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Why is the 3×3 Neutrino Mixing Matrix Almost Unitary in Realistic Seesaw Models?*

XING Zhi-Zhong[1)] ZHOU Shun[2)]

(Institute of High Energy Physics, CAS, Beijing 100049, China)

Abstract A simple extension of the standard model is to introduce n heavy right-handed Majorana neutrinos and preserve its $SU(2)_{\rm L}\times U(1)_{\rm Y}$ gauge symmetry. Diagonalizing the $(3+n)\times(3+n)$ neutrino mass matrix, we obtain an exact analytical expression for the effective mass matrix of $\nu_{\rm e}$, ν_μ and ν_τ. It turns out that the 3×3 neutrino mixing matrix V, which appears in the leptonic charged-current weak interactions, must not be exactly unitary. The unitarity violation of V is negligibly tiny, however, if the canonical seesaw mechanism works to reproduce the correct mass scale of light Majorana neutrinos. A similar conclusion can be drawn in the realistic Type-Ⅱ seesaw models.

Key words neutrino mixing matrix, unitarity, seesaw models

1 Introduction

Recent solar[1], atmospheric[2], reactor[3] and accelerator[4] neutrino oscillation experiments have provided us with very robust evidence that neutrinos are massive and lepton flavors are mixed. This great breakthrough opens a new window to physics beyond the standard model (SM). Indeed, the fact that the masses of neutrinos are considerably smaller than those of charged leptons and quarks remains a big puzzle in particle physics. Although a lot of theoretical models about the origin of neutrino masses have been proposed at either low or high energy scales[5], none of them has proved to be very successful and conceivable.

Within the SM, neutrinos are massless particles and lepton flavor mixing does not exist. The flavor eigenstates of three charged leptons (e, μ, τ) and three neutrinos ($\nu_{\rm e}$, ν_μ, ν_τ), which appear in the leptonic charged-current weak interactions

$$-\mathscr{L}_{\rm cc}=\frac{g}{\sqrt{2}}\,\overline{({\rm e},\ \mu,\ \tau)_{\rm L}}\,\gamma^\mu\begin{pmatrix}\nu_{\rm e}\\ \nu_\mu\\ \nu_\tau\end{pmatrix}_{\rm L} W^-_\mu\ +\ {\rm h.c.}\ , \quad (1)$$

can therefore be identified with their corresponding mass eigenstates. Beyond the SM, neutrinos may gain tiny but non-vanishing masses through certain new interactions at low or high energy scales. In this case, there is the phenomenon of lepton flavor mixing in analogy with that of quark flavor mixing. Identifying the flavor eigenstates of charged leptons with their mass eigenstates, we may express $\nu_{\rm e}$, ν_μ and ν_τ in terms of their mass eigenstates ν_1, ν_2 and ν_3 as follows:

$$\begin{pmatrix}\nu_{\rm e}\\ \nu_\mu\\ \nu_\tau\end{pmatrix}=\begin{pmatrix}V_{\rm e1} & V_{\rm e2} & V_{\rm e3}\\ V_{\mu1} & V_{\mu2} & V_{\mu3}\\ V_{\tau1} & V_{\tau2} & V_{\tau3}\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}\ . \quad (2)$$

The transformation matrix V in Eq. (2) is just the 3×3 lepton flavor mixing matrix, sometimes referred to as the Maki-Nakagawa-Sakata (MNS) matrix[6]. Unlike

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1) E-mail: xingzz@ihep.ac.cn
2) E-mail: zhoush@ihep.ac.cn

the Cabibb-Kobayashi-Maskawa (CKM) quark mixing matrix[7], which is required to be unitary in the SM, the MNS matrix V comes from new physics beyond the SM and its unitarity is not necessarily guaranteed in a specific model. If neutrinos are Majorana particles and V is exactly unitary, one can parametrize V in terms of three mixing angles and three CP-violating phases[8]. If the unitarity of V were significantly violated, more free parameters would in general be needed to describe neutrino mixing. A stringent test of the unitarity of V turns out to be one of the most important goals in the future neutrino factories and super-beam facilities.

The main purpose of this short paper is to show why the 3×3 MNS matrix V is not exactly unitary in a variety of neutrino models incorporated with the famous seesaw mechanism[9]. To be explicit, we extend the SM by including n heavy right-handed Majorana neutrinos and keeping its $SU(2)_{\rm L}\times U(1)_{\rm Y}$ gauge symmetry invariant. After diagonalizing the $(3+n)\times(3+n)$ neutrino mass matrix, we arrive at an exact analytical expression for the effective mass matrix of $\nu_{\rm e}$, ν_μ and ν_τ. Then it becomes obvious that the MNS matrix V, which appears in the leptonic charged-current weak interactions, is not exactly unitary. We find that the unitarity violation of V is negligibly tiny, unless the canonical seesaw mechanism fails to reproduce the correct mass scale of light Majorana neutrinos. A similar conclusion can be drawn in the realistic Type-Ⅱ seesaw mechanism.

