Non-leptonic Weak Interaction with Quark Matter in Magnetic Field*

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Abstract We investigate the non-leptonic weak interaction in magnetic field and discuss the improvement of the previous method to analytically work out the rate for weak field case. Our result easily goes over to the field-free limit. Then we calculate the reaction rate in strong magnetic field where the charged particles are confined to the lowest Landau level. A strong magnetic field strongly suppress the rate, which will be foreseen to affect viscous dynamics in SQM. We also derive a few approximation formulae under given conditions that can be conveniently applied.

Key words quark, magnetic field, weak interaction

1 Introduction

The composition of comparable number of u, d and s-quarks are known as strange quark matter(SQM) would be a stable or metastable configuration of hadronic matter. The bulk SQM is the β-equilibrium system determined by such a series of weak processes: $\mathrm{u}+\mathrm{d}\rightarrow \mathrm{s}+\mathrm{u}$, $\mathrm{d}\rightarrow \mathrm{u}+\mathrm{e}+\overline{\nu}_{\mathrm{e}}$, $\mathrm{s}\rightarrow \mathrm{u}+\mathrm{e}^{-}+\overline{\nu}_{\mathrm{e}}$[1]. There has been a lot of interest in the study of the reaction rate and their astrophysical relevance[2—7]. In the interior of neutron stars, the semi-leptonic reaction is devoted to the cooling of stars while the non-leptonic one to the damping of instability of rapidly rotating stars. We here concern about the non-leptonic process. The previous work[2, 3] were carried out under the usual situation of zero magnetic field. However, the strength of the surface magnetic field of a pulsar is typically of order 10^{12}G[2)] which is concluded from the observation data. Some magnetars are observed to have a magnetic field of 10^{14}—10^{15}G. Considering the flux throughout the stars as a simple trapped primordial flux, the internal magnetic field may go up to 10^{18}G or even more[4, 5]. In spite of the fact that we do not know yet any appropriate mechanism to produce more intense field, the scalar viral theorem indeed allows the field magnitude to be as large as 10^{20}G. Therefore, it is advisable to study the effect of the magnetic field on strange quark matter including the calculation of the rate of the reactions. The quark Urca processes have been discussed in a few of works[8, 9]. Here we focus on the non-leptonic process leading to bulk viscosity in strange quark matter.

$$\mathrm{u}(1)+\mathrm{d}\rightarrow \mathrm{u}(2)+\mathrm{s}\ . \tag{1}$$

In magnetic fields, a classical charged particle will have a transverse cycle motion. In quantum mechanism, the transverse motion is quantized into Landau levels. The quantization effect is important when the magnetic strength is equal to or larger than the critical value $B_m^{(c)}=m_i^2c^3/(q_i\hbar)$ defined by equating the cyclotron energy $qB/(mc)$ to mc^2, where m_i and q_i denote the mass and the charge (absolute value) of the particle[10, 11]. $\hbar$, k and c denote the

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2) 1G=10^{-4}T, in this paper we use G in stead of T

Planck constant, Boltzman constant and velocity of light, respectively, which are taken to be units below. We have considered a wide range of magnetic fields in our study, from "low" to "very high" magnetic fields($B \geqslant 10^{19}$G) called respectively "weak" and "strong" field. Since u-quark mass in magnitude of order is smaller than d-quark and s-quark, we assume the quantization effect on u-quark is important but those of other flavors are negligible[4]. In the weak field strength situation,we deal with the calculation of the rate through some approximations: (i) the unaffected matrix element; and (ii) the approximating free-particle motion direction[12]. In degeneracy, the strong magnetic fields, i.e. $2qB > p_{\mathrm{Fi}}^2$, forces u-quark to occupy the lowest Landau ground state, where p_{Fi} is the Fermi momentum, we should use the exact solution of the Dirac equation in magnetic field to evaluate the reaction rate.

This paper is organized as follows. We solve the dirac equation in Section 2. We consider the case of weak field in Section 3, and then give the reaction rate in strong field in Section 4. Finally, the results are discussed and a summery is made in Section 5.

