

# Study of spin sum rules (and the strong coupling constant at large distances)<sup>\*</sup>

A. Deur<sup>1)</sup>

(Thomas Jefferson National Accelerator Facility, Newport News, VA 23606, USA)

**Abstract** We present recent results from Jefferson Lab on sum rules related to the spin structure of the nucleon. We then discuss how the Bjorken sum rule with its connection to the Gerasimov-Drell-Hearn sum, allows us to conveniently define an effective coupling for the strong force at all distances.

**Key words** Strong coupling constant, QCD spin sum rules, non-perturbative, commensurate scale relations, Schwinger-Dyson, Lattice QCD, AdS/CFT

**PACS** 12.38.Qk, 11.55.Hx

## 1 Introduction

The information on the longitudinal spin structure of the nucleon is contained in the  $g_1(x, Q^2)$  and  $g_2(x, Q^2)$  spin structure functions, with  $Q^2$  the squared four-momentum transferred from the beam to the target, and  $x = Q^2/(2M\nu)$  the Bjorken scaling variable ( $\nu$  is the energy transfer and  $M$  the nucleon mass). The variable  $Q^2$  indicates the space-time scale at which the nucleon is probed and  $x$  is interpreted in the parton model as the fraction of nucleon momentum carried by the struck quark.

Although spin structure functions are the basic observables for nucleon spin studies, considering their integrals taken over  $x$  is advantageous because of resulting simplifications. More importantly, such integrals are at the core of the relation dispersion formalism. Relation dispersions relate the integral over the imaginary part of a quantity to its real part. Expressing the imaginary part in function of the real part using the optical theorem yields sum rules. When additional hypotheses are used, such as a low energy theorem or the validity of Operator Product Expansion (OPE), the sum rules relate the integral to a static property of the target. If the static property is well known, the verification of the sum rule provides a check of the theory and hypotheses used in

the sum rule derivation. When the property is not known because e.g. it is difficult to measure directly, sum rules can be used to access them. In that case, the theoretical framework used to derive the sum rule is assumed to be valid. Details on integrals of spin structure functions and sum rules are given e.g. in the review<sup>[1]</sup>.

Several spin sum rules exist. We will focus on the Bjorken sum rule<sup>[2]</sup> and the Gerasimov-Drell-Hearn (GDH) sum rule<sup>[3]</sup>. In this paper, we will consider the  $n$ -th Cornwall-Norton moments:  $\int_0^1 dx g_1^N(x, Q^2) x^n$ , with  $N$  standing for proton or neutron, and write the first moments as  $\Gamma_1^N(Q^2) \equiv \int_0^1 dx g_1^N(x, Q^2)$ .

## 2 The generalized Bjorken and GDH sum rules

The Bjorken sum rule<sup>[2]</sup> relates the integral over  $(g_1^p - g_1^n)$  to the nucleon axial charge  $g_A$ . This relation has been essential for understanding the nucleon spin structure and establishing, via its  $Q^2$ -dependence, that Quantum Chromodynamics (QCD) describes the strong force when spin is included. The Bjorken integral has been measured in polarized deep inelastic lepton scattering (DIS) at SLAC, CERN and DESY and at moderate  $Q^2$  at Jefferson Lab (JLab),

Received 7 August 2009

<sup>\*</sup> Supported by U.S. Department of Energy (DOE). The Jefferson Science Associates (JSA) operates the Thomas Jefferson National Accelerator Facility for the DOE (DE-AC05-84ER40150)

1) E-mail: deurpam@jlab.org

©2009 Chinese Physical Society and the Institute of High Energy Physics of the Chinese Academy of Sciences and the Institute of Modern Physics of the Chinese Academy of Sciences and IOP Publishing Ltd

see Refs. [4–19]. In the perturbative QCD (pQCD) domain (high  $Q^2$ ) the sum rule reads:

$$\Gamma_1^{p-n}(Q^2) \equiv \int_0^1 dx (g_1^p(x, Q^2) - g_1^n(x, Q^2)) = \frac{g_A}{6} \left[ 1 - \frac{\alpha_s}{\pi} - 3.58 \frac{\alpha_s^2}{\pi^2} - 20.21 \frac{\alpha_s^3}{\pi^3} + \dots \right] + \sum_{i=2}^{\infty} \frac{\mu_{2i}^{p-n}(Q^2)}{Q^{2i-2}}, \quad (1)$$

where  $\alpha_s(Q^2)$  is the strong coupling strength. The bracket term (known as the leading twist term) is mildly dependent on  $Q^2$  due to pQCD soft gluon radiation. The other term contains non-perturbative power corrections (higher twists). These are quark and gluon correlations describing the nucleon structure away from the large  $Q^2$  (small distances) limit.

