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Pion Interferometry for Cylindrical Quark-Gluon Plasma Evolution Sources *

Efaaf M.J.[1)] ZHANG Wei-Ning[2)] Khaliliasr M. HUO Lei JIN En-Pei ZHANG Jing-Bo

(Department of Physics, Harbin Institute of Technology, Harbin 150006, China)

Abstract We examine the two-pion Hanbury-Brown-Twiss(HBT) interferometry for the expanding sources of quark-gluon plasma evolution with the Bjorken cylinder geometry. The two-pion HBT correlation functions are calculated using quantum probability amplitudes in a path-integral formalism and the HBT radius is extracted both for the expanding source and the static source. We find that the HBT radius for the freeze-out emission case is substantially greater than that for the case without absorption of multiple scattering. The expanding velocity of the source leads to a smaller HBT radius. The effect of expanding velocity for the Bjorken cylinder source with zero width of the transition temperature is smaller than those of the corresponding spherical source.

Key words pion interferometry, quark-gluon plasma, Bjorken cylinder source

Two-pion Hanbury-Brown-Twiss(HBT) interferometry has been used extensively to probe the space-time structure of heavy ion collisions[1]. In ultra-relativistic central nucleus-nucleus collisions, soon after the collision of the two nuclei at $(z,t)=(0,0)$, the energy density is expected to be sufficiently high to form a system of quark-gluon plasma(QGP) in the central rapidity region[2,3]. In Bjorken cylinder model[2], the plasma goes to local equilibrium at the proper time τ_0. Then, it evolves according to the laws of hydrodynamics. As the plasma expands, its temperature drops down and the hadronization of the plasma will take place at a later time. The hadrons will stream out of the collision region when the temperature falls below the freeze-out temperature[2,3].

In Refs.[4,5], Wong developed the theory of two-pion interferometry of quantum probability amplitudes in a path-integral formalism and investigated the effects of the collective expansion of source and the multiple scattering of particles on two-pion interferometry. In Refs.[6,7] we examined the two-pion interferometry for the expanding source of hadronic gas with finite baryon density and for the spherical quark-gluon plasma evolution sources using quantum probability amplitudes in a path-integral formalism. In this paper, we shall use the same way to examine the two-pion interferometry for the expanding sources which come from the Bjorken cylinder quark-gluon plasma evolution. We shall restrict our consideration to the particles emitted from the central rapidity region with zero net baryon density and follow Rischke and Gyulassy[8,9] using the equation of state of entropy density suggested by QCD lattice data[10,11] to describe the system of quark-gluon plasma. Once the equation of state and the initial condition are known, the solution of the expansion and hadronization process can be obtained by relativistic hydrodynamics without complicated microscopic details[8,9]. Then, the two-pion correlation function can be calculated by using quantum probability amplitudes in a path-integral formalism, after knowing the dynamical solution[4—7].

At zero net baryon density, the entropy density as a function of temperature can be expressed as[8—11]

$$\frac{s}{s_c}(T)=\left[\frac{T}{T_c}\right]^3\left(1+\frac{d_Q-d_H}{d_Q+d_H}\tanh\left[\frac{T-T_c}{\Delta T}\right]\right), \quad (1)$$

where d_Q and d_H are the degrees of freedom in the quark-gluon plasma phase and the hadronic phase, T_c is the transition

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1) E-mail: mefaaf@yahoo.com

2) E-mail: wnzhang@hit.edu.cn

temperature, s_c is the entropy density at T_c, and ΔT(between 0 and $0.1T_c$) is the width of the transition[8,9]. Once $s(T)$ in Eq.(1) is known one can obtain the pressure p, energy density ϵ, and sound velocity c_S from the following relations as in Refs.[8,9]

$$p = \int_0^T dT' s(T'), \quad \epsilon = Ts - p, \quad c_S^2 = \frac{dp}{d\epsilon}. \tag{2}$$

In our calculations, we take $d_Q = 37$, $d_H = 3$, $T_c = 160$ MeV as in Refs.[8,9], and consider the two QGP systems of $\Delta T = 0$(an exact first-order transition) and $\Delta T = 0.1T_c$, and the ideal pion gas system with the equation of state $p = \epsilon/3$ for comparison.

