Color-Flavor Locked Strange Quark Matter in a Mass Density-Dependent Model*

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Abstract Properties of color-flavor locked (CFL) strange quark matter have been studied in a mass-density-dependent model, and compared with the results in the conventional bag model. In both models, the CFL phase is more stable than the normal nuclear matter for reasonable parameters. However, the lower density behavior of the sound velocity in this model is completely opposite to that in the bag model, which makes the maximum mass of CFL quark stars in the mass-density-dependent model larger than that in the bag model.

Key words superconductivity, quark matter, quark stars

1 Introduction

Because of its theoretical and experimental significance, strange quark matter (SQM) has been investigated for several decades[1]. Dated back in the 1970's, it was already known that proper strangeness fraction could lower the energy per baryon of the system[2, 3]. In 1984, Witten[4] put forward that quark matter with strangeness could be the true ground state of QCD. Immediately after Witten's conjecture, Farhi and Jaffe[5] calculated the properties of SQM in the framework of MIT bag model. The results show that SQM is more stable than normal nuclear matter for a wide range of parameters. Therefore, a great deal of works on SQM emerged[6—9].

Besides the normal unpaired SQM mentioned above, there is another kind of quark matter[10, 11]. Bailin and Love[12] suggested the state of color superconductivity by analogy with the theory of superconductivity in condensed matter physics. Alford, Rajagopal, and Wilczek proposed a new mechanism of paring, the color-flavor locking paring mechanism[13, 14]. There are three massless quarks at high baryon densities, and the color and flavor $SU(3)_{\rm C}\otimes SU(3)_{\rm L}\otimes SU(3)_{\rm R}$ symmetries are broken down to the diagonal subgroup $SU(3)_{\rm C+L+R}$ by the formation of a condensate of quark Cooper pairs. Under such assumptions the form of condensate is

$$\langle q_i^{\alpha} C\gamma^5 q_j^{\beta}\rangle \propto \varepsilon_{ijI}\varepsilon^{\alpha\beta I}, \tag{1}$$

where the Latin indices (i,j) signify flavors and the Greek indices (α,β) stand for colors. The Cooper pairs in this form of condensation are symmetric under simultaneous exchange of color and flavor.

Although it is generally believed that Quantum Chromodynamics (QCD) is the dynamics of strong interaction in the level of quarks, there is still no way to exactly solve the motion equations because of the complexity, and so one has to resort to many phenomenological models to compute the properties of matter. The most commonly used model is the MIT bag model, and many interesting results have

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been obtained[1, 15]. In the bag model, quark masses are constant. As is well known, however, the particle mass varies with environment, i.e., the density and/or temperature. In this paper, we studied the properties of CFL strange quark matter in the frame work of the quark mass density-dependent model, and compared the results with those in the bag model. It is found that the CFL phase is more stable than the normal nuclear matter in both models. However, the lower density behavior of the sound velocity is completely opposite, which makes the maximum mass of CFL quark stars in the mass-density-dependent model much larger than that in the bag model.

The paper is organized as such. In the subsequent section, we describe the thermodynamic potential density of the CFL phase. Then in Sec. 3, we discuss the thermodynamic treatment and calculate the properties of CFL strange quark matter. As an application, the obtained equation of state is applied to calculate the mass-radius relation of CFL quark stars in Sec. 4. Finally, a summary is given in Sec. 5.

2 Thermodynamic potential density of the CFL quark matter

In order to see the difference between CFL and the normal quark matter, we start our consideration from the following thermodynamic potential density for unpaired quark matter:

$$\Omega = -\sum_i \frac{g_i T}{2\pi^2}\int_0^\infty \ln\left[1+\mathrm{e}^{-(\sqrt{p^2+m_i^2}-\mu_i)/T}\right]p^2\mathrm{d}p+B, \tag{2}$$

where g_i is the degeneracy factor (it is 6 for quarks), m_i is the particle mass, and μ_i is the chemical potential, the summation goes over all particle flavors involved. At zero temperature, taking the limit of $T\to 0$ on the right hand side of the above expression gives