The generalized Bjorken sum rule has been derived for small distances. For large distances, in the  $Q^2 \rightarrow 0$  limit, one finds the generalized GDH sum rule. The sum rule was first derived at  $Q^2 = 0$ :

$$\int_{\nu_0}^{\infty} \frac{\sigma_{1/2}(\nu) - \sigma_{3/2}(\nu)}{\nu} d\nu = -\frac{2\pi^2 \alpha \kappa^2}{M_t^2}, \quad (2)$$

where  $\nu_0$  is the pion photoproduction threshold,  $\sigma_{1/2}$  and  $\sigma_{3/2}$  are the helicity dependent photoproduction cross sections for total photon plus target helicities 1/2 and 3/2,  $\kappa$  is the anomalous magnetic moment of the target while  $S$  is its spin and  $M_t$  its mass.  $\alpha$  is

the fine structure constant.

Replacing the photoproduction cross sections by the electroproduction ones generalized the left hand side of Eq. (2) to any  $Q^2$ . Such generalization depends on the choice of convention for the virtual photon flux, see e.g. Ref. [1]. X. Ji and J. Osborne<sup>[20]</sup> showed that the sum rule itself (i.e. the whole Eq. (2)) can be generalized as:

$$\frac{8}{Q^2} \int_0^{x^-} g_1 dx = s_1(0, Q^2), \quad (3)$$

where  $S_1(\nu, Q^2)$  is the spin dependent Compton amplitude. This generalization of the GDH sum rule makes the connection between the Bjorken and GDH generalized sum rules evident:  $\text{GDH} = \frac{Q^2}{8} \times \text{Bjorken}$ .

The connection between the GDH and Bjorken sum rules allows us in principle to compute the moment  $\Gamma_1$  at any  $Q^2$ . Thus, it provides us with a choice observable to understand the transition of the strong force from small to large distances.

### 3 Experimental measurements of the first moments

Results from experimental measurements from SLAC<sup>[6]</sup>, CERN<sup>[8]</sup>, DESY<sup>[9]</sup> and JLab<sup>[10–19]</sup> of the first moments are shown in Fig. 1.

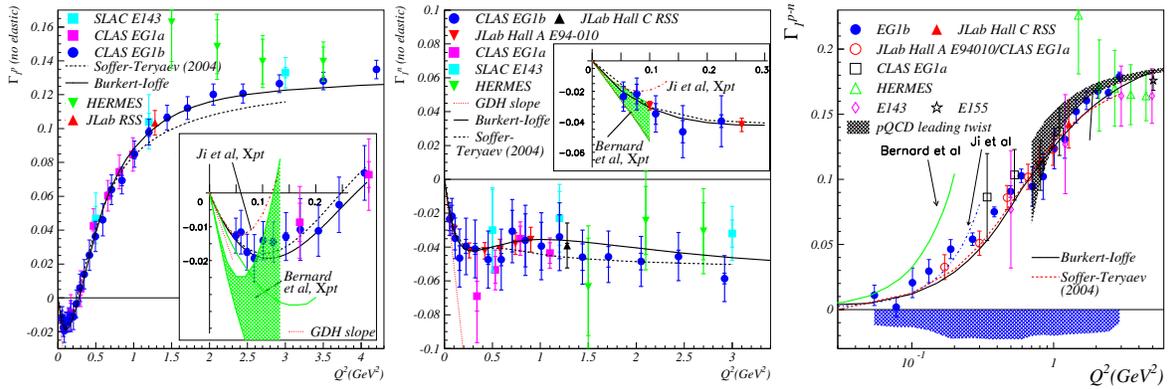


Fig. 1. Experimental data from SLAC, CERN, DESY and JLab at low and intermediate  $Q^2$  on  $\Gamma_1^p$  (left),  $\Gamma_1^n$  (center) and  $\Gamma_1^{p-n}$  (right).

