strength and $V_{ij}^{\text{hyp}}$ stands for the hyperfine potential containing the spin-spin and tensor interactions. This potential can be approximately written as

$$V_{ij} \approx \frac{1}{2}kr_{ij}^2 + \left(U_{ij} + V_{ij}^{\text{hyp}}\right) \equiv V_{ij}^{\text{h.o.}} + (V_{ij}^{\text{pert}}),$$

making clear its similarity to an harmonic oscillator (h. o.) form up to a perturbative correction given by $V_{ij}^{\text{pert}}$. The application of this h. o. quark-quark potential to a baryon (3q system) gives rise to two h. o. potentials corresponding to the two Jacobi coordinates ρ and λ. Then the resulting energy of the baryon is

$$E^{\text{h.o.}} = (2n_\rho + l_\rho + \frac{3}{2})\omega + (2n_\lambda + l_\lambda + \frac{3}{2})\omega \equiv (N+3)\omega,$$

where $n_{\rho,\lambda}$ ($l_{\rho,\lambda}$) are radial (orbital) quantum numbers and $\omega$ is the angular frequency of the oscillator with a value (derived from the fit of the nonstrange baryon spectrum between 1 GeV and 2 GeV) of about 450 MeV. $N \equiv 2(n_\rho + n_\lambda) + l_\rho + l_\lambda$ is called band number since it implies a distribution of the baryon excitations in energy bands corresponding to the values of $N$. By construction $N \geqslant L$, being $L$ the total orbital angular momentum of the 3q system ($\vec{L} = \vec{l_\rho} + \vec{l_\lambda}$). Note also that $N$ determines the parity of the baryon through $(-)^{l_\rho + l_\lambda} = (-)^N$.

Let us now particularize to $\Delta_{D_{35}}(1930)$ which is the lowest energy state with isospin $I = 3/2$ and $J^P = 5/2^-$. As the maximum value for the spin is $S = 3/2$ the minimum value for $L$ is 1. Consequently $N \geqslant 1$ and according to parity $N = 1, 3, 5....$ However $N = 1, L = 1$ is symmetry forbidden since it can not accommodate any spatially symmetric configuration as needed from the isospin symmetric $I = 3/2$ and spin symmetric $S = 3/2$ wave functions. Therefore the minimum value allowed for $N$ is 3. In other words quark Pauli blocking makes the quarks jump in energy to the third energy band. This explains the high mass predicted by $3q^{2b}$ models. The usual configuration assigned to the lowest $\Delta_{D_{35}}$ is the orbitally symmetric $N = 3$, $L^P = 1^-$. It should also be remarked that this is the only $3q^{2b}$ common configuration to $\Delta_{D_{35}}, \Delta_{D_{33}}$ and $\Delta_{S_{31}}$ up to 2 GeV.

## 4 Meson-baryon thresholds

The high $\Delta_{D_{35}}$ mass predicted by $3q^{2b}$ models suggests that other components than the 3q one, such as $4q1\overline{q}$, could be playing a relevant role (alternatively a dynamics including three-quark interactions has been proposed in the literature[12]). In particular we can expect $S$-wave meson-baryon (mB) components with thresholds close above the PDG mass and quite below the $3q^{2b}$ mass prediction to be important (meson-baryon components with thresholds below the PDG mass are expected to contribute mainly to the width). By taking into account that the quite similar masses of $\Delta_{D_{35}}(1930)$, $\Delta_{S_{31}}(1900)(**)$ and $\Delta_{D_{33}}(1940)(*)$ added to the presence of the common 3q configuration may be indicating the same structure in all of them we look for possible common mB components. We find $\omega\Delta(1232)$ with a threshold energy $E_{\rm th}(\omega\Delta)=M_\omega+M_\Delta=2014$ MeV and $\rho\Delta(1232)$ with a lower threshold $E_{\rm th}(\rho\Delta)=M_\rho+M_\Delta=2002$ MeV.

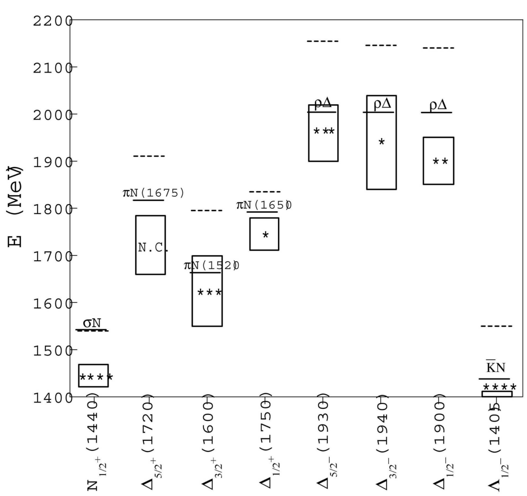


Fig. 1. Mass predictions from Ref. [6] (dotted lines) as compared to experimental mass intervals (boxes). N.C. means non-cataloged resonance, see Ref.[13]. Selected thresholds are indicated by solid lines.

