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Charmonium Formation and Suppression in Nuclear Matter *

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Abstract The coupling Schrödinger equations describing the evolution of $c\bar{c}$ states in nuclear matter are analytically and systematically solved via perturbation method, and the correlation between charmonium formation and nuclear absorption is investigated. After calculating J/Ψ and Ψ' suppression in nucleon-nucleus collisions and comparing with experiment data, it is found that the formation time effect plays an important rule in charmonium suppression, especially in Ψ' suppression.

Key words Charmonium particle, formation, suppression, nucleon matter

1 Introduction

The J/Ψ and Ψ' suppression is widely observed in relativistic heavy ion collisions[1, 2]. The suppression due to nuclear absorption is called normal suppression, and the one beyond normal part is called anomalous suppression[3]. The normal suppression is the background of the anomalous suppression, and the anomalous one is considered as a possible signature of quark-gluon plasma (QGP)[4,5] or hot and dense hadronic matter[6,7] formed in relativistic heavy ion collisions. For a nucleon-nucleus (p-A) process, it is usually difficult to form a hot and dense system, and only normal suppression is taken into account. While the classical Glauber models, in which nuclear absorption is treated as a final state effect without considering charmonium formation time, are used to explain J/Ψ and Ψ' normal suppression in pA processes[8,9], large absorption cross sections must be employed in order to fit the experiment data. From the quantum mechanical uncertainty principle, $\Delta E\Delta t \geqslant 1$, a $c\bar{c}$ pair produced via gluon fusion needs some finite time τ_f to expand into different charmonium states. In fact, there is also a finite formation time τ_c for the color singlet $c\bar{c}$ pair itself. When the incident energy is not very high and Feynman x_F of $c\bar{c}$ is small, it is estimated[1, 10] that only the charmonium formation time effect is important in describing the charmonium suppression, and the color singlet $c\bar{c}$ production can be regarded as instantaneous. Since a $c\bar{c}$ is heavy, one can treat its time evolution in nuclear matter in the frame of non-relativistic quantum mechanics with an optical potential describing charmonium formation and nuclear absorption[10]. In this frame the formation time can be attributed to the transitions between different charmonium states. Normally, in solving the coupling Schrödinger equations for the $c\bar{c}$ states with perturbation method, only first order correction to the J/Ψ and Ψ' survivor probabilities in nuclear collisions is considered[10—12]. In this paper, we investigate the correlation between the charmonium formation and nuclear absorption with perturbation method systematically, give a general method to calculate high order corrections, and point out the importance of the second order improvement numerically. In analogy to the classical Glauber models, we represent the formation time effects with an effective absorption cross section, which is no longer a constant but oscillating in time.

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2 Coupling Equations of $c\bar{c}$ States and Perturbation Solution

The optical model is often used in describing nuclear reactions in quantum mechanics. The real and imaginary part of the optical potential describe, respectively, the particle scattering and absorption. Similarly, we introduce an imaginary potential in the Hamiltonian of a $c\bar{c}$ system

$$H = H_0 + iW \tag{1}$$

to describe its nuclear absorption after production. Suppose the $c\bar{c}$ is produced in a p-A collision at a point with transverse coordinate b and longitudinal coordinate z in the laboratory frame with the center of the target A as the origin point. If we choose the beam motion as the longitudinal direction, b is the impact parameter of the collision. In the rest frame of $c\bar{c}$, its time evolution satisfies the Schrödinger equation

$$i\frac{\partial}{\partial\tau}|\Phi_{c\bar{c}}\rangle = H|\Phi_{c\bar{c}}\rangle. \tag{2}$$

To simplify the problem, we consider in the following only the lowest three states of $c\bar{c}$: J/Ψ, Ψ' and χ labelled by $|0\rangle$, $|1\rangle$ and $|2\rangle$, respectively. In the subspace spanned by the three states,

$$H_0 = \begin{pmatrix} M_{J/\Psi} & 0 & 0 \\ 0 & M_{\Psi'} & 0 \\ 0 & 0 & M_{\chi} \end{pmatrix},$$

$$W(\tau\mid b,z) = -\frac{1}{2}(A-1)v\gamma\rho(b,z+v\gamma\tau)\sigma, \tag{3}$$

