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E2 & M1 Strengths in Deformed Nuclei: Application of the Pseudo-$SU(3)$ Model*

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Abstract An application of the pseudo-$SU(3)$ shell-model to ^{162}Dy illustrates that complex many-body behavior in atomic nuclei can be given a simple geometrical interpretation.

Key words pseudo-$SU(3)$ model, low-lying levels, electro-magnetic transitions

1 Introduction

The identification of new low-lying levels and more precise measurements of E2 and M1 transition strengths continue to challenge our understanding of collective nuclear phenomena. While systems with nucleon numbers greater than about 150 are excellent candidates for probing our understanding of deformation, microscopic calculations for these systems remain illusive. The pseudo-$SU(3)$ model-which capitalizes on good pseudo-spin symmetry in heavy nuclei—is applicable in this domain and has enjoyed considerable success[1,2]. Here we review some recent applications of the pseudo-$SU(3)$ model in this domain, using group theory to give a simple geometrical interpretation of observed phenomena.

The pseudo-$SU(3)$ model builds on the observation that single-particle orbitals with $j = l - \frac{1}{2}$ and $j = (l-2) + \frac{1}{2}$ in the shell η lie close in energy and therefore be labeled as pseudo-spin doublets with quantum numbers $\tilde{j} = j$, $\tilde{\eta} = \eta - 1$, and $\tilde{l} = l - 1$. Its origin has been traced back to the relativistic Dirac equation[3]. In the current version of the pseudo-$SU(3)$ model, the intruder level of opposite (unique) parity in each major shell is relegated to a renormalization role and pseudo-orbital and pseudo-spin quantum numbers are assigned to the remaining (normal parity) single-particle states[4].

2 Model space and interaction

Though only results for ^{162}Dy are reported here, the results extend to other even-even and odd-A rare earth nuclei which are taken to have a closed proton core at $N_\pi = 50$ and a closed neutron core at $N_\nu = 82$. Basis states are built by placing valence nucleons in the open $\eta_\pi = 4$ shell for protons, less the $g_{9/2}$ level which forms part of the proton core plus the unique-parity $h_{11/2}$ intruder level from the shell above, and the $\eta_\nu = 5$ shell for neutrons, less the $h_{11/2}$ level which forms part of the neutron core plus the unique-parity $i_{13/2}$ intruder level from the shell above it. As noted above, nucleons assigned to the unique-parity intruder levels are relegated to a renomalization role. These shells have a complementary pseudo-harmonic oscillator shell structure that is given by $\tilde{\eta}_\sigma(\sigma = \pi, \nu) = \eta_\sigma - 1$ plus the respective intruder levels. Typical applications include about 5 proton and 5 neutron pseudo-

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$SU(3)$ irreducible representations (irreps) with largest values of their respective second order Casimir operators, $\tilde{C}_2^\sigma = (\tilde{Q}^\sigma \cdot \tilde{Q}^\sigma - 3\tilde{L}_\sigma^2)$, and up to about 20 proton-neutron coupled irreps, again with largest combined $\tilde{C}_2$ values where for the latter case $\tilde{C}_2 = (\tilde{Q} \cdot \tilde{Q} - 3\tilde{L}^2)$ where $\tilde{Q} = \tilde{Q}^\pi + \tilde{Q}^\nu$ and $\tilde{L} = \tilde{L}^\pi + \tilde{L}^\nu$.

A realistic pseudo-SU_3 Hamiltonian is used in the calculations:

$$H = \tilde{H}_{sp}^\pi + \tilde{H}_{sp}^\nu - \frac{1}{2}\chi\tilde{Q}\cdot\tilde{Q} - G_\pi \tilde{H}_{pair}^\pi - G_\nu \tilde{H}_{pair}^\nu + aJ^2 + bK_J^2 + a_3\tilde{C}_3 + a_{sym}\tilde{C}_3. \quad (1)$$