2　Analytical results

Let us make a simple extension of the SM by introducing n heavy right-handed Majorana neutrinos N_i (for $i=1,\cdots,n$) and keeping the Lagrangian of electroweak interactions invariant under the $SU(2)_{\rm L}\times U(1)_{\rm Y}$ gauge transformation. In this case, the Lagrangian relevant for lepton masses can be written as

$$-\mathscr{L}_{\rm lepton}=\overline{l_{\rm L}}Y_{\rm l}e_{\rm R}H+\overline{l_{\rm L}}Y_\nu N_{\rm R}H^c+\frac{1}{2}\overline{N^c_{\rm R}}M_{\rm R}N_{\rm R}+{\rm h.c.}\,, \tag{3}$$

where $l_{\rm L}$ denotes the left-handed lepton doublets; $e_{\rm R}$ and $N_{\rm R}$ stand respectively for the right-handed charged-lepton and Majorana neutrino singlets; H is the Higgs-boson weak isodoublet (with $H^c\equiv {\rm i}\sigma_2H^*$); $M_{\rm R}$ is the heavy Majorana neutrino mass matrix; $Y_{\rm l}$ and Y_ν are the coupling matrices of charged-lepton and neutrino Yukawa interactions. After spontaneous gauge symmetry breaking, the neutral component of H acquires the vacuum expectation value $v\approx174$GeV. Then we arrive at the charged-lepton mass matrix $M_{\rm l}=vY_{\rm l}$ and the Dirac-type neutrino mass matrix $M_{\rm D}=vY_\nu$. The overall lepton mass term turns out to be

$$-\mathscr{L}'_{\rm lepton}=\overline{e_{\rm L}}M_{\rm l}e_{\rm R}+\frac{1}{2}\overline{(\nu_{\rm L},N^c_{\rm R})}\begin{pmatrix}0 & M_{\rm D}\\ M^{\rm T}_{\rm D} & M_{\rm R}\end{pmatrix}\begin{pmatrix}\nu^c_{\rm L}\\ N_{\rm R}\end{pmatrix}+{\rm h.c.}\,, \tag{4}$$

where e, $\nu_{\rm L}$ and $N_{\rm R}$ represent the column vectors of $({\rm e},\mu,\tau)$, $(\nu_{\rm e},\nu_\mu,\nu_\tau)_{\rm L}$ and $(N_\alpha,N_\beta,\cdots)_{\rm R}$ fields, respectively. In obtaining Eq. (4), we have used the relation $\overline{\nu_{\rm L}}M_{\rm D}N_{\rm R}=\overline{N^c_{\rm R}}M^{\rm T}_{\rm D}\nu^c_{\rm L}$ as well as the properties of $\nu_{\rm L}$ (or $N_{\rm R}$) and $\nu^c_{\rm L}$ (or $N^c_{\rm R}$)[8]. Note that the scale of $M_{\rm R}$ can naturally be much higher than the electroweak scale v, because those right-handed Majorana neutrinos are $SU(2)_{\rm L}$ singlets and their corresponding mass term is not subject to the magnitude of v.

Without loss of generality, it is convenient to choose a flavor basis in which $M_{\rm l}$ is diagonal, real and positive (i.e., the flavor and mass eigenstates of three charged leptons are identified with each other). Then we concentrate on the $(3+n)\times(3+n)$ neutrino mass matrix in Eq. (4), where $M_{\rm D}$ is a $3\times n$ matrix and $M_{\rm R}$ is an $n\times n$ matrix. The typical number of n is of course n=3, but n=2 is also a very interesting option as discussed in the so-called minimal seesaw models[10]. One may diagonalize the symmetric $(3+n)\times(3+n)$ neutrino mass matrix by use of a unitary transformation matrix:

$$\begin{pmatrix}V & R\\ S & U\end{pmatrix}^\dagger\begin{pmatrix}0 & M_{\rm D}\\ M^{\rm T}_{\rm D} & M_{\rm R}\end{pmatrix}\begin{pmatrix}V & R\\ S & U\end{pmatrix}^*=\begin{pmatrix}\overline{M}_\nu & 0\\ 0 & \overline{M}_{\rm R}\end{pmatrix}\,, \tag{5}$$