where v is the $c\bar{c}$ velocity in the laboratory frame, and γ the corresponding Lorentz factor. The factor $A-1$ instead of A is to ensure no nuclear absorption in a nucleon-nucleon (pp) collision. The nucleon distribution in the target is the Woods-Saxon distribution $\rho(r) = C/(1+e^{(r-R_A)/a})$ with parameters $r_0 = 1.2$ fm, $R_A = r_0A^{1/3}$, $a = 0.5$ fm, the constant C is determined through normalization $\int\rho(r)4\pi r^2dr = 1$. The diagonal elements of the cross section matrix σ are $\sigma_{00} = \langle 0|\hat{\sigma}|0\rangle$, $\sigma_{11} = \langle 1|\hat{\sigma}|1\rangle$, $\sigma_{22} = \langle 2|\hat{\sigma}|2\rangle$, representing the elastic collisions of J/Ψ, Ψ' and χ with nucleons, and the off-diagonal elements like $\sigma_{01} = \langle 0|\hat{\sigma}|1\rangle$, $\sigma_{10} = \langle 1|\hat{\sigma}|0\rangle$ describe the conversion processes between different charmonium states during propagation through nuclear matter. The inelastic cross sections turn out to be nearly as big as the elastic ones[1,12]. In the following numerical calculations, we take[1,12] $\sigma_{00} = 2$ mb, $\sigma_{11} = 1.5\sigma_{00}$, and $\sigma_{10} = \sigma_{01} = 0.95\sigma_{00}$. Since is weakly coupled with J/Ψ and Ψ', the equation on J/Ψ and Ψ' formation and absorption can be further simplified in the two dimensional subspace spanned by J/Ψ and Ψ'. We ignore the formation time of χ and set[1] $\sigma_{22} = 9$ mb.

Substituting the expansion of the $c\bar{c}$ state in the 2-dimensional space,

$$|\Phi_{c\bar{c}}\rangle = C_0(\tau)e^{-iM\tau_{J/\Psi}}|\Phi_{J/\Psi}\rangle + C_1(\tau)e^{-iM'\tau_{\Psi}}|\Phi_{\Psi'}\rangle \tag{4}$$

into the Schrödinger equation (2), we obtain

$$\frac{\partial}{\partial_\tau}\begin{pmatrix} C_0(\tau) \\ C_1(\tau) \end{pmatrix} = -\frac{1}{2}(A-1)v\gamma\rho(b,z+v\gamma\tau)\times \begin{pmatrix} \sigma_{00} & \sigma_{01}\epsilon e^{-i\omega\tau} \\ \sigma_{10}\epsilon e^{i\omega\tau} & \sigma_{11} \end{pmatrix}\begin{pmatrix} C_0(\tau) \\ C_1(\tau) \end{pmatrix} = \begin{pmatrix} W_{00} & W_{01} \\ W_{10} & W_{11} \end{pmatrix}\begin{pmatrix} C_0(\tau) \\ C_1(\tau) \end{pmatrix}, \tag{5}$$

with $\omega = \Delta M = M_{\Psi'} - M_{J/\Psi}$. This is the evolution equation for J/Ψ and Ψ' probability amplitudes $C_0(\tau\mid b,z)$ and $C_1(\tau,\mid b,z)$ of $c\bar{c}$ produced at the point (b,z) in the target. Its initial condition is determined through pp process[1] and normalization,

$$\left|\frac{C_1(0,\mid b,z)}{C_0(0,\mid b,z)}\right|^2 = 0.21,$$

$$|C_1(0,\mid b,z)|^2 + |C_0(0,\mid b,z)|^2 = 1. \tag{6}$$

In order to show the difference between quantum mechanical method and classical method analytically, we solve the evolution equation (5) by perturbation method. We first ignore the formation time effect by dropping the off diagonal elements in the matrix W. In this case the equation turns into

$$\frac{\partial}{\partial_\tau}\begin{pmatrix} C_0(\tau) \\ C_1(\tau) \end{pmatrix} = \begin{pmatrix} W_{00} & 0 \\ 0 & W_{11} \end{pmatrix}\begin{pmatrix} C_0(\tau) \\ C_1(\tau) \end{pmatrix}. \tag{7}$$

As there is no more coupling between C_0 and C_1, the equation can be easily integrated with the solution

$$C_0^{(0)}(\tau\mid b,z) = C_0(0,\mid b,z)e^{-\frac{1}{2}\sigma_{00}v\gamma(A-1)\int_0^\tau\rho(b,z+v\gamma\tau')d\tau'},$$

$$C_1^{(0)}(\tau\mid b,z) = C_1(0,\mid b,z)e^{-\frac{1}{2}\sigma_{11}v\gamma(A-1)\int_0^\tau\rho(b,z+v\gamma\tau')d\tau'}. \tag{8}$$