Along this line of thinking a generalized study of light-quark baryon states with $3q^{2b}$ masses significantly higher than the corresponding PDG mass averages has been carried out in reference[13]. In Fig. 1 we show graphically the situation indicating for each resonance the PDG mass, the $3q^{2b}$ mass prediction from[6] and the selected mB threshold $E_{\rm th(mB)}=M_{\rm m}+M_{\rm B}$. Furthermore a schematic model for the coupling of the 3q and mB channels has been worked out. This model can be considered to a certain extent as a crude simplification of the EBAC data analysis[14] where bare $B^*$ states are introduced to represent the quark-core components of the baryon resonances (the bare masses are parameters that may be identified with constituent quark model predictions). Our model considers a $2\times2$ hamiltonian matrix

$$[H]\simeq\begin{pmatrix} M_{\rm m}+M_{\rm B} & a \\ a^* & M_{3q^{2b}}\end{pmatrix},$$

where a parametrizes the 3q-mB coupling. The corresponding eigenvalues, $M_\pm$, are given by

$$M_\pm=\left(\frac{M_{3q^{2b}}+(M_{\rm m}+M_{\rm B})}{2}\right)\pm\sqrt{\left(\frac{M_{3q^{2b}}-(M_{\rm m}+M_{\rm B})}{2}\right)^2+|a|^2}.$$

Then by choosing the lowest eigenvalues $M_-$, from a universal $|a|\sim85$ MeV, all the baryon state masses can be well reproduced, see Fig. 2 (the assignment to $M_+$ states has been detailed in[13] and will not be considered here). Moreover the diagonalization provides the probabilities of the 3q and mB components entering in each state. In particular for $\Delta_{D_{35}}(1930)$ and its partners the probability of a $\rho\Delta$ component is bigger than 80%.

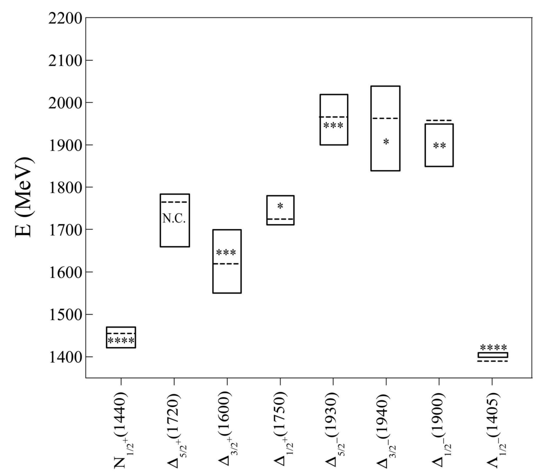


Fig. 2. Mass predictions (dotted line) from a schematic hybrid model (see text) as compared to experimental mass intervals.

## 5 $\Delta_{D_{35}}(1930)$ as a $\rho\Delta$ bound state

The large $\rho\Delta$ probability for $\Delta_{D_{35}}(1930)$, $\Delta_{S_{31}}(1900)$ and $\Delta_{D_{33}}(1940)$, related to the fact that the $\rho\Delta$ threshold energy is much lower than the $3q^{2b}$ mass prediction but above the PDG mass, may be indicating that these states actually correspond to $\rho\Delta$ bound states. To analyze this indication we consider the $\rho\Delta\to\rho\Delta$ interaction at the hadronic level. Effective meson exchange lagrangian approaches to deal with $\rm mB\to mB$ processes have been developed by different working groups (Mainz, Giessen, KVI, Bonn, Juelich, Valencia,...).The hidden gauge formalism, used by the Valencia group, is particularly suited for the interaction of vector mesons with baryons, see[15] and references threrein. The vertices in the $VB \to VB$ diagram of Fig. 3, with $V$ standing for a vector meson in the $SU(3)$ flavor nonet and $B$ for a baryon in the $SU(3)$ flavor decuplet, are determined by the chiral lagrangians

$$\mathcal{L}_{\text{III}}^{(3V)} = \mathrm{i}g\langle(\partial_\mu V_\nu - \partial_\nu V_\mu)V^\mu V^\nu\rangle,$$

where $g = \dfrac{M_V}{2f}$ with $f$ the pion decay constant and

$$\mathcal{L}^{(VBB)} = -\mathrm{i}\overline{B}_\mu^{abc}\gamma_\nu D^\nu B_{abc}^\mu\,,$$

where $B_{abc}^\mu$ is the spin decuplet field ($abc$ stand for the $SU(3)$ indices). The covariant derivative is defined as

$$D^\nu B_{abc}^\mu = \partial^\nu B_{abc}^\mu + (\Gamma^\nu)_a^d B_{dbc}^\mu + (\Gamma^\nu)_b^d B_{adc}^\mu + (\Gamma^\nu)_c^d B_{abd}^\mu$$

with

$$\Gamma^\nu = -\frac{1}{4f^2}(V^\mu\partial^\nu V_\mu - \partial^\nu V_\mu V^\mu)$$