The energy momentum tensor of a thermalized fluid cell in the center-of-mass frame of the source is[8,9,12,13]

$$T^{\mu\nu}(x) = [\epsilon(x) + p(x)]u^{\mu}(x)u^{\nu}(x) - p(x)g^{\mu\nu}, \tag{3}$$

where x is the space-time coordinate, $u^{\mu} = \gamma(1, \boldsymbol{v})$ is the 4-velocity of the cell, and $g^{\mu\nu}$ is the metric tensor. From the local conservation of energy and momentum, one can get the equations for the Bjorken cylinder geometry as[8,9]

$$\partial_t E + \partial_r[(E + p)v] = -F, \tag{4}$$

$$\partial_t M + \partial_r(Mv + p) = -G, \tag{5}$$

where r indicates the transverse radial coordinate of the cylinder, $E \equiv T^{00}$, $M \equiv T^{0r}$,

$$F = \left(\frac{v}{r} + \frac{1}{t}\right)(E + p), \quad G = \left(\frac{v}{r} + \frac{1}{t}\right)M. \tag{6}$$

For the Bjorken cylinder, the above equations describe the system's evolution in the transverse direction ("T" direction) at $z = 0$, and due to the assumption of longitudinal boost invariance[2], the hydrodynamical solution for arbitrary z can be easily obtained by a Lorentz boost with rapidity $\eta = \mathrm{Artanh}(z/t)$[3,8,9].

We assume the initial conditions as[8]

$$\epsilon(\tau_0, r) = \begin{cases} \epsilon_0, & r < r_0, \\ 0, & r > r_0, \end{cases} \quad v(\tau_0, r) = \begin{cases} 0, & r < r_0, \\ 1, & r > r_0, \end{cases} \tag{7}$$

and take $\epsilon_0 = 1.875T_c s_c$, $\tau_0 = 0.1r_0$ as in Ref.[8], and choose $r_0 = 6.0$ fm in our calculations. In order to solve Eqs.(4) and (5), we first use the HLLE scheme[8,9] to obtain the solutions of the equations for $F = G = 0$. Then, we employ the Sod's operator splitting method[8,9,14] for each time step to obtain the complete time evolution of the system. The grid spacing for the HLLE scheme is taken as $\Delta r = 0.01r_0$, the time step width for the HLLE scheme and Sod's method corrector step is $\Delta t = 0.99\Delta r$[8]. Figs. 1(1), (2), and (3) show the temperature profiles for the evolution systems of $\Delta T = 0$, $\Delta T = 0.1T_c$, and the ideal gas. Figs. 1(1′), (2′), (3′) show the transverse velocity profiles, and Figs. 1(1″), (2″), (3″) show the isotherms for the three systems, respectively. The temperature profiles and the isotherms are consistent with the results of Ref.[8](see the Fig.8 of Ref.[8]).

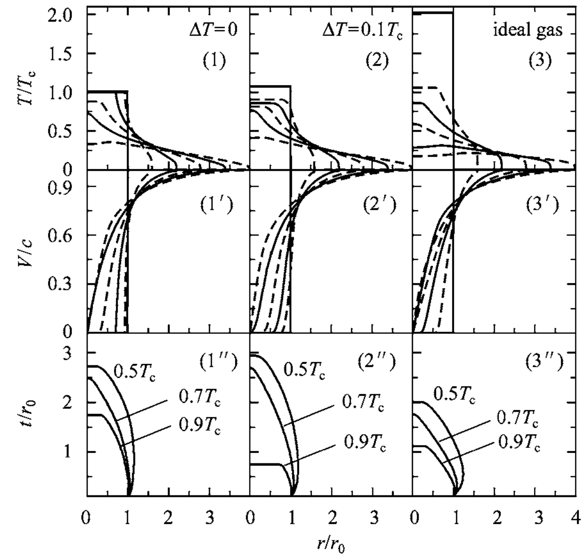

Fig.1. (1,1′),(2,2′), and (3,3′) are the temperature and transverse velocity profiles for the systems of $\Delta T = 0$, $\Delta T = 0.1T_c$, and the ideal gas at different times $t_n = \tau_0 + 0.6n\lambda r_0$, respectively, where $\tau_0 = 0.1r_0$, $n = 0,1,2,3,4,5$, $\lambda = 0.99$. (1″), (2″) and (3″) show the isotherms for the three systems.