2 The solution of Dirac function in magnetic field

We first solve the Dirac equation in the presence of a magnetic field $\boldsymbol{B}$. Let the uniform magnetic field $\boldsymbol{B}$ along the z-axis, and we choose the asymmetric Landau gauge

$$\boldsymbol{A} = (0, Bx, 0), \tag{2}$$

so that the four-dimensional polarized wave function can be expressed in term of stationary states in the normalization volume $V = L_x L_y L_z$. In the magnetic field, the u-quark wave function for fermions ultra-relativistically reads

$$\psi_+(t,\boldsymbol{x}) = \frac{\exp[-\mathrm{i}Et + \mathrm{i}p_y y + \mathrm{i}p_z z]}{\sqrt{2E(E+m)L_y L_z}} \times \begin{pmatrix} 0 \\ (E+m)I_{\nu;p_z}(x) \\ p_z I_{\nu;p_z}(x) \\ -\mathrm{i}\sqrt{2\nu qB} I_{\nu-1;p_z}(x) \end{pmatrix}, \tag{3}$$

for spin up, and

$$\psi_-(t,\boldsymbol{x}) = \frac{\exp[-\mathrm{i}Et + \mathrm{i}p_y + \mathrm{i}p_z]}{\sqrt{2E(E+m)L_y L_z}} \times \begin{pmatrix} 0 \\ (E+m)I_{\nu-1;p_z}(x) \\ \mathrm{i}\sqrt{2\nu qB} I_{\nu;p_z}(x) \\ -p_z I_{\nu-1;p_z}(x) \end{pmatrix} \tag{4}$$

for spin down cases, where the energy $E = \sqrt{p_z^2 + m^2 + 2\nu qB}$, q is the charge of u-quark, and ν is the Landau level. These states are degenerate and can be described as other quantum numbers, such as

$$\nu = l + \frac{1}{2}(1-s), \tag{5}$$

where l is the orbital quantum number and s is the spin quantum number.

And then we get

$$\xi = \sqrt{qB}\left(-x + \frac{p_y}{qB}\right) \tag{6}$$

and

$$I_{\nu;p_z} = \left(\frac{qB}{\pi}\right)^{\frac{1}{4}} \exp\left(-\frac{\xi^2}{2}\right) \times \frac{1}{\sqrt{2^\nu \nu!}} H_\nu(\xi), \tag{7}$$

where H_ν is the Hermite polynomial.

3 The rate in weak magnetic field

If the matrix element for the reaction (1) is determined, we can express the rate per volume of reaction (1) as[2, 3]:

$$\Gamma(\mathrm{u_1 d} \rightarrow \mathrm{su_2}) = \frac{36}{2}\left[\prod_i \int \frac{\mathrm{d}^3 p_i}{(2\pi)^3 2E_i}\right] |M_{\mathrm{s}}|^2 \times S(2\pi)^4 \delta^4(P_1 + P_{\mathrm{d}} - P_2 - P_{\mathrm{s}}), \tag{8}$$

where the phase space integration will be carried out over all particle states, the statistical distribution function can be written as $S = f_1 f_{\mathrm{d}}(1-f_2)(1-f_{\mathrm{s}})$ and the quark distribution in reaction(1) is described by the Fermi-Dirac distributions in the form of

$$f_{\mathrm{i}}(E_{\mathrm{i}}) = \left[1 + \exp\left(\frac{E_{\mathrm{i}} - \mu_{\mathrm{i}}}{T}\right)\right]^{-1}, \mathrm{i} = 1, 2, \mathrm{d}, \mathrm{s}, \tag{9}$$

with μ_{i} bring the chemical potential.

We consider the reaction (1) when the magnetic field is not strong enough to force the quark getting into the lowest Landau level. Previous study[5] showed that the matrix element for the weak process remains unaffected and only the phase factor is modified.

The matrix element summed over final spins and averaged over initial spins is given by [2]:

$$|M_{\mathrm{s}}|^2 = 64G_{\mathrm{F}}^2 \sin^2\theta_{\mathrm{C}} \cos^2\theta_{\mathrm{C}} (P_1 \cdot P_{\mathrm{d}})(P_2 \cdot P_{\mathrm{s}}), \tag{10}$$

here $P_i = (E_i - \boldsymbol{p}_i)$ is the four-momentum of the quark i, $G_{\mathrm{F}} = 1.166 \times 10^{-11}\mathrm{MeV}^{-2}$ is the Fermi constant, and θ_{C} is the Cabibbo angle ($\cos^2\theta_{\mathrm{C}} = 0.948$). We neglect the masses of the up and down quarks, then $E_1 = p_1$, $E_2 = p_2$, $E_{\mathrm{d}} = p_{\mathrm{d}}$, $E_{\mathrm{s}} = (p_{\mathrm{s}}^2 + m_{\mathrm{s}}^2)^{1/2}$. So the four-momentum products can be written as[2, 3]

$$(P_1 \cdot P_{\mathrm{d}})(P_2 \cdot P_{\mathrm{s}}) = E_1 E_2 E_{\mathrm{d}} E_{\mathrm{s}} (1 - \cos\theta_{1\mathrm{d}}) \times \left(1 - \frac{p_{\mathrm{s}}}{E_{\mathrm{s}}} \cos\theta_{2\mathrm{s}}\right), \tag{11}$$

where θ_{ij} denotes the angle between the quarks i and j.