$$\Omega = -\sum_i \frac{g_i}{2\pi^2}\int_0^{\nu_i}\left(\mu_i-\sqrt{p^2+m_i^2}\right)p^2\mathrm{d}p+B, \tag{3}$$

where ν_i is the Fermi momentum for flavor i. It can be linked to the chemical potential μ_i by

$$\nu_i = \sqrt{\mu_i^2-m_i^2}. \tag{4}$$

The properties of normal unpaired strange quark matter have been investigated for more than two decades since the pioneer works of many authors[2—4]. In the case of CFL phase, due to the energy gap Δ, determined by solving the gap equation[13, 14, 16], a new term should be added in the above expression. The thermodynamic potential density for CFL quark matter is then[14, 17]

$$\Omega_{\mathrm{CFL}} = -\sum_i \frac{g_i}{2\pi^2}\int_0^{\nu}\left(\mu_i-\sqrt{p^2+m_i^2}\right)p^2\mathrm{d}p - \frac{3\Delta^2\mu^2}{\pi^2}+B, \tag{5}$$

where the $\mu\equiv(\mu_{\mathrm{u}}+\mu_{\mathrm{d}}+\mu_{\mathrm{s}})/3$ is the average potential of quarks. The common Fermi momentum ν is a fictional intermediate parameter. To have maximum paring, the common Fermi momentum is determined by minimizing the thermodynamic potential, i.e.,

$$\frac{\partial\,\Omega_{\mathrm{CFL}}}{\partial\nu}=0. \tag{6}$$

In the MIT bag model, m_{u}, m_{d}, and m_{s} are, respectively, the current mass of u, d, and s quarks. Because m_{u} and m_{d} are extremely small, they can be treated as zero. In this case, Eq. (6) gives

$$\nu = 2\mu-(\mu^2+m_{\mathrm{s}}^2/3)^{1/2}. \tag{7}$$

Therefore, in the CFL phase, Eq. (4) is no longer valid.

Because our purpose in this paper is to study how the density-dependence of quark masses will influence the properties of CFL quark matter, the u and d quark masses can not be treated as zero. Generally, the common Fermi momentum ν is obtained by solving

$$\sum_i\sqrt{\nu^2+m_i^2}=3\mu. \tag{8}$$

The integration in Eq. (5) can be easily performed out, giving

$$\Omega_{\mathrm{CFL}} = -\sum_i\frac{g_i}{48\pi^2}\left\{\nu\left[8\mu_i\nu^2- 3(2\nu^2+m_i^2)\sqrt{\nu^2+m_i^2}\right]+ 3m_i^4\ln\frac{\nu+\sqrt{\nu^2+m_i^2}}{m_i}\right\}-\frac{3\Delta^2\mu^2}{\pi^2}+B. \tag{9}$$

If replacing the ν^2 in this expression with $\mu_i^2-m_i^2$, one immediately gets the Eq. (2) of Ref. [17], with a global difference of minus sign (we believe that a global minus sign was lost there). However, as we mentioned after Eq. (7) that the relation $\nu=\sqrt{\mu_i^2-m_i^2}$ is no longer valid in the CFL phase, the expression (2) in Ref. [17] is surely not correct. In this paper, we use the correct expression in Eq. (9) to calculate the properties of CFL strange quark matter.

3 Thermodynamical treatment in the mass density-dependent model

In the following, we briefly discuss the thermodynamic formulas corresponding to the mass density-dependent model we use. The detailed derivation can be found in the literature[7, 18].

From Eq. (9), we can easily get the number density for each quark flavor from the relation $n_i=-\partial\Omega_{\rm CFL}/\partial\mu_i$. The result is

$$n_{\rm u}=n_{\rm d}=n_{\rm s}=(\nu^3+2\Delta^2\mu)/\pi^2, \tag{10}$$

which explicitly shows that the numbers of u, d, and s quarks are all equal to each other. Therefore, the CFL strange quark matter is always naturally neutralized, without requiring any electrons[19].

The energy density E is also easy to get, i.e.,

$$E=\Omega_{\rm CFL}+\sum_i\mu_i n_i=\Omega_{\rm CFL}+3n_{\rm b}\mu, \tag{11}$$

where $n_{\rm b}\equiv(n_{\rm u}+n_{\rm d}+n_{\rm s})/3=n_{\rm u}=n_{\rm d}=n_{\rm s}$ is the baryon number density.