where R, S, U and V are the $3\times n$, $n\times3$, $n\times n$ and 3×3 sub-matrices, respectively; $\overline{M}_\nu$ and $\overline{M}_{\rm R}$ denote the diagonal 3×3 and $n\times n$ mass matrices with eigenvalues m_i and M_j (for $i=1,2,3$ and $j=1,\cdots,n$), respectively. Eq. (5) yields

$$S^\dagger M^{\rm T}_{\rm D}R^*+V^\dagger M_{\rm D}U^*+S^\dagger M_{\rm R}U^*=0\,, \tag{6}$$

and

$$\overline{M}_\nu = S^\dagger M_D^T V^* + V^\dagger M_D S^* + S^\dagger M_R S^* ,$$
$$\overline{M}_R = U^\dagger M_D^T R^* + R^\dagger M_D U^* + U^\dagger M_R U^* . \quad (7)$$

With the help of Eq. (6), $S^\dagger$ can be expressed as

$$S^\dagger = -V^\dagger M_D M_R^{-1}\left[1 + M_D^T R^* (U^*)^{-1} M_R^{-1}\right]^{-1} . \quad (8)$$

Combining Eqs. (7) and (8), we arrive at

$$\begin{aligned} V\overline{M}_\nu V^T &= -M_D M_R^{-1} M_D^T + \Delta_V , \\ U\overline{M}_R U^T &= M_R + \Delta_U , \end{aligned} \quad (9)$$

where

$$\begin{aligned} \Delta_V = &\ M_D M_R^{-1} M_D^T R^* R^T - \\ &\ M_D M_R^{-1} (U^\dagger)^{-1} R^\dagger M_D S^* V^T , \\ \Delta_U = &\ M_D^T R^* U^T - M_R S^* S^T . \end{aligned} \quad (10)$$

It is worth remarking that we have made no approximation in obtaining Eqs. (9) and (10). Because the $(3+n)\times(3+n)$ transformation matrix in Eq. (5) is unitary, its four sub-matrices satisfy the following conditions:

$$\begin{aligned} V^\dagger V + S^\dagger S = VV^\dagger + RR^\dagger = 1 , \\ U^\dagger U + R^\dagger R = UU^\dagger + SS^\dagger = 1 ; \end{aligned} \quad (11)$$

and

$$\begin{aligned} V^\dagger R + S^\dagger U = VS^\dagger + RU^\dagger = 0 , \\ R^\dagger V + U^\dagger S = SV^\dagger + UR^\dagger = 0 . \end{aligned} \quad (12)$$

Obviously, U, V, R and S are in general not unitary.

Note that V is just the MNS neutrino mixing matrix. To see this point more clearly, one may re-express $\mathscr{L}_{cc}$ in Eq. (1) by using the mass eigenstates of three charged leptons and those of $(3+n)$ neutrinos. The latter can be denoted as ν_i (for $i = 1, 2, 3$) and N_n (for $i = 1, \cdots, n$), which are related to $(\nu_e, \nu_\mu, \nu_\tau)$ through

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_L = V \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L + R \begin{pmatrix} N_1 \\ \vdots \\ N_n \end{pmatrix}_L . \quad (13)$$

Then $\mathscr{L}_{cc}$ reads

$$-\mathscr{L}_{cc} = \frac{g}{\sqrt{2}}\left[\overline{(e,\ \mu,\ \tau)_L}\ V\gamma^\mu \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L W_\mu^- + \overline{(e,\ \mu,\ \tau)_L}\ R\gamma^\mu \begin{pmatrix} N_1 \\ \vdots \\ N_n \end{pmatrix}_L W_\mu^-\right] + \text{h.c.} . \quad (14)$$

We observe that V enters the charged-current interactions between three charged leptons (e, μ, τ) and three well-known light neutrinos (ν_1, ν_2, ν_3), while R is relevant to the charged-current interactions between (e, μ, τ) and $(N_1, \cdots, N_n)$. Thus V is the MNS matrix. The unitarity of V is naturally violated, due to the presence of non-vanishing R and S. A preliminary upper bound on the matrix elements of R is at the $\mathscr{O}(10^{-3})$ level, extracted from some precise electroweak data[11]. In the limit of $R \to 0$ and $S \to 0$, V turns out to be exactly unitary.