Now we can calculate the rate of weak process in the weak magnetic field by replacing the u-quark phase space factor[5]

$$2\int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3} \longrightarrow \frac{qB}{(2\pi)^2} \sum_{\nu=0}^{\nu_{\max}} (2 - \delta_{\nu,0}) \int \mathrm{d}p_z \ , \tag{12}$$

so the reaction rate can be written as:

$$\begin{aligned} \Gamma(\mathrm{u}_1\mathrm{d} \to \mathrm{su}_2) = {} & \frac{18}{(2\pi)^6} G_{\mathrm{F}}^2 \sin^2\theta_{\mathrm{C}} \cos^2\theta_{\mathrm{C}} (eB)^2 \times \\ & \sum_{\nu_1=0}^{\nu_{\max}} (2 - \delta_{\nu_{1,0}}) \sum_{\nu_2=0}^{\nu_{\max}} (2 - \delta_{\nu_{2,0}}) \times \\ & \int p_{\mathrm{d}}^2 \mathrm{d}p_{\mathrm{d}} p_{\mathrm{s}}^2 \mathrm{d}p_{\mathrm{s}} \int \mathrm{d}p_{1z} \int \mathrm{d}p_{2z} \times \\ & S\delta(E_1 + E_{\mathrm{d}} - E_2 - E_{\mathrm{s}}) I \ , \end{aligned} \tag{13}$$

where

$$\begin{aligned} I = {} & \int (\prod_i^{\mathrm{d,s}}) \mathrm{d}\Omega_i (1 - \cos\theta_{1\mathrm{d}}) \left(1 - \frac{p_{\mathrm{s}}}{E_{\mathrm{s}}} \cos\theta_{2\mathrm{s}}\right) \times \\ & \delta^3(\boldsymbol{p}_1 + \boldsymbol{p}_{\mathrm{d}} - \boldsymbol{p}_2 - \boldsymbol{p}_{\mathrm{s}}). \end{aligned} \tag{14}$$

As we know, the angle between two vectors is a function of the respective inclinations and azimuth angle in spherical coordinates, i.e.

$$\begin{aligned} \cos\theta_{1\mathrm{d}} &= \cos\theta_1 \cos\theta_{\mathrm{d}} + \sin\theta_1 \sin\theta_{\mathrm{d}} \cos(\varphi_1 - \varphi_{\mathrm{d}}), \\ \cos\theta_{2\mathrm{s}} &= \cos\theta_2 \cos\theta_{\mathrm{s}} + \sin\theta_2 \sin\theta_{\mathrm{s}} \cos(\varphi_2 - \varphi_{\mathrm{s}}). \end{aligned} \tag{15}$$

In the magnetic field, the angles of polarized quarks, $\theta_1, \theta_2, \varphi_1, \varphi_2$ vary with the Landau level (ν_1, ν_2) and z-component of momentum (p_{1z}, p_{2z}). Therefore the integrations and summations in Eq. (12) become very difficult due to the coupling of variables ν and p_z into the integrated function. We need to make an improvement on Chakrabaty's approach. Fortunately, we can approximately regard the angles as independent variables in the weak field situation, and then the integrations and summations are decoupled, because the kinetic direction of quark should only deviates slightly from that in the field-free case, although the modification of the absolute value of the momentum is considered. It completely coincides with the free-particle's matrix approximation described by Eq. (10).

We immediately have:

$$2\int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3} \longrightarrow \frac{qB}{(2\pi)^3} {\sum_{\nu=0}^{\nu_{\max}}}' (2 - \delta_{\nu,0}) \int' dp_z \int' \mathrm{d}\Omega \ , \tag{16}$$

where $\sum'$ and $\int'$ denote the summation and integrations are independent of the angles. In this approximation, which may be called free-particle direction average(FDA), Eq. (13) and Eq. (14) read:

$$\begin{aligned} \Gamma(\mathrm{u}_1\mathrm{d} \to \mathrm{su}_2) = {} & \frac{18}{(2\pi)^8} G_{\mathrm{F}}^2 \sin^2\theta_{\mathrm{C}} \cos^2\theta_{\mathrm{C}} (eB)^2 \times \\ & \sum_{\nu_1=0}^{\nu_{1\max}} (2 - \delta_{\nu_{1,0}}) \sum_{\nu_2=0}^{\nu_{2\max}} (2 - \delta_{\nu_{2,0}}) \times \\ & \int p_{\mathrm{d}}^2 \mathrm{d}p_{\mathrm{d}} p_{\mathrm{s}}^2 \mathrm{d}p_{\mathrm{s}} \int \mathrm{d}p_{1z} \int \mathrm{d}p_{2z} \times \\ & S\delta(E_1 + E_{\mathrm{d}} - E_2 - E_{\mathrm{s}}) I' \ , \end{aligned} \tag{17}$$