For the pressure, however, one should be very careful. To let the energy minimum appears exactly at zero pressure, which is a general requirement of thermodynamics[18], an additional term should appear in the pressure expression:

$$P=-\Omega_{\rm CFL}+n_{\rm b}\sum_i\frac{\partial\Omega_{\rm CFL}}{\partial m_i}\frac{\partial m_i}{\partial n_{\rm b}}. \tag{12}$$

The second term is due to the density-dependence of quark masses.

As for the density-dependent mass m_i, it can be devided into two parts as

$$m_i=m_{i0}+m_{\rm I}. \tag{13}$$

The meaning of Eq. (13) is clear. The first term represents current quark mass, and the second term $m_{\rm I}$, independent of flavors and densities, represents the effect due to the interaction between quarks. In principle, the density dependence of $m_{\rm I}$ should be determined from QCD. As mentioned before, however, there is no way to exactly solve QCD presently. Therefore, the density dependence is normally given phenomenologically. It has been shown that the following parametrization is reasonable[7, 20]

$$m_i=m_{i0}+\frac{D}{n_{\rm b}^{1/3}}, \tag{14}$$

where D is a fixed constant determined by stability argument. Such a form satisfies $\lim_{n_{\rm b}\to 0}m_{\rm I}=\infty$ and $\lim_{n_{\rm b}\to\infty}m_{\rm I}=0$, which are just the requirements of quark confinement and asymptotic freedom.

Because weak equilibrium is always reached in SQM by the reactions like $\rm d,s\leftrightarrow u+e+\bar{\nu}_e$ and $\rm s+u\leftrightarrow u+d$, relevant chemical potentials satisfy

$$\mu_{\rm d}=\mu_{\rm s}, \tag{15}$$

$$\mu_{\rm d}+\mu_{\nu}=\mu_{\rm u}+\mu_{\rm e}. \tag{16}$$

The CFL matter is naturally neutral and the number of three quark flavors are equal, we therefore have $\mu_{\rm e}=0$. At the same time, we can write $\mu_{\nu}=0$ due to the fact that neutrinos enter and leave the system freely. Consequently, we have

$$\mu_{\rm u}=\mu_{\rm d}=\mu_{\rm s}. \tag{17}$$

Therefore, only one chemical potential is independent.

For a given baryon number density $n_{\rm b}$, we can solve the common Fermi momentum ν and the only independent chemical potential by solving Eqs. (8) and (10). The energy density and pressure can then be calculated, respectively, from Eqs. (11) and (12).

Because the current mass of u/d quarks is very small, we simply take $m_{\rm u0}=m_{\rm d0}=0$. But for the current mass of s quarks, we take $m_{\rm s0}$=120MeV. The paring parameter is taken to be Δ=100MeV. In the pure bag model calculation, the bag constant is taken to be (170MeV)^4. In the calculation with quark mass density-dependent model, we take $\sqrt{D}$=120MeV. But in this case, B should be smaller, and we use $B^{1/4}$=140MeV, according to the stability arguements[5]. If one takes a zero bag constant in this case, $\sqrt{D}$ should be 156MeV[18]. However, the presure balance would not be reached for CFL phase.

In Fig. 1, we show the energy per baryon as a function of density. Please note, the lowest energy per baryon (the full dot) corresponds exactly to the zero pressure (open cicle). For comparison, we have also shown the result in the bag model. It is obviously seen that the energy per baryon in the density-dependent model(QMDD) is lower than that in the bag model. The corresponding equation of state is given in Fig. 2.

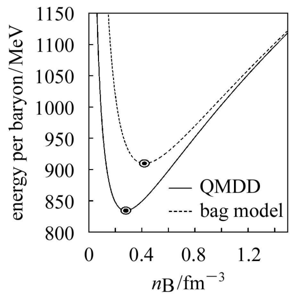

Fig. 1. The energy per baryon as a function of density. The minimum is 834MeV, located at the density 0.28fm^{-3} where the pressure is zero. The result from the bag model is also shown for comparison.