This agrees with the classical case[1] without considering the formation time effect. We use now (8) as the zeroth order approximation and let

$$C_0 = C_0^{(0)} + C_0^{(1)},$$
$$C_1 = C_1^{(0)} + C_1^{(1)}. \tag{9}$$

Substituting it into equation (5), we have the evolution equation for the first order probability amplitudes,

$$\frac{\partial C_0^{(1)}}{\partial \tau} = W_{01}(\tau) C_1^{(0)}(\tau),$$
$$\frac{\partial C_1^{(1)}}{\partial \tau} = W_{10}(\tau) C_0^{(0)}(\tau). \quad (10)$$

with the solution

$$C_0^{(1)}(\tau) = \int_0^\tau W_{01}(\tau') C_1^{(0)}(\tau') d\tau',$$
$$C_1^{(1)}(\tau) = \int_0^\tau W_{10}(\tau') C_0^{(0)}(\tau') d\tau'. \quad (11)$$

Again, we let

$$C_0 = C_0^{(0)} + C_0^{(1)} + C_0^{(2)},$$
$$C_1 = C_1^{(0)} + C_1^{(1)} + C_1^{(2)}, \quad (12)$$

and obtain the equations to the second order approximation,

$$\frac{\partial C_0^{(2)}}{\partial \tau} = W_{00}(\tau) C_0^{(1)}(\tau) + W_{01}(\tau) C_1^{(1)}(\tau),$$
$$\frac{\partial C_1^{(2)}}{\partial \tau} = W_{11}(\tau) C_1^{(1)}(\tau) + W_{10}(\tau) C_0^{(1)}(\tau). \quad (13)$$

with the solution

$$C_0^{(2)}(\tau) = \int_0^\tau (W_{00}(\tau') C_0^{(1)}(\tau') + W_{01}(\tau') C_1^{(1)}(\tau')) d\tau',$$
$$C_1^{(2)}(\tau) = \int_0^\tau (W_{11}(\tau') C_1^{(1)}(\tau') + W_{10}(\tau') C_0^{(1)}(\tau')) d\tau'. \quad (14)$$

For incident nucleon energy $E_{lab} = 450$ GeV and target nucleon number $A = 200$, the probabilities $|C_0(\tau)|^2$ and $|C_1(\tau)|^2$ of $c\bar{c}$ produced at $b = 0$ and $z = -R_A$ and with $x_F = 0.15$ are shown in Fig.1.

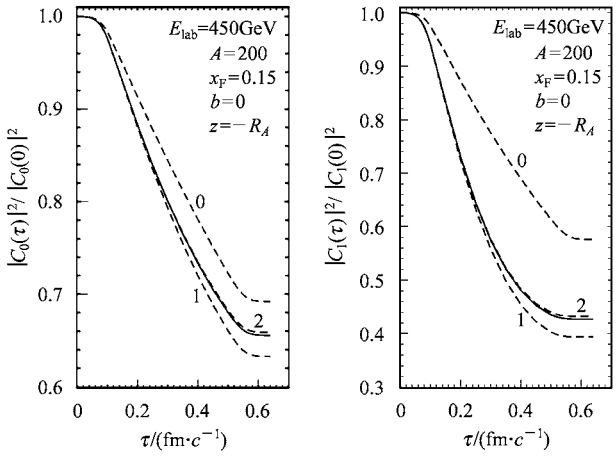

Fig.1. The probabilities $|C_0(\tau)|^2$(left) and $|C_1(\tau)|^2$ (right) of the $c\bar{c}$ produced at $b = 0$ and $z = -R_A$ and with $x_F = 0.15$. The dashed lines labelled by 0,1,2 indicate, respectively, classical, first-order and second-order calculation, and the solid line is the exact numerical calculation.