where

$$\begin{aligned} I' = {} & \int' \left(\prod_i^{1,2,\mathrm{d,s}} \right) \mathrm{d}\Omega_i (1 - \cos\theta_{1\mathrm{d}}) \left(1 - \frac{p_{\mathrm{s}}}{E_{\mathrm{s}}} \cos\theta_{2\mathrm{s}}\right) \times \\ & \delta^3(\boldsymbol{p}_1 + \boldsymbol{p}_{\mathrm{d}} - \boldsymbol{p}_2 - \boldsymbol{p}_{\mathrm{s}}). \end{aligned} \tag{18}$$

Since $\mu_i \gg T$, only those fermions whose momenta lie close to their respective Fermi surface can take part in the reaction. We use the method in Refs. [6,7] to complete partially the integral of I' through

$$\delta^3(\boldsymbol{p}_1 + \boldsymbol{p}_{\mathrm{d}} - \boldsymbol{p}_2 - \boldsymbol{p}_{\mathrm{s}}) = \int \frac{\mathrm{d}^3\boldsymbol{x}}{(2\pi^3)} \exp(\mathrm{i}\boldsymbol{p} \cdot \boldsymbol{x}) \tag{19}$$

and then

$$I' = \frac{2^7\pi^2}{p_{\mathrm{F_1}}p_{\mathrm{F_2}}p_{\mathrm{F_d}}p_{\mathrm{F_s}}}\int\frac{\mathrm{d}x}{x^2}\Bigg[\prod_i^{1,2,\mathrm{d,s}}\sin(p_{\mathrm{F_i}}x)+ a\sin(p_{\mathrm{F_1}}x)\sin(p_{\mathrm{F_d}}x)f(p_{\mathrm{F_2}}x)f(p_{\mathrm{F_s}}x)+ \sin(p_{\mathrm{F_2}}x)\sin(p_{\mathrm{F_s}}x)f(p_{\mathrm{F_1}}x)f(p_{\mathrm{F_d}}x)+ a\prod_i^{1,2,\mathrm{d,s}}f(p_{\mathrm{F_i}}x)\Bigg] \tag{20}$$

$$\equiv \frac{2^7\pi^2}{p_{\mathrm{F_1}}p_{\mathrm{F_2}}p_{\mathrm{F_s}}}J, \tag{21}$$

where $a=\frac{p_{\mathrm{F_s}}}{\mu_{\mathrm{s}}}$, $f(p_{\mathrm{Fi}}x)=\cos(p_{\mathrm{Fi}}x)-\frac{\sin(p_{\mathrm{Fi}}x)}{p_{\mathrm{Fi}}x}$ and the integral J is defined through Eq. (20) and Eq. (21), which can be calculated numerically.

The net rate of transforming d-quark into s-quark is[2]

$$\Gamma(\mathrm{d}\to\mathrm{s})=\left[1-\exp\left(\frac{\mu_{\mathrm{d}}-\mu_{\mathrm{s}}}{T}\right)\right]\Gamma(\mathrm{u_1d}\to\mathrm{su_2}). \tag{22}$$

Using the method in Refs. [7, 12] and substituting Eqs. (17), (18) and (21) into (22), we can get

$$\Gamma(\mathrm{d}\to\mathrm{s})= \frac{3}{2\pi^6}G_{\mathrm{F}}^2\sin^2\theta_{\mathrm{C}}\cos^2\theta_{\mathrm{C}}(qB)^2\sum_{\nu1=0}^{\nu_{1\max}}(2-\delta_{\nu1,0})\times \sum_{\nu=0}^{\nu_{2\max}}(2-\delta_{\nu2,0})\frac{\mu_{\mathrm{d}}^2\mu_{\mathrm{s}}}{\sqrt{\mu_1^2-2\nu_1qB}\sqrt{\mu_2^2-2\nu_2qB}}\times \Delta\mu(\Delta\mu^2+4\pi^2T^2)J, \tag{23}$$

where

$$\nu_{i\max}=\mathrm{Int}\left(\frac{\mu_i^2}{2qB}\right),\ i=1,2,\ \Delta\mu=\mu_{\mathrm{d}}-\mu_{\mathrm{s}}\ . \tag{24}$$

When $B\to 0$, the sum can be replaced by integral of ν and then

$$\Gamma(\mathrm{d}\to\mathrm{s}) = \frac{6}{\pi^6}G_{\mathrm{F}}^2\sin^2\theta_{\mathrm{C}}\cos^2\theta_{\mathrm{C}}\mu_{\mathrm{d}}^2\mu_{\mathrm{u}}^2\mu_{\mathrm{s}}\times \Delta\mu(\Delta\mu^2+4\pi^2T^2)J, \tag{25}$$

which is just that in the field-free case.