3 Approximation and illustration

For simplicity, we denote the mass scales of M_R (or $\overline{M}_R$) and M_D as M_0 and m_0, respectively. Of course, $M_0 \gg v$ and $m_0 \lesssim v$ are naturally expected in almost all the reasonable extensions of the SM. The smallness of m_0/M_0 implies that the sub-matrices R and S are strongly suppressed in magnitude. This point can straightforwardly be observed from Eq. (8), which approximates to

$$S^\dagger \approx -VM_D M_R^{-1} \sim \mathscr{O}(m_0/M_0) . \quad (15)$$

On the other hand, Eq. (5) yields

$$R = +M_D U^* \overline{M}_R^{-1} \sim \mathscr{O}(m_0/M_0) . \quad (16)$$

These results, together with Eqs. (11) and (12), lead to

$$\begin{aligned} V^\dagger V \approx VV^\dagger \approx 1 , \\ U^\dagger U \approx UU^\dagger \approx 1 , \end{aligned} \quad (17)$$

which hold up to $\mathscr{O}(m_0^2/M_0^2)$. Then we arrive at the light Majorana neutrino mass matrix

$$M_\nu \equiv V\overline{M}_\nu V^T \approx -M_D M_R^{-1} M_D^T \quad (18)$$

and the heavy Majorana neutrino mass matrix $M_R \approx U\overline{M}_R U^T$ from Eq. (9) as two good approximations.

Eq. (18) is just the well-known (Type-I) seesaw relation between M_ν and M_R[9]. It indicates that the mass scale of three light neutrinos is of $\mathscr{O}(m_0^2/M_0)$. In other words, the smallness of three left-handed neutrino masses is essentially attributed to the largeness of n right-handed neutrino masses.

To illustrate how the unitarity of V or U is slightly violated in a more explicit way, let us consider the simplest seesaw model with only a single heavy right-handed Majorana neutrino (i.e., $n=1$). In this special case, $M_R=M_0$ holds1). The 3×1 matrix R and the 1×3 matrix S can be written as

$$R=\begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix}, \qquad S^{\rm T}=\begin{pmatrix} s_x \\ s_y \\ s_z \end{pmatrix}. \tag{19}$$

Then we obtain

$$\begin{aligned} R^\dagger R &= |r_x|^2+|r_y|^2+|r_z|^2 \equiv |r|^2 , \\ SS^\dagger &= |s_x|^2+|s_y|^2+|s_z|^2 \equiv |s|^2 . \end{aligned} \tag{20}$$

Note that $|r| = |s| \sim m_0/M_0$ holds. In view of Eq. (11), the departure of $U^\dagger U$ or $UU^\dagger$ from unity is at the $\mathscr{O}(m_0^2/M_0^2)$ level. On the other hand,

$$\begin{aligned} RR^\dagger &= \begin{pmatrix} |r_x|^2 & r_x r_y^* & r_x r_z^* \\ r_x^* r_y & |r_y|^2 & r_y r_z^* \\ r_x^* r_z & r_y^* r_z & |r_z|^2 \end{pmatrix}, \\ S^\dagger S &= \begin{pmatrix} |s_x|^2 & s_x^* s_y & s_x^* s_z \\ s_x s_y^* & |s_y|^2 & s_y^* s_z \\ s_x s_z^* & s_y s_z^* & |s_z|^2 \end{pmatrix}. \end{aligned} \tag{21}$$

It becomes obvious that the magnitude of each matrix element of $RR^\dagger$ or $S^\dagger S$ is at most of $\mathscr{O}(|r|^2)$ or $\mathscr{O}(|s|^2)$. Hence the deviation of $V^\dagger V$ or $VV^\dagger$ from the 3×3 identity matrix is also at the $\mathscr{O}(m_0^2/M_0^2)$ level.

Given $m_0 \sim 100$GeV and $m_0^2/M_0 \sim 0.1$eV, one may easily obtain $M_0 \sim 10^{14}$GeV. The latter is just the typical mass scale of heavy right-handed Majorana neutrinos in most of the realistic seesaw models. This estimate implies that the magnitude of R or S is of $\mathscr{O}(m_0/M_0) \sim \mathscr{O}(10^{-12})$. Hence the above-obtained seesaw formula is valid up to a high accuracy of $\mathscr{O}(m_0^2/M_0^2) \sim \mathscr{O}(10^{-24})$. Noticeably, the unitarity of the 3×3 MNS matrix is only violated at the $\mathscr{O}(10^{-24})$ level in such a canonical seesaw scenario. It is therefore very safe to neglect the extremely tiny $\mathscr{O}(m_0^2/M_0^2)$ correction to both M_ν and V.