There is an excellent mapping of the moments at intermediate  $Q^2$  and enough data points a low  $Q^2$  to start testing the Chiral Perturbation Theory ( $\chi PT$ ), the effective theory strong force at large distances. In particular, the Bjorken sum is important for such test because the (p-n) subtraction cancels the  $\Delta_{1232}$  resonance contribution which should make

the  $\chi PT$  calculations significantly more reliable<sup>[21]</sup>. The comparison between the data at low  $Q^2$  and  $\chi PT$  calculations<sup>[22, 23]</sup> can be seen more easily in the insert in each plot of Fig. 1. The calculations assume the  $\Gamma_1$  slope at  $Q^2=0$  from the GDH sum rule prediction. Consequently,  $\chi PT$  calculates the deviation from the slope and this is what one should test. A meaning-

ful comparison is provided by fitting the lowest data points using the form  $\Gamma_1^N = \frac{\kappa_N^2}{8M^2}Q^2 + aQ^4 + bQ^6\dots$  and compare the obtained value of  $a$  to the values calculated from  $\chi PT$ . Such comparison has been carried out for the proton, deuteron<sup>[16]</sup> and the Bjorken sum<sup>[18]</sup>. These fits point out the importance of including a  $Q^6$  term for  $Q^2 < 0.1$  GeV<sup>2</sup>. The  $\chi PT$  calculations seems to agree best with the measurement of the Bjorken sum, in accordance with the discussion in<sup>[21]</sup>. Phenomenological models<sup>[24, 25]</sup> are in good agreement with the data over the whole  $Q^2$  range.

## 4 The strong coupling at large distances

A primary goal of the JLab experiments was to map precisely the intermediate  $Q^2$  range in order to shed light on the transition from short distances (where the degrees of freedom pertinent to the strong force are the partonic ones) to large distances where the hadronic degrees of freedom are relevant to the strong force. One feature seen on Fig. 1 is that the transition from small to large distances is smooth, e.g. without sign of a phase transition. This fact can be used to extrapolate the definition of the strong force effective coupling to large distances. Before discussing this, we first review the QCD coupling and the issues with calculating it at large distances.

In QCD, the magnitude of the strong force is given by the running coupling constant  $\alpha_s$ . At large  $Q^2$ , in the pQCD domain,  $\alpha_s$  is well defined and is given by the series:

$$\mu \frac{\partial \alpha_s}{\partial \mu} = 2\beta(\alpha_s) = -\frac{\beta_0}{2\pi}\alpha_s^2 - \frac{\beta_1}{4\pi^2}\alpha_s^3 - \frac{\beta_2}{64\pi^3}\alpha_s^4 - \dots, \quad (4)$$

where  $\mu$  is the energy scale, to be identified to  $Q$ . The first terms of the  $\beta$  series are:  $\beta_0 = 11 - \frac{2}{3}n$  with  $n$  the number of active quark flavors,  $\beta_1 = 51 - \frac{19}{3}n$  and  $\beta_2 = 2857 - \frac{5033}{9}n + \frac{325}{27}n^2$ . The solution of the differential equation 4 is:

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda_{\text{QCD}}^2)} \left[ 1 - \frac{2\beta_1}{\beta_0^2} \frac{\ln[\ln(\mu^2/\Lambda_{\text{QCD}}^2)]}{\ln(\mu^2/\Lambda_{\text{QCD}}^2)} + \frac{4\beta_1^2}{\beta_0^4 \ln^2(\mu^2/\Lambda_{\text{QCD}}^2)} \times \left( \left( \ln[\ln(\mu^2/\Lambda_{\text{QCD}}^2)] - \frac{1}{2} \right)^2 + \frac{\beta_2\beta_0}{8\beta_1^2} - \frac{5}{4} \right) \right]. \quad (5)$$

Eq. (5) allows us to evolve the different experimental

determinations of  $\alpha_s$  to a conventional scale, typically  $M_{z_0}^2$ . The agreement between the  $\alpha_s$  obtained from different observables demonstrates its universality and the validity of Eq. (4). One can obtain  $\alpha_s(M_{z_0}^2)$  with doubly polarized DIS data and assuming the validity of the Bjorken sum. Solving Eq. (1) using the experimental value of  $\Gamma_1^{\text{p-n}}$ , and then using Eq. (5) provides  $\alpha_s(M_{z_0}^2)$ .