4 The rate of weak process in strong magnetic field

Now we consider the strong magnetic field effect on the non-leptonic weak interaction in this section. In the case of strong magnetic field where $B\geqslant B_m^{(\mathrm{u})}$, all u-quarks occupy the lowest Landau ground state with the u-quark spins pointing in the direction of the magnetic field. We treat other flavor particles in this process as free particles which are not affected by the magnetic field. The matrix element for the reaction reads:

$$M=\frac{G_{\mathrm{F}}\sin\theta_{\mathrm{C}}\cos\theta_{\mathrm{C}}}{\sqrt{2}}\int\bar{\psi}_2\gamma_\mu(1-\gamma_5)\psi_{\mathrm{d}}\bar{\psi}_{\mathrm{s}}\gamma^\mu(1-\gamma_5)\psi_1\mathrm{d}^4x, \tag{26}$$

when d and s quark are treated as free particles, then we get

$$\psi_{\mathrm{i}}=\frac{1}{V^{\frac{1}{2}}}\exp(-\mathrm{i}P_{\mathrm{i}}\cdot r)U_{\mathrm{i}}\ , \tag{27}$$

$$U_{\mathrm{i}}=\sqrt{\frac{E_{\mathrm{i}}+m_{\mathrm{i}}}{2E_{\mathrm{i}}}}\begin{pmatrix}1\\0\\ \frac{p_{z_{\mathrm{i}}}}{E_{\mathrm{i}}+m_{\mathrm{i}}}\\ \frac{p_{x_{\mathrm{i}}}+\mathrm{i}p_{y_{\mathrm{i}}}}{E_{\mathrm{i}}+m_{\mathrm{i}}}\end{pmatrix}, \tag{28}$$

where P_{i} denotes four-dimensional momentum, i= d, s.

Consider $\nu=0$, the wave function of u-quark in Eqs. (3) and (4) can be written as

$$\psi_1 = \frac{1}{\sqrt{L_yL_z}}\exp(-\mathrm{i}E_1t+\mathrm{i}p_{y_1}y_1+\mathrm{i}p_{z_1}z_1)\times \left(\frac{qB}{\pi}\right)^{\frac{1}{4}}\exp\left[-\frac{qB}{2}\left(-x+\frac{p_{y_1}}{qB}\right)^2\right]U_1\ , \tag{29}$$

$$\psi_2 = \frac{1}{\sqrt{L_yL_z}}\exp(-\mathrm{i}E_2t+\mathrm{i}p_{y_2}y_2+\mathrm{i}p_{z_2}z_2)\times \left(\frac{qB}{\pi}\right)^{\frac{1}{4}}\exp\left[-\frac{qB}{2}\left(-x+\frac{p_{y_2}}{qB}\right)^2\right]U_2\ , \tag{30}$$

where

$$U_1=\frac{1}{\sqrt{2E_1(E_1+m_{u_1})}}\begin{pmatrix}E_1+m\\0\\p_{z_1}\\0\end{pmatrix}, \tag{31}$$

$$U_2=\frac{1}{\sqrt{2E_2(E_2+m_{u_2})}}\begin{pmatrix}E_2+m\\0\\p_{z_2}\\0\end{pmatrix} \tag{32}$$

for spin up.

We now calculate the squared matrix element which is summed over the initial state and averaged over the final state. We use the Eqs. (29), (32) and (33) to express the corresponding matrix element:

$$|M_{\rm s}|^2 = \left[\bar{U}_2\gamma_\mu(1-\gamma_5)U_{\rm d}\bar{U}_{\rm s}\gamma^\mu(1-\gamma_5)U_1\right]\times \\ [\bar{U}_2\gamma_\mu(1-\gamma_5)U_{\rm d}\bar{U}_{\rm s}\gamma^\mu(1-\gamma_5)U_1]^\dagger. \quad (33)$$