The classical solution describes nuclear absorption of J/Ψ and Ψ′ independently to each other, with absorption section σ_{00} and σ_{11}. In quantum mechanical approach, transitions between the states $|0\rangle$ and $|1\rangle$ and their absorption by nuclear matter happen at the same time, and are treated systematically in the frame of time-dependent Schrödinger equation. As W_{01} describes the transition from $|\Psi'\rangle$ to $|J/\Psi\rangle$, the first order correction $C_0^{(1)}$ indicates the contribution of this transition to the survival probability amplitude of J/Ψ. Different from pure transitions, when the transitions between $|\Psi'\rangle$ and $|J/\Psi\rangle$ happens, they suffer from nuclear absorption simultaneously. It is the accompanying transitions that the time evolution of J/Ψ and Ψ′ cannot be solely determined by the constant absorption cross sections σ_{00} and σ_{11}. For the second order correction $C_0^{(2)}(14)$, the first term $W_{00} C_0^{(1)}$ in the integration describes the nuclear absorption of the J/Ψ coming from the transition of Ψ′, and the second term $W_{01} C_1^{(1)}$ represents the transition process $|J/\Psi) \to |\Psi'\rangle \to |J/\Psi\rangle$. Higher orders of correction, which stand for even more complex entanglement of transition and nuclear absorption, can be interpreted in the same way.

Our calculation shows that there are big difference between the classical and quantum mechanical solutions. The classical solution (dashed lines labelled by 0) is good only in the very beginning of the evolution, and the first order quantum correction (dashed lines labelled by 1) still exhibit con-

siderable difference from the exact, numerical result (solid lenes), especially in the final stage of the evolution. With second order correction, our result (dashed lines labelled by 2) well approximates to the exact ones. Higher orders of correction have little contribution and can be safely ignored.

In order to better illustrate the formation time effect, we introduce a time dependent effective absorption cross section $\sigma^{\rm eff}(\tau)$ in place of the classical constant absorption cross section, and put all the quantum mechanical effects in $\sigma^{\rm eff}(\tau)$. In classical case,

$$\left|\frac{C_i^{(0)}(\tau \mid b,z)}{C_i^{(0)}(0 \mid b,z)}\right|^2 = {\rm e}^{-v\gamma\sigma_{ii}(A-1)\int_0^\tau \rho(b,z+v\gamma\tau')\,{\rm d}\tau'}. \quad (15)$$

If we write the solution of the Schrödinger equation (5) in the form of

$$\left|\frac{C_i(\tau \mid b,z)}{C_i(0 \mid b,z)}\right|^2 = {\rm e}^{-v\gamma\sigma_{ii}^{\rm eff}(A-1)\int_0^\tau \rho(b,z+v\gamma\tau')\,{\rm d}\tau'}, \quad (16)$$

$\sigma_{ii}^{\rm eff}$is right the effective absorption cross section. From the comparison of (15) and (16), we obtain

$$\sigma_{ii}^{\rm eff}(\tau) = \sigma_{ii}\frac{\ln \mid C_i(\tau \mid b,z) \mid^2}{\ln \mid C_i^{(0)}(\tau \mid b,z) \mid^2}. \quad (17)$$

Fig. 2 shows the time evolution of the effective absorption cross sections with $x_{\rm F}=0.15$. The two curves correspond to $c\bar{c}$ produced in a central collision with $b=0$, $z=-R_A$ and in a peripheral collision with $b=R_A, z=-R_A$, respectively. In central collisions, $\sigma_{ii}^{\rm eff}(\tau)$ fluctuates violently with time, indicating larger formation time effect. The characteristic time of the fluctuations is described by the mass difference, $\Delta\tau\approx 1/\omega$, and the variation of the magnitude is about $\Delta\sigma_{ii}^{\rm eff}/\sigma_{ii}\approx 15\%$. In peripheral collisions, the variation of $\sigma_{ii}^{\rm eff}(\tau)$ is much more smooth.

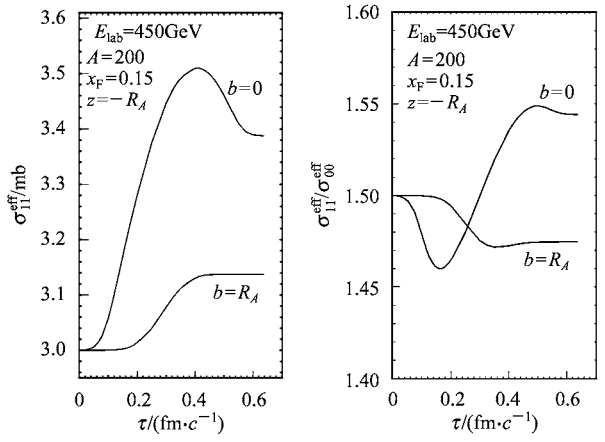

Fig. 2. The Ψ′ effective cross section (left) and the ratio of Ψ′ to J/Ψ effective cross sections (right) as functions of proper time.

