Neutrino energy loss by electron capture in magnetic field at the crusts of neutron stars*

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Abstract Based on the p-f shell model, the effect of strong magnetic field on neutrino energy loss rates by electron capture is investigated. The calculations show that the magnetic field has only a slight effect on the neutrino energy loss rates in the range of 10^8—10^{13} G on the surfaces of most neutron stars. But for some magnetars, the range of the magnetic field is 10^{13}—10^{18} G, and the neutrino energy loss rates are greatly reduced, even by more than four orders of magnitude due to the strong magnetic field.

Key words strong magnetic field, neutrino energy loss, neutron star

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1 Introduction

According to some observation data, the burst of the neutrino is the first signal from the explosion. With the escape of the neutrino, a large quantity of energies would be set free during the explosion process. At the same time, lots of messages are taken away from the star by neutrinos. The researches on neutrino and the neutrino energy loss (hereafter NEL) rates therefore have been the hotspot and former-border issue in astrophysics and particle physics. In the last decades, some authors had investigated extensive results of their calculation of the NEL rates. For example, Fuller, Flower and Newman[1—4], based on the simple shell model, accomplished many pioneering work, in which they made detailed calculation of the NEL rates due to electron capture on a lot of nucleus in the stars. Naoki Itoh et al.[5, 6] analyzed the NEL rates from the pair, photo-, and plasma bremsstrahlung, and recombination neutrino processes based on the Weinberg-salam theory. S.Esposito et al.[7] also reviewed the energy loss rate of the stellar plasma. However they paid no attention to the NEL rates in a strong magnetic field (hereafter SMF). In 2006, the neutrino bremsstrahlung process was considered in the presence of SMF by Indranath Bhattacharyya[8] and the calculations for this process in the absence of magnetic field were also carried out simultaneously.

As is well known, the strength of the surface magnetic field of most neutron stars is 10^8—10^{13} G[9], concluded from some researches and observation data. Some magnetars whose density is 10^4—10^{11} g/cm^3 and temperature is 10^5—10^9 K, are observed to have a magnetic field of 10^{13}—10^{18} G[10—12]. Under the strong magnetic fields, higher density and temperature, the electron capture always plays an important role. Some researches show that an amount of neutrinos are produced during this process and the hot neutron star turns cool speedily. It is due to the fact that a great deal of energy is taken away from the star by the neutrino synchronously. Thus it is very interesting to work on the neutrino energy loss rates by electron capture due to SMF.

In this paper, based on the p-f shell model, the NEL rates for some nucleus by electron capture are investigated in SMF. The remainder of the article is arranged as follows: in the next section, we analyze the NEL rates in a SMF. In Section 3, we discuss the affection of SMF on the NEL rates . And some conclusions are given in Section 4.

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2 The NEL rates in a SMF

The strong magnetic field B is considered to be along the z-axis. The Dirac equation can be solved exactly. The positive energy levels are given by[13]

$$\varepsilon_n=(p_z^2+1+2n\Theta)^{\frac{1}{2}} \quad (n=0,1,2,3,\cdots), \qquad (1)$$

where $\Theta=\dfrac{h_e B}{m_e^2 c}=0.0225B_{12}$, $B=10^{12}B_{12}$ and p_z is the electron momentum along the field. Note that in this paper all of the energies and the moment are in units of $m_e c^2$ and $m_e c$, where m_e is the electron mass and c is the light velocity, correspondingly. The respective spinors will be given by Kaminker and Yakovlev[14]. The electron chemical potential is found by inverting the expression for the lepton number density:

$$n_e=\rho/\mu_e=\frac{\Theta}{2\pi^2\lambda_e^3}\sum_{n=0}^{\infty}q_{n0}\int_0^{\infty}(f_{-e}-f_{+e})\,dp_z\,, \qquad (2)$$

where ρ is the density in g/cm^3, μ_e is the average molecular weight. $\lambda_e=h/m_e c$ is Compton wavelength, $q_{n0}=2$–δ_{n0} is the electron degenerate number,

$$f_{-e}=\left[1+\exp\left(\frac{\varepsilon_n-U_F-1}{kT}\right)\right]^{-1}$$

and

$$f_{+e}=\left[1+\exp\left(\frac{\varepsilon_n+U_F+1}{kT}\right)\right]^{-1}$$

are the electron, positron distribution functions respectively, the k is the Boltzmann constant. T is the electron temperature; U_F is the electron chemical potential.