The two-particle HBT correlation function is defined as the ratio of the two-particle momentum distribution $P(k_1, k_2)$ to the product of the single-particle momentum distribution $P(k_1)P(k_2)$. Using quantum probability amplitudes in a path-integral formalism, the correlation function $C(k_1, k_2) = P(k_1, k_2)/P(k_1)P(k_2)$ can be written as [4—7]

$$C(k_1, k_2) = 1 + \left| \int d^4x \, e^{i(k_1 - k_2)\cdot x + i\phi_c(x, k_1 k_2)} \rho_{\mathrm{eff}}(x; k_1 k_2) \right|^2, \tag{8}$$

where $\phi_c(x, k_1 k_2)$ is a phase from the collective expansion of the source and ρ_{eff} is the effective density

$$\rho_{\mathrm{eff}}(x; k_1 k_2) = \frac{e^{-2\mathrm{Im}\bar{\phi}_s(x)} \sqrt{f_{\mathrm{init}}(\kappa_1, x) f_{\mathrm{init}}(\kappa_2, x)}}{\sqrt{P(k_1)P(k_2)}}. \tag{9}$$

In Eq.(9), $e^{-2\mathrm{Im}\bar{\phi}_s(x)}$ is the absorption factor due to multiple scattering, $f_{\mathrm{init}}(\kappa, x) = \rho(x)A^2(\kappa(x), x)$ is the phase-space distribution of the pion-emitting source, which is proportional to the Bose-Einstein distribution in the local frame, $\rho(x)$ is the pion-source density, and $A(\kappa(x), x)$ is the magnitude of the amplitude for producing a pion with momentum κ at x. It can be seen that the effective density is related to the phase-space distribution of the pion-emitting source, modified by an absorption factor arising from multiple scattering[4—7]. The two extreme cases of the absorption of multiple scattering are the pions without absorption after production and with a strong absorption which leads to a freeze-out emission[6]. In this paper we calculate the correlation functions for the three kinds of cylindrical evolution sources, $\Delta T = 0$, $\Delta T = 0.1T_c$, and the ideal gas, and consider only the two extreme cases of the absorption of multiple scattering as in Ref.[7] for the spherical expanding sources. We first construct the two-dimension correlation function $C(q, q_0)$, ($q = |(\boldsymbol{k}_1 - \boldsymbol{k}_2)_T|$, $q_0 = |E_1(\boldsymbol{k}_1) - E_2(\boldsymbol{k}_2)|$), from $P(k_1, k_2)$ and $P(k_1)P(k_2)$ by summing over $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ for each (q, q_0) bin with the limitation $|(\boldsymbol{k}_1 - \boldsymbol{k}_2)_z| \leqslant 15$ MeV/c. Then, we obtain the one-dimension transverse correlation function $C(q)$ by integrating $C(q, q_0)$ over q_0 within $q_0 \leqslant 20$MeV.

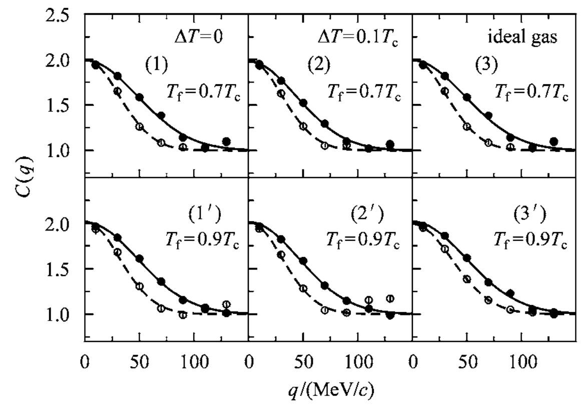

Fig.2. The two-pion correlation functions for the Bjorken cylinder expanding sources of $\Delta T = 0$(1 and 1′), $\Delta T = 0.1T_c$(2 and 2′), and the ideal gas (3 and 3′) for the freeze-out temperatures $T_f = 0.7T_c$ and $0.9T_c$. The symbols of bullets and circles are for the cases without absorption and with the freeze-out emission, respectively.

Figs.2(1,2,3) and (1′,2′,3′) show the transverse correlation function $C(q)$ of the three kinds of cylindrical expanding sources for the freeze-out temperatrue $T_f = 0.7T_c$ and $0.9T_c$, respectively. The symbols of bullets and circles are for the two cases without absorption and with the freeze-out emission. The solid and dashed lines are the corresponding fitted curves with the parametrized correlation function as