And then we immediately get:

$$|M_{\rm s}|^2 = \frac{1}{2E_1^2E_2^2E_{\rm d}E_{\rm s}}(E_1-p_{z_1})^2(E_2+p_{z_2})^2\times \\ (E_{\rm d}+p_{z_{\rm d}})(E_{\rm s}-p_{z_{\rm s}}), \quad (34)$$

where we set $m_{\rm d}=m_{\rm u}=0$, $p_{z_1}<0$ and $p_{z_2}>0$, otherwise $|M_{\rm s}|^2=0$. Carrying out the integration, we obtain

$$|M|^2 = \frac{G_{\rm F}^2\sin\theta_{\rm C}^2\cos\theta_{\rm C}^2}{2VL_x^2(L_yL_y)^3}(2\pi)^3|M_{\rm s}|^2\times \\ \exp\left[\frac{-(p_{y_1}-p_{y_2})^2-(p_{x_{\rm d}}-p_{x_{\rm s}})^2}{2qB}\right]\times \\ \delta(E_2+E_{\rm s}-E_1-E_{\rm d})\delta(p_{y_1}+p_{y_{\rm d}}-p_{y_2}-p_{y_{\rm s}})\times \\ \delta(p_{z_1}+p_{z_{\rm d}}-p_{z_2}-p_{z_{\rm s}}). \quad (35)$$

The rate per volume of the reaction is given by:

$$\Gamma({\rm u_1d\rightarrow su_2}) = \frac{n_1n_{\rm d}}{2}\int\frac{V{\rm d}^3\boldsymbol{p}_{\rm d}}{(2\pi)^3}\int\frac{V{\rm d}^3\boldsymbol{p}_{\rm s}}{(2\pi)^3}\int_{\frac{-qBL_x}{2}}^{\frac{qBL_x}{2}}\times \\ \frac{L_y}{2\pi}{\rm d}p_{y_1}\int_{\frac{-qBL_x}{2}}^{\frac{qBL_x}{2}}\frac{L_y}{2\pi}{\rm d}p_{y_2}\int_{-\infty}^{\infty}\frac{L_y}{2\pi}{\rm d}p_{z_1}\times \\ \int_{-\infty}^{\infty}\frac{L_y}{2\pi}{\rm d}p_{z_2}|M|^2f_1f_{\rm d}(1-f_2)(1-f_{\rm s}), \quad (36)$$

where the factor $n_1=n_{\rm d}=6$ comes from 2 spins and 3 colors. Since only the left-hand helicity state of the $\rm u_1$-quark couples to W^- (W^- is the mediate of the reaction), there should be a factor of $\frac{1}{2}$ in Eq. (35). The integration over ${\rm d}p_{y_1}$ and ${\rm d}p_{y_2}$ can be carried out by using the delta function of the y-component of the momentum. The integration over ${\rm d}p_{z_1}$ and ${\rm d}p_{z_2}$ can be converted into ${\rm d}E_1$ and ${\rm d}E_2$ respectively[4, 5].

Since $\mu_{\rm s}\gg T$, only those momenta which lie close to their respective Fermi surfaces can take part in the reaction. As the conservation of the z-component of the momentum, we can get

$$p_{\rm F_1}+p_{\rm F_d}\cos\theta_{\rm d}-p_{\rm F_2}-p_{\rm F_s}\cos\theta_{\rm s}=0. \quad (37)$$

We approximately set $p_{\rm F_u}=p_{\rm F_d}=p_{\rm F_s}$ near the equilibrium. Then we can get $\cos\theta_{\rm d}-\cos\theta_{\rm s}=2$, so we can carry out the integration over the momentum space of the d-quark and s-quark.

Substituting Eq. (35) into Eq. (22), we obtain

$$\Gamma({\rm d\rightarrow s}) = \frac{G_{\rm F}^2\sin^2\theta_{\rm C}\cos^2\theta_{\rm C}(qB)}{4\pi^5}\times \\ \exp\left[\frac{(2\mu_{\rm u})^2-(\mu_{\rm d}+p_{F_{\rm s}})^2}{2qB}\right](3\mu_{\rm s}+6\mu_{\rm u}-\mu_{\rm d})\times \\ \mu_{\rm d}^2\Delta\mu(\Delta\mu^2+4\pi^2T^2), \quad (38)$$

where $\Delta\mu=\mu_{\rm d}-\mu_{\rm s}$, $p_{\rm F_s}=\sqrt{\mu_{\rm s}^2-m_{\rm s}^2}$.