Strengths of the quadrupole-quadrupole ($\tilde{Q}\cdot\tilde{Q}$) and pairing interactions ($\tilde{H}_{pair}^\sigma$) are taken to be fixed, respectively, at values typical of those used by other authors, namely, $\chi = 35A^{5/3}$ MeV, $G_\pi = 21/A$ MeV and $G_\nu = 19/A$ MeV. The spherical single-particle terms in this expression have the form

$$\tilde{H}_{sp}^\sigma = \sum_{i_\sigma} (C_\sigma \tilde{l}_{i_\sigma} \cdot \tilde{s}_{i_\sigma} + D_\sigma \tilde{l}_{i_\sigma}^2). \quad (2)$$

Calculations are carried out with the single-particle spin-orbit ($\tilde{l}_\sigma \cdot \tilde{s}_\sigma$) and orbit-orbit ($\tilde{l}_\sigma^2$) interaction strengths fixed by systematics[5]. The four 'free' parameters a, b, a_3, a_{sym} are fixed by requiring a best fit to the low-energy spectra. (For odd-mass systems, the a_3 parameter was set to be zero because it has very little effect on the overall results.) No other parameters, except for effective charges in the definition of the E2 operator, enter into the theory; E2 and M1 transitions are not part of the fitting procedure.

3 Typical results for even-even systems

Two sets of results have been reported in the literature for even-even systems. The earliest were for the Gd isotopes[6]. The most recent, and hence most sophisticated in terms of the number of $SU(3)$ irreps included in the basis states and the nature of the Hamiltonian, are for the ^{160,162,164}Dy and ^{168}Er nuclei[7]. In what follows we review some of the most recent results.

3.1 Excitation spectra

The even-even Gd, Dy, and Er nuclei exhibit well-developed ground-state rotational bands as well as bands that are built on low-lying $K^\pi = 0^+, 2^+$ and even 4^+ states. Relative excitation energies for states with angular momentum 0^+ are determined mainly by the quadrupole-quadrupole interaction. The single-particle terms and the pairing interactions mix these states. The 0_2^+ states lie close to their experimental counterparts while the 0_3^+ states (not fit to the data) usually lie slightly above their experimental counterparts. Of the four 'free' parameters in the Hamiltonian, a is adjusted to reproduce the moment of inertia of the ground state band, a_3 is varied to yield a best fit to the energy of the second 0^+ state, a_{sym} is adjusted to give a best fit to the first 1^+ state, and b is determined by the band-head of the $K^\pi = 2^+$ band.

Fig.1(a) shows the calculated and experimental $K^\pi = 0^+$, $K^\pi = 2^+$ and the first and second excited $K^\pi = 0^+$ bands for ^{162}Dy[8]. For the first three bands, the calculated numbers are within 7% of the measured energies. The model predicts a continuation of the first excited $K^\pi = 0^+$ band with two additional states of angular momentum 6 and 8. The calculated second excited $K^\pi = 0^+$ band (not included in the fitting procedure) lies about 0.5MeV higher than the experimental one.

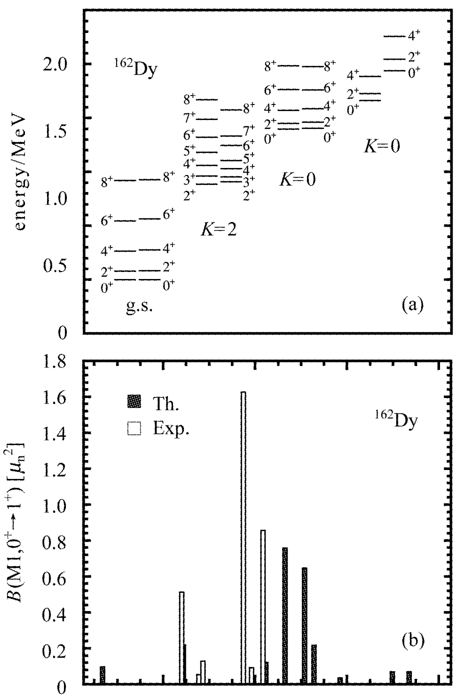

Fig.1. Energy spectra of ^{162}Dy obtained using Hamiltonian(1) is given on the left, insert (a). 'Exp.' represents the experimental results and 'Th.' the calculated ones. Insert (b) gives the theoretical and experimental M1 transition strengths from the $J^\pi = 0^+$ ground state to $J^\pi = 1^+$ states.