Eq. (5) leads to an infinite coupling at large distances, when  $Q^2$  approaches  $\Lambda_{\text{QCD}}^2$ . This is not a conceptual problem since we are out of the validity domain of pQCD on which Eq. (5) is based. But since data show no sign of discontinuity or phase transition when crossing the intermediate  $Q^2$  domain, one should be able to define an effective coupling  $\alpha_s^{\text{eff}}$  at any  $Q^2$  that matches  $\alpha_s$  at large  $Q^2$  but stays finite at small  $Q^2$ .

The Bjorken Sum Rule can be used to define  $\alpha_s^{\text{eff}}$  at low  $Q^2$ . Defining  $\alpha_s^{\text{eff}}$  from a pQCD equation truncated to first order (in our case Eq. (1):  $\Gamma_1^{\text{p-n}} \equiv \frac{1}{6}(1 - \alpha_{s,g1}/\pi)$ ), offers advantages. In particular,  $\alpha_s^{\text{eff}}$  does not diverge near  $\Lambda_{\text{QCD}}$  and is renormalization scheme independent. However,  $\alpha_s^{\text{eff}}$  becomes dependent on the choice of observable employed to define it. If  $\Gamma_1^{\text{p-n}}$  is used as the defining observable, the effective coupling is noted  $\alpha_{s,g1}$ . Relations, called commensurate scale relations<sup>[26]</sup>, link the different effective couplings so in principle one effective coupling is enough to describe the strong force and the theory retains its predictive power. These relations are defined for short distances and whether they extrapolate to large distances remains to be investigated.

The choice of defining the effective charge with the Bjorken sum has many advantages: low  $Q^2$  data exist and near real photons data from JLab is being analyzed<sup>[27, 28]</sup>. Furthermore, sum rules constrain  $\alpha_{s,g1}$  at both low and large  $Q^2$ , as will be discussed in the next paragraph. Another advantage is that, as discussed for the low  $Q^2$  domain, the simplification arising in  $\Gamma_1^{\text{p-n}}$  makes a quantity well suited to be calculated at any  $Q^2$ <sup>[21]</sup>. These simplifications are manifest at large  $Q^2$  when comparing the validities of the Bjorken and Ellis-Jaffe sum rules. It also simplifies Lattice QCD calculations in the intermediate  $Q^2$  domain. Finally, it may be argued that  $\alpha_{s,g1}$  might be more directly comparable to theoretical calculations than other effective couplings extracted from other observables: part of the coherent response of the nucleon is suppressed in the Bjorken sum, e.g. the  $\Delta$  resonance, so the non-resonant background, akin to the pQCD incoherent scattering process, contributes especially importantly to the Bjorken sum. This

argument is reinforced if global duality works, a credible proposal since the  $\Delta$  resonance is suppressed.

The effective coupling definition in terms of pQCD evolution equations truncated to first order was proposed by Grunberg<sup>[29]</sup>. Grunberg's definition is meant for short distances but one can always extrapolated this definition and see how the resulting coupling compares to calculation of  $\alpha_s$  at large distances. Using Grunberg's definition at large distances entails including higher twists in  $\alpha_{s,g1}$  in addition to the higher terms of the pQCD series.

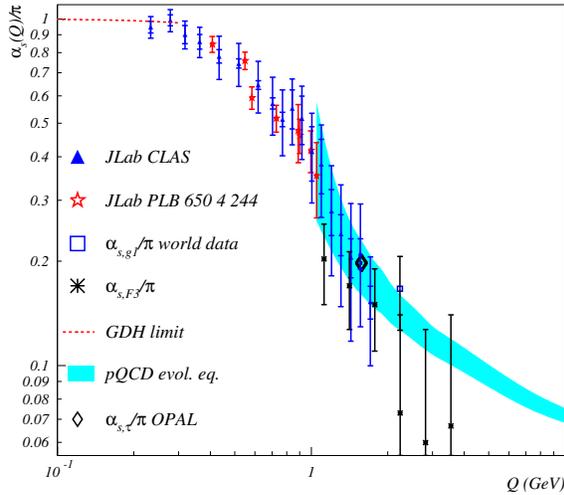


Fig. 2. Effective couplings extracted from different observables, see text for details. The gray band indicates  $\alpha_{s,g1}$  extracted from the pQCD expression of the Bjorken sum at leading twist and third order in  $\alpha_s$ . The values of  $\alpha_{s,g1}/\pi$  extracted using the GDH sum rule is given by the red dashed line.