5 Discussion and conclusion

We calculate the rate of non-leptonic quark weak process in the magnetic field and give analytic solutions under the weak-field and the strong-field approximations, respectively. Based on these results, we express the net rate of transforming the d-quark to the s-quark in a unified form:

$$\Gamma({\rm d\rightarrow s})=\Gamma_k(n_{\rm b},qB)\Delta\mu(\Delta\mu^2+4\pi^2T^2), \quad (39)$$

where k can be 0, L and H, which denote the zero-field, weak-field and strong-field cases, respectively. In accordance with the formula (25), the result goes to that in the field-free case, when the magnetic field strength vanishes. We thus have

$$\Gamma_0(n_{\rm b})=\frac{16}{5}G_{\rm F}^2\sin^2\theta_{\rm C}\cos^2\theta_{\rm C}\left(\frac{n_{\rm b}}{\pi}\right)^{\frac{5}{3}}, \quad (40)$$

meanwhile, for $2q_{\rm u}B<\mu_{\rm u}^2$, the $\Gamma_{\rm L}$ reads simply from the Eq. (23)

$$\Gamma_{\rm L}(n_{\rm b},qB) = \frac{144}{\pi^4}G_{\rm F}^2\sin^2\theta_{\rm C}\cos^2\theta_{\rm C}q_{\rm u}Bn_{\rm b}\times \\ J\frac{\nu_m^{\frac{3}{2}}}{3\nu_m^{\frac{1}{2}}+4(\nu_m-1)^{\frac{3}{2}}+8\nu_m^{\frac{3}{2}}}, \quad (41)$$

where J as a function of μ is defined in Fig. 1. We find that $\Gamma_{\rm L}$ has a small deviation from that in the field-free case. The comparison is made in Fig. 2.

The simplified formula of $\Gamma_{\rm H}(n_{\rm b},qB)$ need slightly complicated calculations for $2q_{\rm u}B\geqslant\mu_{\rm u}^2$.

In the referred SQM ($\mu_i\gg m_i$, only u-quark is polarized), the number densities of various quark components read

$$n_{\rm d,s}=\frac{\mu_{\rm d,s}^3}{\pi^2},$$

$$n_{\rm u} = \frac{3q_{\rm u}B\mu_{\rm u}}{2\pi^2}, \tag{42}$$

the β-equilibrium

$$\mu_{\rm d} = \mu_{\rm s} = \mu, \quad \mu_{\rm u} = \mu - \mu_{\rm e}, \tag{43}$$

the charge neutrality

$$2n_{\rm u} - n_{\rm d} - n_{\rm s} - 3n_{\rm e} = 0, \tag{44}$$

the baryon number density conservation

$$n_{\rm b} = \frac{1}{3}(n_{\rm d} + n_{\rm u} + n_{\rm s}), \tag{45}$$

should be satisfied. Then we solve these equations numerically to obtain the chemical potentials of quarks with respective to the magnetic field.

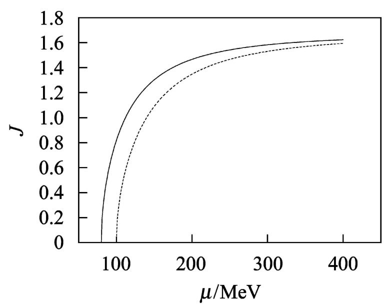

Fig. 1. Numerical result of J, given by Eqs. (20) and (21), as a function of μ for various values of the parameter $m_{\rm s}$. The solid curve is for $m_{\rm s}$=80MeV, and the dotted curve is for $m_{\rm s}$=100MeV.

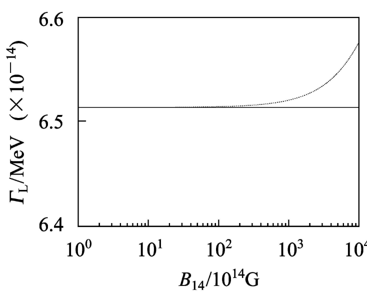

Fig. 2. Derivation of $\Gamma_{\rm L}$ from Γ_0 with B when $n_{\rm b}$=0.2fm^{-3}.