3.2 Electromagnetic transitions

Theoretical and experimental B(E2) transitions strengths between the states in the ground state band in ^{162}Dy are shown in Table 1[8]. The overall agreement between the calculated and experimental numbers is also reasonable. The $B(E2;\ 2_1 \rightarrow 4_1)$ is within 1% of the experimental value, and the last two calculated B(E2) values differ from the experimental values by less than 0.1 $e^2 b^2$ which is well within the experimental error. As with the excitation spectra, these results are typical of what one finds for other nuclei in this region. Contributions to the quadrupole moments from the nucleons in the unique parity orbitals are parameterized through an effective charge, e_f, with $e_\nu = e_f$, and $e_\pi = 1 + e_f$, so the E2 operator is given by: $E2 \rightarrow \tilde{Q}_\mu = e_\pi \tilde{Q}_\pi + e_\nu \tilde{Q}_\nu$[9]. Predictions were also made for the inter-band E2 transitions between members of the excited bands as well as for various intra-band E2 transitions.

Table 1. Experimental and theoretical B(E2) strengths for ^{162}Dy.

$J_i \rightarrow J_f$	$B(E2; J_i \rightarrow J_f)(e^2 b^2)$	
	exp.	theory
$0_1 \rightarrow 2_1$	5.134 ± 0.155	5.134
$2_1 \rightarrow 4_1$	2.675 ± 0.102	2.635
$4_1 \rightarrow 6_1$	2.236 ± 0.127	2.325
$6_1 \rightarrow 8_1$	2.341 ± 0.115	2.201

Another test of the theory is M1 transition strengths, mediated by the operator $M^1_\mu = \sqrt{\frac{3}{4\pi}}\mu_N \{ g^o_\pi L^\pi_\mu + g^s_\pi S^\pi_\mu + g^o_\nu L^\nu_\mu + g^s_\nu S^\nu_\mu \}$ where the $g^{o,s}_\alpha$ factors are the respective orbit (o) and spin(s) gyromagnetic ratios for protons ($\alpha = \pi$) and neutrons ($\alpha = \nu$). This is called the scissors mode because it can be pictured in lowest order as the rotation of the proton and neutron distributions relative to one another, like the opening and closing of the blades of a scissors[10,11]. A description of this mode within the framework of the interacting boson model (IBM) led to its detection in ^{156}Gd using high-resolution inelastic electron scattering techniques[12,13]. Systematic studies employing nuclear resonance fluorescence scattering measurements followed[14]. The non-observation of these low-energy M1 excitations in inelastic proton scattering has served to confirmed its orbital character[15,16]. Over the past two decades an impressive wealth of information about the scissors mode in even-even nuclei has been obtained and analyzed[17].

The pseudo-SU(3) model offers a very similar, but even richer interpretation of the scissors mode[18]. According to the Littlewood rules for coupling Young diagrams, the allowed product pseudo-SU(3) configuration can be expressed in mathematical terms by using three integers (m, l, k): $(\lambda_\pi, \mu_\pi) \otimes (\lambda_\nu, \mu_\nu) = \oplus_{m,l,k} (\lambda_\pi + \lambda_\nu - 2m + l, \mu_\pi + \mu_\nu - 2l + m)^k$, where the parameters l and m are defined in a fixed range given by the values of the initial SU_3 representations[19]. In this formulation, k serves to distinguish between multiple occurrences of equivalent (λ, μ) irreps in the tensor product. The number of k values is equal to the outer multiplicity, ρ_{max} $(k = 1, 2, \cdots, \rho_{max})$. The l and m labels can be identified with excitation quanta of a two-dimensional oscillator involving relative rotations (θ, the angle between the principal axes of the proton and neutron system, and ϕ, the angle between semi-axes of the proton and neutron system) of the proton-neutron system, $m = n_\theta$, $l = n_\phi$[20]. These correspond to two distinct type of 1^+ motion, the scissors and twist modes, and their realization in terms of the pseudo-SU(3) model. The pseudo-SU(3) irreps obtained from the tensor product that contain a $J^\pi = 1^+$ state are those corresponding to $(m, l, k) = (1,0,1), (0,1,1), (1,1,1)$, and $(1,1,2)$. A pure pseudo-SU(3) picture gives rise of a maximum of four 1^+ states that are associated with the scissors, twist, and double degenerate scissors-plus-twist modes [(1, 1,1) and (1,1,2)][20,21].