The NEL rates (in $m_e c^2/s^1$) due to electron capture for the k-th nucleus (Z,A) in thermal equilibrium at temperature T is given by a sum over the initial parent states i and the final daughter states f[15]

$$\lambda_k^{\nu}=\sum_i\frac{(2J_i+1)\,e^{-E_i/kT}}{G(Z,A,T)}\sum_f\lambda_{if}^{\nu}\,, \qquad (3)$$

where J_i and E_i are the spin and excitation energy of the parent states, $G(Z,A,T)$ is the nuclear partition function. The NEL rates associated with the electron capture from one of the initial states to all possible final states are λ_{if}^{ν}, $\lambda_{if}^{\nu}=\dfrac{\ln 2}{(ft)_{if}}f_{if}$ with the relation

$$\frac{1}{(ft)_{if}}=\frac{1}{(ft)_{if}^{F}}+\frac{1}{(ft)_{if}^{GT}}.$$

The ft-values and the corresponding Gamow-Teller or Fermi transition matrix elements are related with the equations

$$\frac{1}{(ft)_{if}^{GT}}=\frac{10^{3.596}}{|M_{GT}|_{if}^2},\quad \frac{1}{(ft)_{if}^{F}}=\frac{10^{3.79}}{|M_F|_{if}^2}\,. \qquad (4)$$

The phase space factor in SMF will be found in Ref. [16] and defined as

$$f_{if}^{B}=\frac{\Theta}{2}\sum_{n=0}^{\infty}\theta_n\,, \qquad (5)$$

where

$$\theta_n=q_{n0}\int_{q_n}^{\infty}(Q_{if}+\varepsilon_n)^3\,\frac{F(z,\varepsilon_n)}{1+\exp\left(\dfrac{\varepsilon_n-U_F-1}{kT}\right)}dp_z$$

where $Q_{if}=Q_{00}+E_i-E_f$ is the EC threshold energy and $Q_{00}=M_p c^2-M_d c^2$, M_p and M_d are the mass of the parent nuclear and the daughter nucleus respectively, E_i, E_f are the excitation energies of the nuclear i-th states and f-th state corresponding, the ε_n is total of rest mass and kinetic energy, $F(z,\varepsilon_n)$ is the coulomb wave correction function[1]. We assume that a SMF will have no effect on $F(z,\varepsilon_n)$, which is valid only under the condition that the electron wavefunctions are locally approximated by the plane-wave functions[16]. The condition requires that the Fermi wavelength $\lambda_F\sim h/P_F$ (P_F is the Fermi momentum without a magnetic field) be smaller than the radius $\sqrt{2}\boldsymbol{\rho}'$ (where $\boldsymbol{\rho}'=\lambda e/\sqrt{\Theta}$) of the cylinder which corresponds to the lowest Landau level[17].

$$F(z,\varepsilon_n)=2(1+\gamma)\left(\frac{2p_e R}{\hbar}\right)^{-2(1-\gamma)}e^{\pi\nu}\frac{|\Gamma(\gamma+i\nu)|^2}{|\Gamma(2\gamma+1)|^2}\,, \qquad (6)$$

where $\nu=Z\alpha(E_e/pc)$, $\gamma=[1-(Z\alpha)^2]^{1/2}$, R is the nuclear radius, Γ is the gamma function and Z is the nuclear charge of the parent state and α is the fine-structure constant. The q_n is defined as

$$q_n=\begin{cases}(Q_{if}^2-1-2n\Theta)^{\frac{1}{2}}, & Q_{if}<-(1+2n\Theta)^{\frac{1}{2}},\\ 0, & \text{otherwise.}\end{cases} \qquad (7)$$

3 The effect on NEL rates of some nucleus in SMF

Figure 1 show the NEL rates of the nuclide ^{56}Co and ^{56}Ni as a function of magnetic field due to electron capture process which include Gamow-Teller and Fermi transition at the different temperature of $T=7\times10^9$ K, 5×10^9 K, 3×10^9 K and the density of $\rho/\mu_e=3.16\times10^8$ mol/cm^3. From Fig. 1, one can find that the magnetic field has only a slight effect on the NEL rates when $B<10^{13}$ G. However the influences are increased by SMF when $B>10^{14}$ G. The NEL rates in a SMF are greatly decreased. For example, the NEL rates of ^{56}Co are decreased by three orders of magnitude and by four orders of magnitude for ^{56}Ni at 3×10^9 K. Under the same conditions as Fig. 1 and at the temperature 3×10^9 K, Fig. 2 show us that the affection of SMF on the NEL rates of some important nucleus. One can find that when $B < 10^{14}$ G, the magnetic field has only a minor influence on NEL rates. However the NEL rates are greatly influenced when $B > 10^{14}$ G. For instance, the NEL rates of ^{54}Cr, ^{53}Fe, ^{58}Co, ^{47}V and ^{48}V may be almost reduced by three orders of magnitude due to SMF and even by more than four orders of magnitude for the nuclide ^{55}Co and ^{57}Ni.