It is common to fold the dynamics due to forces (here the Higher Twists) into an effective parameter so that the particle can be treated as free. It is interesting to review quickly the characteristics of such effective parameters, e.g. in the field of quantum electronics. There, near the energy extrema of electrons moving in a crystal, the effects of external forces applied to the crystal are folded into effective masses and the electron motions can be described using the free Schrodinger equation. Then, the (effective) mass of an electron becomes a tensor  $m_{ij}^*$  (that depends on the electron energy) rather than a scalar since the crystal lattice is not isotropic and the total acceleration depends on the lattice forces.  $m_{ij}^*$  depends on the material and, near an energy maximum,  $m_{ii}^*$  is negative. Holes also have effective masses of opposite signs as for electrons. Effective masses are useful to

determine quantities of interest of a material, such as the quantum state densities, the speed of electric signals, or the surface of isoenergy. This illustrates the relevance of effective parameters, but also that we should not be shocked if our effective couplings depends on reactions or are negative.

Effective couplings have been extracted from different observables, see Fig. 2. Values of  $\alpha_{s,g1}/\pi$  extracted from the world data on the Bjorken sum at  $Q^2 = 5 \text{ GeV}^2$ <sup>[7]</sup> and from JLab data<sup>[10, 18]</sup> have been compared using the commensurate scale relations<sup>[30]</sup> to  $\alpha_{s,\tau}$  extracted from the OPAL data on  $\tau$  decay<sup>[26]</sup>, and  $\alpha_{s, \text{GLS}}$  extracted using the Gross-Llewellyn Smith sum rule<sup>[31]</sup> and its measurement by the CCFR collaboration<sup>[32]</sup>. There is good agreement between  $\alpha_{s,g1}$ ,  $\alpha_{s,F3}$  and  $\alpha_{s,\tau}$ .

The GDH and Bjorken sum rules can be used to extract  $\alpha_{s,g1}$  at small and large  $Q^2$  respectively<sup>[30]</sup>. This, together with the JLab data at intermediate  $Q^2$ , provides for the first time a coupling at any  $Q^2$ . A striking feature of Fig. 2 is that  $\alpha_{s,g1}$  becomes scale invariant at small  $Q^2$ . This was predicted by a number of calculations and it is known that color confinement leads to an infrared fixed point<sup>[33]</sup>, but it is the first time it is seen experimentally. A fit of the  $\alpha_{s,g1}$  has been performed and is shown on Fig. 3 (plain black line).

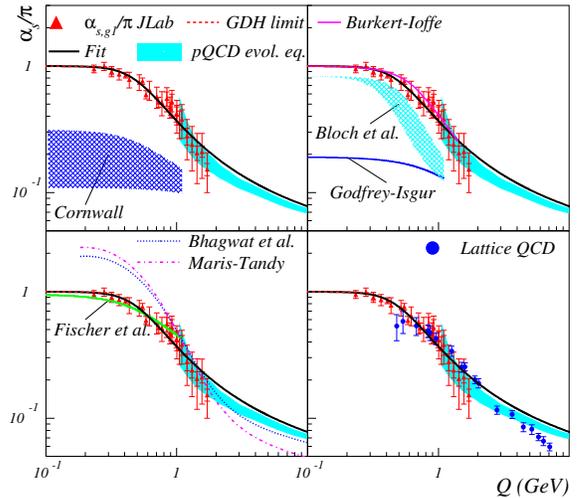


Fig. 3. The effective coupling  $\alpha_{s,g1}$  compared to  $\alpha_s$  calculations.