Results for the Dy isotopes, assuming a pure pseudo-SU_3 scheme, are given in Table 2.

Table 2. B(M1) transition strengths [μ_N^2] in the pure symmetry limit of the pseudo SU_3 model.

nucleus	[(λ_π, μ_π)	(λ_π, μ_π)	$(\lambda, \mu)]_{gs}$	$(\lambda, \mu)_1^+$	B(M1)	mode
$^{160-162}$Dy	(10,4)	(18,4)	(28,8)	(29,6)	0.56	t
				(26,9)	1.77	s
				$(27,7)^1$	1.82	s + t
				$(27,7)^2$	0.083	t + s
^{164}Dy	(10,4)	(20,4)	(30,8)	(31,6)	0.56	t
				(28,9)	1.83	s
				(29,7)	1.88	s + t
				(29,7)	0.09	t + s

The strong coupled pseudo-SU_3 irrep $(\lambda, \mu)_{gs}$ for the ground state is given with its proton and neutron sub-irreps and the irreps associated with the 1^+ states, $(\lambda', \mu')_1^+$. In addition, each transition is labeled as a scissors (s) or twist (t) or combination mode.

The experimental results shown in Fig.1(b) suggest a much larger number of 1^+ states with non-zero B(M1) transition strengths from the 0^+ ground state[8]. The SU(3) breaking residual interactions lead to a fragmentation in the M1 strength distribution, since the ground state 0^+ is then a combination of several SU(3) irreps, each with allowed M1 transitions to other SU(3) irreps. For ^{162}Dy the summed M1 strength, 4.24 μ_N^2 and 2.29 μ_N^2, respectively, for the pure

$SU(3)$ limit of theory and as determined using Hamiltonian (1), is in reasonable agreement with the experimental number ($3.29\ \mu_N^2$). Results for ^{160}Dy are similar to those for ^{162}Dy but the summed strength for ^{164}Dy is less than reported. However, since the latter exceeds the pure $SU(3)$ sumrule limit, it could either be a bad number or an indication that spin mixing is not being taken into account properly. In any case, it is important to note that unlike the $E2 \to \tilde{Q}$ case, real, not effective, gyromagnetic ratios are used to define the M1 operator.

4 Conclusions

The results for ^{162}Dy illustrate the current level of applicability of the pseudo-$SU(3)$ shell model. First and foremost, one can say that it can be used to reproduce excitation spectra: in addition to the ground-state ($K^\pi = 0_1^+$) and gamma ($K^\pi = 2_\gamma^+ = 2_1^+$) bands in even-even systems, it offers a reasonable description of the first two exited 0^+ bands ($K^\pi = 0_{2,3}^+$) as well as the $K^\pi = 4_1^+$ band. Other studies show that it can also be used to reproduce several (for example, seven for ^{163}Dy) bands in odd-mass systems[22,23,25—27].

The theory, which now accomodates realistic one-body terms (single-particle energies) and two-body interactions (for example, pairing), also allows one to calculate E2 and M1 transition strengths. Agreement with existing experimental numbers is reasonable for both inter-and intra-band transitions. In the case of the E2 operator, effective charges are required because the theory as currently employed still does not include its logical symplectic [$SU(3) \to Sp(3, R)$] extension and the so-called opposite parity intruder states are relegated to a renormalization role. Work to build both of these aspects into the theory is underway.

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形变核中的 E2 和 M1 强度:赝-SU(3)模型的应用*

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摘要　通过赝-SU(3)壳模型对^{162}Dy 核的描述,揭示了原子核的复杂多体行为能通过简单的几何图像来解释.

关键词　赝-SU(3)模型　低激发能级　电磁跃迁

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