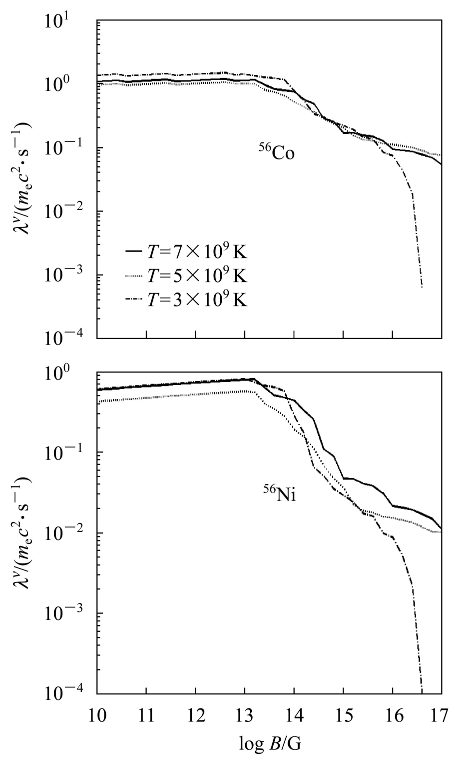

Fig. 1. The NEL rates as a function of magnetic field at the density of $\rho/\mu_e = 3.16\times10^8$ mol/cm^3 and different temperature.

From two Figures, the results tell us that at the same density, the SMF has different effects on NEL rates at different temperatures. The lower the temperature is, the larger the influence on NEL is when $B > 10^{13}$ G, because the electron energy is so low at the lower temperature that SMF affects the electron capture so greatly. At the same time, the number of the neutrinos produced from the electron capture process is so large that the NEL rates would be affected so much. On the other hand, the SMF has different effects on NEL rates at different densities. It is due to the fact that the higher the density is, the larger the electron Fermi energy is. It may affect the electron capture greatly due to SMF. The detailed discussions on it can be found in Ref. [16].

By analyzing the effect on NEL rates due to SMF for different nuclides, one can see that the SMF has different effects on NEL rates for different nuclides due to the difference of the nuclide's threshold energy in the EC reaction. It may greatly affect the EC rates and the NEL rates.

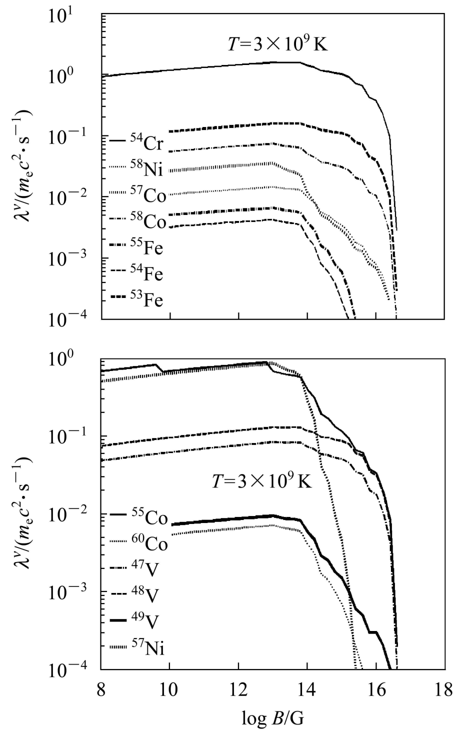

Fig. 2. The NEL rates as a function of magnetic field at the density of $\rho/\mu_e = 3.16\times10^8$ mol/cm^3 and the temperature of 3×10^9 K.

In summary, one can conclude that the influence of the SMF on NEL rates is very obvious. Due to the electron gas quantum domino effect, the curve has some gurgitation during the degressive process in the four pictures.

4 Conclusion

The NEL due to electron capture is investigated in a SMF. We draw the following conclusion that the magnetic field has only a slight effect on the NEL rates in a range of 10^8—10^{13} G on the surfaces of most neutron stars. But for some magnetars, the range of the magnetic field is 10^{13}—10^{18} G, and the NEL rates are greatly reduced or even by four orders of magnitude by SMF.

As is well known, with the escape of a great number of neutrinos by electron capture, the neutrino energy loss gives one of the main contributions to the cooling of stellar interior in the late stages of star evolution. It is helpful to the collapse and the explosion of the supernova[18]. The conclusion obtained in this study may have significant influence on further research of nuclear astrophysics and neutrino astrophysics, especially the research of late evolution of neutron stars and r-process nucleosynthesis in the neutron star systems.

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