There are several techniques used to predict  $\alpha_s$  at small  $Q^2$ , e.g. lattice QCD, solving the Schwinger-Dyson equations, or choosing the coupling in a constituent quark model so that it reproduces hadron spectroscopy. However, the connection between these

$\alpha_s$  is unclear, in part because of the different approximations used. In addition, the precise relation between  $\alpha_{s,g_1}$  (or any effective coupling defined using<sup>[29]</sup> or Ref. [26]) and these computations is unknown. Nevertheless, one can still compare them to see if they share common features. In Fig. 3,  $\alpha_{s,g_1}$  extracted from JLab data, its fit, and its extraction using the Burkert and Ioffe<sup>[24]</sup> model to obtain  $\Gamma_1^{p-n}$  are compared to  $\alpha_s$  calculations. The methods used are solving the Schwinger-Dyson equations (Top left: Cornwall<sup>[34]</sup>; Top right: Bloch<sup>[35]</sup>; Bottom left: Maris-Tandy<sup>[37]</sup>, Fischer, Alkofer, Reinhardt and Von Smekal<sup>[38]</sup>, and Bhagwat et al.<sup>[39]</sup>),  $\alpha_s$  used in a quark constituent model (Godfrey-Isgure<sup>[36]</sup>) and Lattice QCD<sup>[40]</sup> (bottom right). The calculations and  $\alpha_{s,g_1}$  present a similar behavior. Some calculations, in particular the lattice one, are in excellent agreement with  $\alpha_{s,g_1}$ .

These works show that  $\alpha_s$  is scale invariant (conformal behavior) at small and large  $Q^2$  (but not in the transition region between the fundamental description of QCD in terms of quarks and gluons degrees of freedom and its effective one in terms of baryons and mesons). The scale invariance at large  $Q^2$  is the well known asymptotic freedom. The conformal behavior at small  $Q^2$  is essential to apply a property of conformal field theories (CFT) to the study of hadrons: the Anti-de-Sitter space/Conformal Field Theory (AdS/CFT) correspondence of Maldacena<sup>[41]</sup>, that links a strongly coupled gauge field to weakly

coupled superstrings states. Perturbative calculations are feasible in the weak coupling AdS theory. They are then projected on the AdS boundary, where they correspond to the calculations that would have been obtained with the strongly coupled CFT. This opens the possibility of analytic non-perturbative QCD calculations<sup>[42]</sup>.

## 5 Summary and perspectives

We discussed the JLab data on moments of spin structure functions, in particular at large distances where we compared them to  $\chi PT$ , the strong force effective theory at large distances. The smoothness of  $Q^2$ -dependence of the moments when transiting from perturbative to the non-perturbative domain allows to extrapolate the definitions of effective strong couplings from short to large distances. Thanks to the data on nucleon spin structure and to spin sum rules, the effective strong coupling  $\alpha_{s,g_1}$  can be extracted in any regime of QCD. The question of comparing it with theoretical calculations of  $\alpha_s$  at low  $Q^2$  is open, but such comparison exposes a similarity between these couplings. Apart for the parton-hadron transition region, the coupling shows that QCD is approximately a conformal theory. This is a necessary ingredient to the application of the AdS/CFT correspondence that may make analytical calculations possible in the non-perturbative domain of QCD.