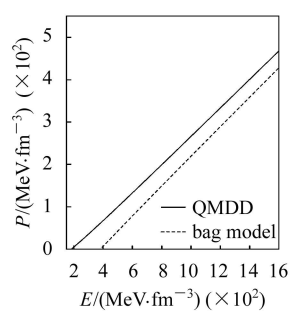

Fig. 2. Equation of state

4 Mass-radius relation of CFL quark stars

For a long time, it is believed that some of the neutron stars are composed of quark matter, and thus, in fact, they are quark stars. In this section, we assume the compact star is a spherically symmetric object consisting of CFL strange quark matter. The static properties of such strange stars are obtained by solving the Tolman-Oppenheimer-Volkov equation[21]

$$\frac{\mathrm{d}p}{\mathrm{d}r}=-\frac{GmE}{r^2}\frac{(1+P/E)(1+4\pi r^3P/m)}{1-2Gm/r}, \tag{18}$$

with the subsidiary condition

$$\mathrm{d}m/\mathrm{d}r=4\pi r^2E, \tag{19}$$

where $G=6.707\times10^{-45}\mathrm{MeV}^{-2}$ is the gravitational constant, r is the distance from the core of the star, $E=E(r)$ is the energy density, $P=P(r)$ is the pressure.

In Fig. 3, we plot the mass-radius relation of CFL strange stars. It is found that the maximum mass is about 2 times the solar mass in the density-dependent model, but only 1.5 times in the bag model. This different is caused by the fact that the sound velocity in the CFL quark matter is quite different in both models. To understand this, we plot the velocity of sound in Fig. 4. At higher densities, the results in both models approach to the ultra-relativistic case, as expected. At lower densities, however, the density behavior is obviously opposite.The sound velocity In the bag model goes up with decreasing densities, and it may even exceed the speed of light. This is naturally not reasonable from the viewpoint of the theory of special relativity. While in the density- dependent model, the sound velocity goes down to zero at zero density. The difference in lower densities might thus be an indication that the density-dependent mechanism is more suitable for the description of CFL phase than the conventional bag model.

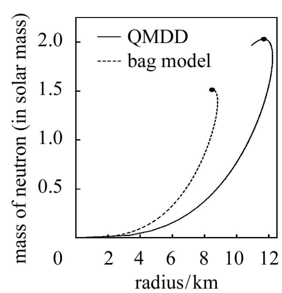

Fig. 3. The mass-radius relation of CFL quark stars. The maximum mass is about 2 times the solar mass in the mass-density-dependent model, but only 1.5 times in the bag model.

It should be noted that the model calculations here are oversimplified. Many important factors are not considered, e.g., when the density becomes lower, CFL phase may transit to unpaired quark phase, which will be studied in the future. So the concrete values in this paper should not be taken seriously, and further studied are needed.

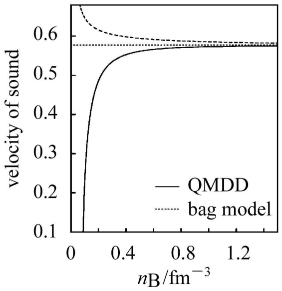

Fig. 4. Velocity of sound in the bag model and in the mass-density-dependent model. At higher densities, both approach to the ultra-relativistic case. At lower densities, however, the density behavior is obviously opposite.

5 Summary

We have studied the color-flavor locked strange quark matter within the framework of the quark mass-density-dependent model, and compared the results with those in the MIT bag model. It is found that the lower density behavior of the sound velocity is completely opposite in both models. The maximum mass of the CFL quark stars is thus bigger in the mass-density-dependent model than in the bag model.

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质量密度相关模型研究CFL奇异夸克物质性质*

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摘要 在质量–密度相关模型下研究了CFL奇异夸克物质, 并将结果与传统的袋模型结果进行比较. 两个模型均表明, 在合理的参数范围内, CFL相比正常核物质更稳定. 然而, 低密度时声速的行为完全相反, 这使得CFL夸克星的最大质量在质量–密度相关模型下比袋模型大.

关键词 色超导 奇异夸克物质 夸克星

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