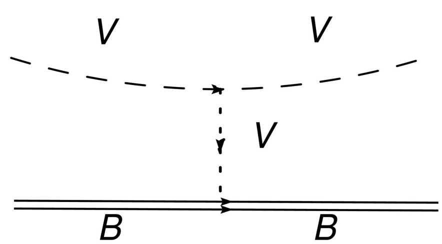


Fig. 3. Diagram obtained in the Effective Chiral Lagrangian approach for the interaction of a vector meson with a baryon.

Then from Fig. 3 a potential $\overline{V}$ can be derived and from it a $T$-matrix

$$T = \frac{\overline{V}}{1-\overline{V}G}\,,$$

where $G$ is the loop function for a vector meson and a baryon which is regularized with a cutoff. In order to take into account the widths of $V$ and $B$, $G$ is convoluted with the mass distributions of $V$ and $B$.

The ρΔ case has been thoroughly studied in Ref. [16]. The interaction potential turns out to be attractive for isospin $I=3/2$ and $I=1/2$ (the more attractive) and repulsive for the exotic $I=5/2$. For both $I=3/2$ and $I=1/2$ the corresponding $J^P=1/2^-$, $3/2^-$, $5/2^-$ states are degenerate. In Fig. 4 (without $V$ and $B$ widths) and Fig. 5 (with $V$ and $B$ widths), taken from this latter work, the dependence $|T|^2(\sqrt{s})$ for $I=3/2$ and different values of the cutoff (these values are chosen about the so called natural value of 600—700 MeV[17]) is plotted. A pole (bound state) shows up in the energy interval 1940—1980 MeV. The small variation of the pole position, corresponding to a quite large cutoff interval of 150 MeV, makes us fully confident on the presence of such a pole. This confidence is reinforced by the calculation of the small additional contribution coming from intermediate ωΔ states involving the anomalous coupling $\rho \to \omega\pi$ and by the expectation (qualitative) of small contributions coming from πN and πΔ intermediate states. Note though that these and other possible additional contributions might have some effect enlarging the width (imaginary part of the pole) whilst hardly varying the pole mass (real part).

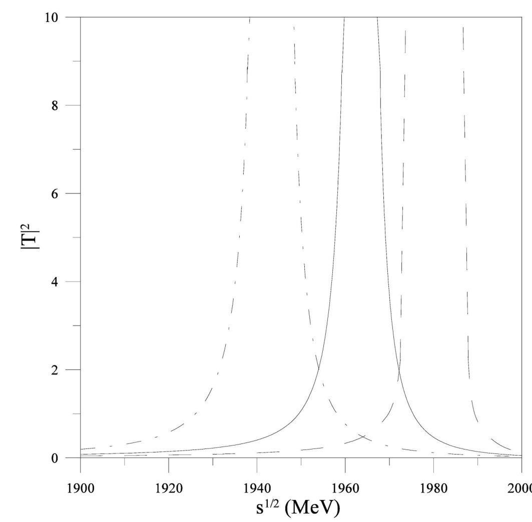


Fig. 4. $|T|^2$ for $\rho\Delta \to \rho\Delta$ in the $I=3/2$ channel for several values of the cutoff $q_{\max}$: solid line $q_{\max}=770$ MeV, dashed line $q_{\max}=700$ MeV, dashed-dotted line $q_{\max}=630$ MeV.

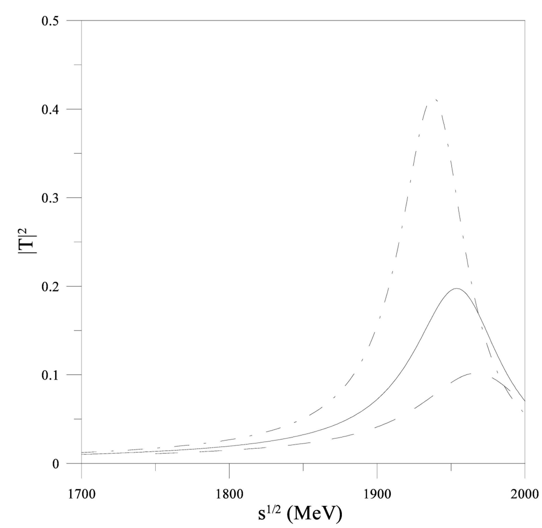


Fig. 5. $|T|^2$ for $\rho\Delta \to \rho\Delta$ in the $I=3/2$ channel for several values of the cutoff $q_{\max}$ including ρ and Δ mass distributions: solid line $q_{\max}=770$ MeV, dashed line $q_{\max}=700$ MeV, dashed-dotted line $q_{\max}=630$ MeV.