The accuracy of Eq. (18) should be highlighted, because this seesaw formula was naively regarded as an approximation of $\mathscr{O}(m_0/M_0)$. Our instructive analysis shows that its validity is actually up to $\mathscr{O}(m_0^2/M_0^2)$. Furthermore, the unitarity violation of V or U can only take place at the $\mathscr{O}(m_0^2/M_0^2)$ level. That is why the 3×3 MNS neutrino mixing matrix is almost unitary in the realistic seesaw models.

4 Concluding remarks

Note that Eq. (18) is usually referred to as the Type-Ⅰ seesaw relation. A somehow similar relation, the so-called Type-Ⅱ seesaw formula, can be derived from the generalized lepton mass term

$$-\mathscr{L}''_{\rm lepton}=\overline{e_{\rm L}}M_l e_{\rm R}+\frac{1}{2}\overline{(\nu_{\rm L},N_{\rm R}^c)}\begin{pmatrix} M_{\rm L} & M_{\rm D} \\ M_{\rm D}^{\rm T} & M_{\rm R}\end{pmatrix}\begin{pmatrix}\nu_{\rm L}^c \\ N_{\rm R}\end{pmatrix}+{\rm h.c.}, \tag{22}$$

where M_L may result from a new Yukawa interaction term which violates the $SU(2)_L\times U(1)_Y$ gauge symmetry[5]. The mass scale of M_L is likely to be much lower than the electroweak scale v. Following the strategies outlined above, one may diagonalize the $(3+n)\times(3+n)$ neutrino mass matrix in Eq. (22) and arrive at the effective light Majorana neutrino mass matrix

$$M_\nu \equiv V\overline{M}_\nu V^{\rm T} \approx M_{\rm L}-M_{\rm D}M_{\rm R}^{-1}M_{\rm D}^{\rm T}, \tag{23}$$

where V is the 3×3 MNS neutrino mixing matrix. This result is just the Type-Ⅱ seesaw relation. Since the mass scale of M_L is expected to be smaller than that of M_D in those realistic models[5], Eq. (23) is valid up to the accuracy of $\mathscr{O}(m_0^2/M_0^2)$. The unitarity of V is also violated at the $\mathscr{O}(m_0^2/M_0^2)$ level, analogous to the Type-Ⅰ seesaw case.

It is worthwhile to mention that a quite novel (recursive expansion) recipe has been developed by

1) Because the rank of M_R equals one, the seesaw relation in Eq. (18) implies that M_ν is also a rank-one neutrino mass matrix. Thus two of its three mass eigenvalues must vanish, leading to a vanishing neutrino mass-squared difference. This result is certainly in contradiction with current solar and atmospheric neutrino oscillation experiments. In other words, the canonical seesaw model with a single heavy right-handed Majorana neutrino is not realistically viable.

Grimus and Lavoura[12] to calculate the effective mass matrices of three light neutrinos and three heavy neutrinos with arbitrary precision in the seesaw mechanisms. Their language, which can be related with ours by certain transformations, is less transparent in describing the 3×3 MNS matrix and discussing its tiny unitarity violation.

We conclude that the 3×3 MNS matrix V, which appears in the leptonic charged-current weak interactions, must not be exactly unitary in the canonical (Type-Ⅰ) and Type-Ⅱ seesaw models. Its unitarity violation is extremely small, as required by the models themselves to reproduce the correct mass scale of light Majorana neutrinos. Nevertheless, the unitarity of V could be more significantly violated by other sources of new physics (e.g., the existence of additional heavy charged leptons or light sterile neutrinos[13]). We remark that testing the unitarity of V, both its normalization conditions and its orthogonality relations[14], is one of the important experimental tasks to be fulfilled in the future neutrino factories and super-beam facilities.

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跷跷板模型中三阶中微子混合矩阵的幺正性破坏*

邢志忠1)　周顺2)

(中国科学院高能物理研究所　北京　100049)

摘要　对标准模型的一种简单扩充就是引入n个重的右手中微子且保持其$SU(2)_{\rm L}\times U(1)_{\rm Y}$规范对称性. 通过对角化$(3+n)\times(3+n)$阶中微子质量矩阵, 得到关于$\nu_e$, ν_μ和ν_τ的有效质量矩阵的精确的解析表达式. 结果表明, 在轻子带电弱流中出现的3×3中微子混合矩阵V必须不是严格幺正的. 如果通过跷跷板机制产生正确的轻的中微子的质量标度, 那么V的幺正性破坏的程度非常小, 几乎可以忽略. 类似的结论同样可以在第二类跷跷板模型中得到.

关键词　中微子混合矩阵　幺正性　跷跷板模型

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1) E-mail: xingzz@ihep.ac.cn

2) E-mail: zhoush@ihep.ac.cn