## References

- 1 CHEN J P, Deur A, Meziani Z E. Mod. Phys. Lett. A, 2005, **20**: 2745
- 2 Bjorken J D. Phys. Rev., 1966, **148**: 1467; Phys. Rev. D, 1970, **1**: 465; Phys. Rev. D, 1970, **1**: 1376
- 3 Drell S D, Hearn A C. Phys. Rev. Lett., 1966, **16**: 908; Gerasimov S, Sov. J. Nucl. Phys., 1966, **2**: 430
- 4 Anthony P L et al. Phys. Rev. Lett., 1993, **71**: 959
- 5 Abe K et al. Phys. Rev. Lett., 1995, **74**: 346; 1995, **75**: 25; 1996, **76**: 587; Phys. Lett. B, 1995, **364**: 61; Phys. Rev. D, 1998, **58**: 112003
- 6 Abe K et al. Phys. Rev. Lett., 1997, **79**: 26
- 7 Anthony P L et al. Phys. Lett. B 1999, **458**: 529; 1999, **463**: 339; 2000, **493**: 19; 2003 **553**: 18
- 8 Adeva D et al. Phys. Rev. D, 1998, **58**: 112001
- 9 Ackerstaff K et al. Phys. Lett. B, 1998, **404**: 383; 1998, **444**: 531; Airapetian A et al. Phys. Lett. B, 1998, **442**: 484; Phys. Rev. Lett., 2003, **90**: 092002; Eur. Phys. J. C, 2003, **26**: 527; Phys. Rev. D, 2007, **75**: 012007
- 10 Fatemi R et al. Phys. Rev. Lett., 2003, **91**: 222002
- 11 Yun J et al. Phys. Rev. C, 2003, **67**: 055204
- 12 Amarian M et al. Phys. Rev. Lett., 2002, **89**: 242301
- 13 Amarian M et al. Phys. Rev. Lett., 2004, **92**: 022301
- 14 Amarian M et al. Phys. Rev. Lett., 2004, **93**: 152301
- 15 Dharmawardane V et al. Phys. Lett. B, 2006, **641**: 11
- 16 Prok Y et al. Phys. Lett. B, 2009, **672**: 12
- 17 Deur A et al. Phys. Rev. Lett., 2004, **93**: 212001-1
- 18 Deur A et al. Phys. Rev. D, 2004, **78**: 032001
- 19 Slifer K, arXiv:0812.0031, 2009
- 20 Ji X et Osborne J, J. of Phys. G, 2001, **27**: 127
- 21 Burkert V D, Phys. Rev. D, 2001, **63**: 097904
- 22 Bernard V, Hemmert T R, Meissner Ulf-G. Phys. Rev. D, 2003, **67**: 076008
- 23 Ji X, KAO C W, Osborne J. Phys. Lett. B, 2000, **472**: 1
- 24 Burkert V D, Ioffe B L. Phys. Lett. B, 1992, **296**: 223; J. Exp. Theor. Phys., 1994, **78**: 619
- 25 Soffer J, Teryaev O V. Phys. Lett. B, 2002, **545**: 323; Phys. Rev. D, 2004, **70**: 116004
- 26 Brodsky S J, LU H J. Phys. Rev. D, 1995, **51**: 3652; Brodsky S J, Gabadadze G T, Kataev A L, LU H J. Phys. Lett. B, 1996, **372**: 133; See also Brodsky S J. hep-ph/0310289, Brodsky S J, Menke S, Merino C, Rathsman J. Phys Rev D, 2003, **67**: 055008
- 27 CHEN J P, Deur A, Garibaldi F. JLab experiment E97-110
- 28 Battaglieri M, Deur A, De Vita R, Ripani M. JLab exper-

- iment E03-006
- 29 Grunberg G. Phys. Lett. B, 1980, **95**: 70; Phys. Rev. D, 1984, **29**: 2315; Phys. Rev. D, 1989, **40**: 680
- 30 Deur A, Burkert V, CHEN J-P, Korsch W. Phys. Lett. B, 2007, **650**: 4 244; Deur A, Burkert V, CHEN J-P, Korsch W. Phys. Lett. D, 2008, **665**: 349
- 31 Gross D J, Llewellyn Smith C H. Nucl. Phys B, 1969, **14**: 337
- 32 Kim J H et al. Phys. Rev. Lett., 1998, **81**: 3595
- 33 Brodsky S J, Shrock R. Phys. Lett. B, 2008, **666**: 95; Brodsky S J, de Teramond G, Shrock R. arXiv:0807.2484
- 34 Cornwall J M. Phys. Rev. D, 1982, **26**: 1453
- 35 Bloch J C R. Phys. Rev. D, 2002, **66**: 034032
- 36 Godfrey S, Isgur N. Phys. Rev. D, 1985, **32**: 189
- 37 Maris P, Tandy P C. Phys. Rev. C, 1999, **60**: 055214
- 38 Fischer C S, Alkofer R. Phys. Lett. B, 2002, **536**: 177; Fischer C S, Alkofer R, Reinhardt H. Phys. Rev. D, 2002, **65**: 125006; Alkofer R, Fischer C S, Von Smekal L. Acta Phys. Slov., 2002, **52**: 191
- 39 Bhagwat M S et al. Phys. Rev. C, 2002, **68**: 015203
- 40 Furui S, Nakajima H. Phys. Rev. D, 2004, **70**: 094504
- 41 Maldacena J M. Adv. Theor. Math. Phys., 1998, **2**: 252; Int. J. Theor. Phys. 1999, **38**: 1113
- 42 See for ex. Polchinski J, Strassler M J. Phys. Rev. Lett., 2002, **88**: 031601; JHEP, 2003, **0305**: 012; Brodsky S J, de Teramond G F. Phys. Rev. Lett., 2005, **94**: 201601; Phys. Rev. Lett., 2006, **96**: 201601. Karch A, Katz E, Son D T, Stephanov M A. Phys. Rev. D, 2006, **74**: 015005