When comparing our results for $I=3/2$ with data we find a good quantitative agreement for the masses of our degenerate $J^P=1/2^-$, $3/2^-$, $5/2^-$ states with $\Delta_{S_{31}}(1900)$, $\Delta_{D_{33}}(1940)$ and $\Delta_{D_{35}}(1930)$. Concerning their widths the calculated value, $\sim 50$ MeV, is significantly smaller than data following the expected tendency. Therefore our calculation provides some support to the existence of these not well established resonances. Moreover, for $\Delta_{D_{35}}$, the absence of any 3q configuration close in energy allows the identification of $\Delta_{D_{35}}(1930)$ as a pure $\rho\Delta$ bound state and the interpretation of the 3q bound state as another resonance lying $\sim 300$ MeV above. For $\Delta_{S_{31}}$ and $\Delta_{D_{33}}$ the presence of 3q configurations around 2000 MeV (excitations of the low lying states in the $N=1$ band), which could contribute with a significant probability, prevents making the same definite assignment.

## 6 Extraction of $\Delta_{D_{35}}(1930)$ from data analyses

As detailed in section 2 most light-quark baryon resonances are extracted from data through a multichannel scattering matrix including effective inelastic channels. It seems then reasonable to think that the implementation of $\rho\Delta$ as one of these inelastic channels may be helpful in the extraction of $\Delta_{D_{35}}(1930)$ as a distinctive resonance. This may explain why the CMB and KSU data analyses explicitly containing $\rho\Delta$ find a mass about 1930 MeV whereas the VPI-GWU analysis not including $\rho\Delta$ misses such a mass. Actually the mass found by VPI-GWU, $2233\pm 53$ MeV, might well correspond to the lowest $\Delta_{D_{35}(3q)}$ state.

The same type of study can be done for $\Delta_{S_{31}}(1900)$ and $\Delta_{D_{33}}(1940)$ and the same explanation, based on the need of including the relevant $\rho\Delta$ channel, may be inferred from it. Therefore we can tentatively conclude that the implementation of $\rho\Delta$ as a quasi two-body channel may be an essential ingredient for the proper extraction of these resonances from data. Furthermore the introduction of $\rho\Delta$ could also be indispensable to properly satisfy unitarity once data involving $\pi\pi\pi$N in the final state are incorporated into the fit.

## 7 Summary

We have performed an extensive theoretical and phenomenological study of $\Delta_{D_{35}}(1930)(***)$. The impossibility to describe it (altogether with the rest of the light-quark baryon spectrum) from constituent 3q models based on two-body interactions has been understood as a consequence of quark Pauli blocking that forces the quarks to acquire two extra units of orbital angular momentum to satisfy the symmetry requirements. This makes feasible that $4q\overline{q}$ terms in the form of $S$-wave meson-baryon components are energetically dominant. We have centered on the study of $\rho\Delta$ as it has the lower threshold above the experimental average mass. To evaluate the $\rho\Delta$ effect we have used first a schematic model containing $\rho\Delta$ as well as 3q components. From it a clear conclusion has been extracted: $\Delta_{D_{35}}(1930)$ is mostly a $\rho\Delta$ state. Second we have used the hidden gauge formalism combined with unitary techniques to show that $\Delta_{D_{35}}(1930)$ can be dynamically generated as a bound state from the $\rho\Delta\rightarrow\rho\Delta$ interaction described in terms of an effective chiral lagrangian at the hadron level. Thus the key role played by a $\rho\Delta$ component is confirmed. Moreover the absence of 3q configurations close in energy takes us to interpret $\Delta_{D_{35}}(1930)$ as being essentially a $\rho\Delta$ bound state. This interpretation allows for a solution of the long-standing puzzle concerning the description of this resonance in constituent quark models.

From the experimental standpoint the $\rho\Delta$ nature of $\Delta_{D_{35}}(1930)$ gets also support from a revision of the inputs in the phenomenological analyses employed to extract the resonance properties. It turns out that a mass close to the average comes out from those analyses including $\rho\Delta$ as an effective inelastic channel. On the contrary when this channel is misssing a much higher mass is obtained. This points out to the implementation of a quasi two-body $\rho\Delta$ channel to properly give account of the resonance. Notice that this implementation may also play an important role in the unitarized description of data from processes like $(\pi,\gamma,e)N\rightarrow\pi\pi\pi N$ that should be carried out in a future. We encourage an effort along this line.

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