$$C(q) = 1 + \lambda e^{-q^2R^2}. \tag{10}$$

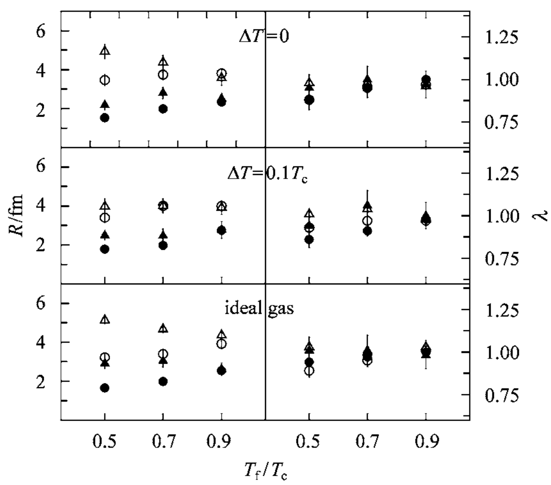

Fig.3. Two-pion HBT results for the three kinds of sources of $\Delta T = 0$, $\Delta T = 0.1T_c$, and the ideal gas. The symbols of bullets and circles are the results of the expanding sources for the cases without absorption and with the freeze-out emission. The symbols of solid triangles and open triangles are the results of the corresponding "static" sources for the two cases, respectively.

Fig.3 gives the fitted results of R and λ in Eq.(10), where the symbols of bullets and circles are the results of the expanding sources for the two cases without absorption and with the freeze-out emission, respectively. In order to investigate the effect of expanding velocity of source on the HBT radius, we also examine the two-pion interferometry for the corresponding "static" sources, which have the same density distributions as the expanding sources but the expanding velocities of the sources are forced to be zero[7]. In Fig.3, the symbols of solid triangles and open triangles are the results of these "static" sources, for the cases without absorption and with the freeze-out emission, respectively. It can be seen that the HBT radius for the freeze-out emission case is substantially greater than that for the corresponding case without absorption, as for the spherical evolution sources[7]. Also as we observed for the spherical evolution sources[7], the expanding velocity leads to a smaller HBT radius[15], and this effect is greater at the lower freeze-out temperature because the average expanding velocity of the source is larger at the lower freeze-out temperature than that at the higher freeze-out temperature.

For the spherical source of $\Delta T = 0$[7], one can find that there is a shock velocity during the transition from QGP to the hadronic phase (see Fig.1(1′) of Ref. [7]), and it leads to a greater effect of the expanding velocity on HBT radius at the lower freeze-out temperature (see Fig.3 of Ref.[7]). However, for the Bjorken cylinder source of $\Delta T = 0$ there is not the shock velocity, so the effect of the expanding velocity on HBT radius is smaller. Because the average expanding velocity of the Bjorken cylinder source of the ideal gas is larger than those of the other two, the effect of expanding velocity for the ideal gas source at the lower freeze-out temperature is greater. It is different from the results of the spherical sources[7].

In summary, we examine the two-pion interferometry for the Bjorken cylinder evolution sources of the QGP with the equation of state of entropy density and the ideal pion gas with the equation of state $p = \epsilon/3$. The evolution of the sources is described by relativistic hydrodynamics and the HBT radius is obtained using quantum probability amplitudes in a path-integral formalism. We find that as for the spherical evolution sources[7], the HBT radius for the freeze-out emission case is substantially greater than that for the case without absorption. The expanding velocity of source leads to a smaller HBT radius and this effect is important when the freeze-out temperature is low. The influence of the expanding velocity for the Bjorken cylinder source of $\Delta T = 0$ is smaller than that for the ideal gas source, which is different from the results of the corresponding spherical source. An advantage of the two-pion interferometry of quantum probability amplitudes in a path-integral formalism is that it can consider the effects of the collective expansion of the source and the multiple scattering of the particles[4—6]. For the multiple scattering we only considered two extreme cases in this paper. A more detailed investigation for this effect will be of great interest.

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柱形夸克-胶子等离子体演化源的 π 干涉学分析 *

Efaaf M.J.[1)] 张卫宁[2)] Khaliliasr M. 霍雷 金恩培 张景波

(哈尔滨工业大学物理系 哈尔滨 150006)

摘要 本文对 Bjorken 柱形夸克–胶子等离子体演化膨胀源进行了 2π Hanbury-Brown-Twiss(HBT)干涉学分析.利用量子几率振幅的路径积分公式计算 2π HBT 关联函数并得到对膨胀和静态源的 HBT 半径.研究发现,在冻结发射情况下的 HBT 半径要明显大于没有考虑多重散射吸收情况下的结果,源的膨胀速度导致 HBT 半径变小.对相变温度宽度为零的 Bjorken 柱形源,膨胀速度的影响要小于对应的球形源的结果.

关键词 π 干涉学 夸克–胶子等离子体 Bjorken 柱形源

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1) E-mail:mefaaf@yahoo.com

2) E-mail:wnzhang@hit.edu.cn