3 J/Ψ and Ψ′ suppression in NA collisions

We discussed above the time evolution of $c\bar{c}$ produced at a fixed point in the target in order to elucidate the mechanism of J/Ψ and Ψ′ formation time in the framework of quantum mechanics. To compare with experiment data of pA collisions, we must integrate over all the $c\bar{c}$'s produced in the target. Since the measurable survival rate of J/Ψ and Ψ′ correspond to the time limit $\tau\to\infty$, we define

$$K_i(b,z) = \left|\frac{C_i(\tau\to\infty \mid b,z)}{C_i(0 \mid b,z)}\right|^2. \quad (18)$$

Integrating over the $c\bar{c}$ production point (b, z), we obtain the survival rate of J/Ψ and Ψ′,

$$s_i = \int {\rm d}^2 b \int {\rm d}z \rho(b,z) K_i(b,z). \quad (19)$$

After the formation and nuclear absorption, Ψ′ and χ can decay into J/Ψ, the observed J/Ψ originates with probability 60% from directly formed J/Ψ, probability 10% and 30% from the decay of Ψ′ and χ respectively[1],

$$\begin{aligned} S_{J/\Psi} &= 0.6 s_{J/\Psi} + 0.3 s_{\Psi'} + 0.1 s_\chi, \\ S_{\Psi'} &= s_{\Psi'}, \\ S_\chi &= s_\chi. \end{aligned} \quad (20)$$

As the formation time of χ is ignored, the K factor of χ is given by

$$K_2(b,z) = \left|\frac{C_2(\tau\to\infty \mid b,z)}{C_2(0 \mid b,z)}\right|^2 = {\rm e}^{-v\gamma\sigma_{22}(A-1)\int_0^\tau \rho(b,z+v\gamma\tau')\,{\rm d}\tau'}. \quad (21)$$

We calculated the J/Ψ and Ψ′ survival rate in different pA processes in both the classical and quantum mechanical approach. The target mass dependence of the survival rate is shown in Fig. 3. Our quantum mechanical calcula-

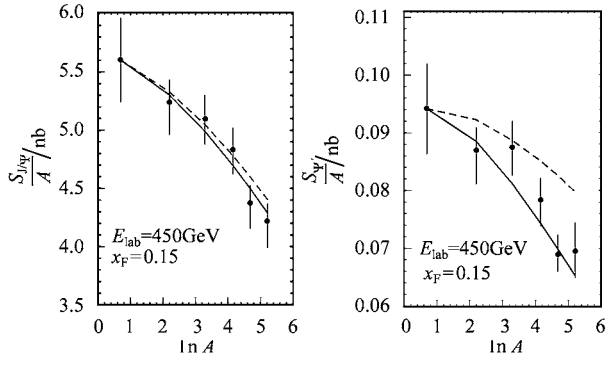

Fig. 3. The target mass dependence of J/Ψ and Ψ′ suppression in pA collisions at beam energy $E_{\rm lab}$ = 450 GeV and with $x_{\rm F}$ = 0.15. The dashed lines indicate the classical result, and the solid lines represent the quantum calculation to the second order. The experiment data are from NA50 collaboration.

tion to the second order correction well fit the experiment data[8], while the classical result deviates from the data explicitly, especially for the Ψ'.

4 Conclusions

The quantum equation on time evolution of $c\bar{c}$ in nuclear matter is solved with perturbation method. The effect of quantum transitions between different $c\bar{c}$ states on charmonium nuclear absorption is discussed and the normal suppression in pA collisions is calculated. We have the following conclusions:

(1) The mechanism of J/Ψ and Ψ' formation time can be attributed to the quantum transitions between different charmonium states. The time scale characterizing the fluctuations of the effective absorption cross section is approximately $1/(m_{\Psi'} - m_{J/\Psi})$, and the variation in magnitude of the effective cross section is approximately 15%.

(2) The classical approach without consideration of quantum transition can only describe the evolution of $c\bar{c}$ in the very beginning, and the first order quantum correction is still not enough in the final stage of the evolution. The perturbation calculation to the second order can well describe the normal J/Ψ and Ψ' suppression in pA collisions.

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核物质中粲偶素粒子的形成与压低*

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摘要 用微扰方法系统求解了 $c\bar{c}$ 各态之间的耦合 Schroedinger 方程,讨论了粲粒子的形成时间和核吸收的关联.通过计算 J/Ψ 及 Ψ' 在不同核子-核过程中的压低并与实验结果比较,发现形成时间对 J/Ψ,特别是 Ψ' 的核吸收过程有重要的影响.

关键词 粲偶素粒子 形成 压低 核物质

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