Figure 3 shows that the chemical potentials of quarks are nearly equal under this situation. Combining Eqs. (41), (42), (44) and (45) in the approximation of $\mu_{\rm u} \approx \mu_{\rm d} = \mu_{\rm s} = \mu$, μ can be solved analytically through the algebraic equation

$$\mu^3 + \frac{3}{4}q_{\rm u}B\mu - \frac{\pi^2}{2}n_{\rm b} = 0 \tag{46}$$

and then $\Gamma_{\rm H}(n_{\rm b}, qB)$ becomes

$$\Gamma_{\rm H} = \frac{G_{\rm F}^2 \sin^2\theta_{\rm C}\cos^2\theta_{\rm C}(qB)}{4\pi^5} \times \left[\frac{q_{\rm u}B}{\left(-6n_{\rm b}\pi^2 + \sqrt{36n_{\rm b}^2\pi^4 + (q_{\rm u}B)^3}\,\right)^{\frac{1}{3}}} - \left(-6n_{\rm b}\pi^2 + \sqrt{36n_{\rm b}^2\pi^4 + (q_{\rm u}B)^3}\,\right)^{\frac{1}{3}}\right]^3. \tag{47}$$

Figure 4 gives a comparison of the results obtained in Eq. (47) and Eq. (37), respectively. They fit each other well with a small error.

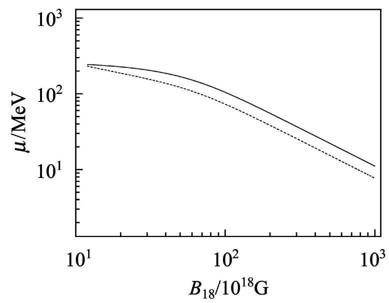

Fig. 3. The numerical result of μ as a function of $\boldsymbol{B}$ which we get from Eqs. (46), (47) and (48). The dotted curve is for $\mu_{\rm u}$ when $n_{\rm b}$=0.2fm^{-3}. The solid curve is for the case where $\mu = \mu_{\rm d} = \mu_{\rm s}$.

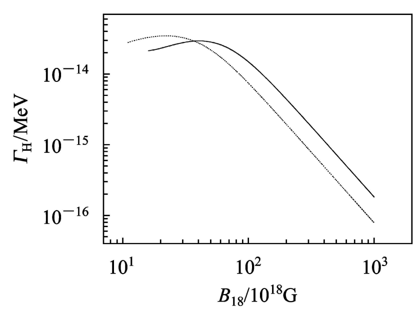

Fig. 4. $\Gamma_{\rm H}$ as a function of B when $n_{\rm b}$= 0.2fm^{-3}. The solid curve is for the result of Eq. (37) and the dot curve is for the result of Eq. (47).

The formula (47) will be reduced to

$$\Gamma_{\rm H} = \frac{24G_{\rm F}^2\sin^2\theta_{\rm C}\cos^2\theta_{\rm C}}{11\pi^3}q_{\rm u}Bn_{\rm b} \tag{48}$$

for $2qB \sim \mu^2$, and it tends to the limit

$$\Gamma_{\rm H} = \frac{16\pi G_{\rm F}^2\sin^2\theta_{\rm C}\cos^2\theta_{\rm C}n_{\rm b}^3}{(q_{\rm u}B)^2} \tag{49}$$

for $2qB \gg \mu^2$.

In Fig. 5 we compare the result of Eq. (47) with those from Eq. (48) and Eq. (49). We find that they could be good approximations in the future realistic application.

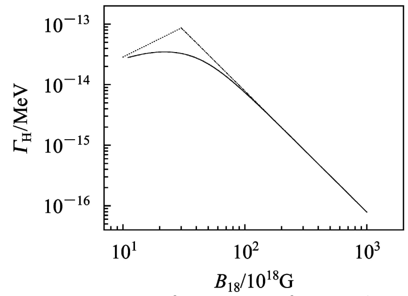

Fig. 5. $\Gamma_{\rm H}$ as a function of B when $n_{\rm b}=0.2{\rm fm}^{-3}$. The solid curve is for the result of Eq. (47), the dotted curve is for the result of Eq. (48) and the dash dot curve is for the result of Eq. (49).

Under the consideration that the u-quark is polarized but the effects of other flavors are negligible, we investigate the influence of the magnetic field on the non-leptonic reaction rate. Although the result for the weak field case has a small deviation from the field-free case, we give an analytical treatment of the weak reaction which can be extended to the calculation of other reaction process. However, the strong magnetic field can extremely suppress the rate. It is possible to induce a decrease of bulk viscosity in the magnetized SQM. This may have serious implications for compact star and pulsar dynamics. In fact, we should also distinguish the d-quark and the s-quark in calculations to obtain refined result for applications. That is our future work.

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磁场中夸克物质的非轻子弱作用过程*

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摘要 讨论了磁场中夸克物质的非轻子弱作用过程. 改进了在弱磁场情况下的近似计算方法, 分别给出了强弱磁场下非轻子过程的反应率的表达式, 以及一定条件下的近似表达式. 结果显示, 强磁场极大的抑制反应率的大小, 进而影响奇异夸克物质的粘滞性.

关键词 夸克 磁场 弱作用过程

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