Calculation of Rare Decay $B^+ \to D_s^+ \bar{K}^{*0}$ in Perturbative QCD Approach*

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Abstract The rare decay $B^+ \to D_s^+ \bar{K}^{*0}$ can occur only via annihilation diagrams in the Standard Model. We calculate the branching ratio in perturbative QCD approach based on k_T factorization theorem. We find that the branching ratio of this decay is about of order 10^{-8}, which may be sensitive to new physics.

Key words perturbative QCD, annihilation rare decay, wave function

1 Introduction

Rare B decays are very important in particle physics, because they are important windows in testing the standard model (SM) and they are sensitive to new physics. As a rather simple method, factorization approach is accepted, because it explained many decay branching ratios successfully[1]. Recently, some efforts have been made to improve its theoretical application. One of these methods is the perturbative QCD approach (PQCD). Many rare decay branching ratios such as $B \to K\pi$[2], $B \to \pi\pi$[3], $B \to \pi\rho$[4] were predicted, using this approach.

In recent calculations, $B \to D_s^{(*)} K^{(*)+}$, $B^+ \to D_s^{(*)+} \bar{K}^0$ have been analyzed in the PQCD approach[5—7], while $B^+ \to D_s^+ \bar{K}^{*0}$ is not calculated. In decay $B^+ \to D_s^+ \bar{K}^{*0}$, none of quarks in the final states is the same as those quarks in the B meson. This decay is a pure annihilation type decay, which is described as B meson annihilating into vacuum and D_s^+, $\bar{K}^{*0}$ being produced from vacuum afterwards. Detailed information about PQCD picture can be got in Ref.[8].

In PQCD, the decay amplitude is usually separated into soft (Φ), hard (H) and harder (C) dynamics by different scales. the factorization theorem allows us to write the decay amplitude as convolution,

$$\text{Amplitude} \sim \int d^4k_1 d^4k_2 d^4k_3 \; Tr[\,C(t)\Phi_B(k_1)\cdot \Phi_{D_s}(k_2)\Phi_{K^*}(k_3)H(k_1,k_2,k_3,t)\,]. \quad (1)$$

In Eq.(1), $k_i\,(i=1,2,3)$ are momenta of light quarks in each meson. $C(t)$ is Wilson coefficient which comes from the QCD radiative corrections to the four quark operators. Φ_M is the wave function which describes the inner information of meson M. H describes the four-quark operator and the quark pair from the sea connected by a hard gluon whose scale is at the order of M_B, so H can be perturbatively calculated. The hard part H is channel dependent, while Φ_M is independent of the specific processes.

Some analytic formulas for the decay amplitudes of $B^+ \to D_s^+ \bar{K}^{*0}$ decays will be given in the next section. In section 3, we give the numerical results and discussion. Finally, we conclude this study in section 4.

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2 $B^+ \to D_s^+ \bar{K}^{*0}$ amplitudes

For simplicity, we consider B meson at rest. In the light-cone coordinate, the B meson momentum P_1, D_s^+ momentum P_2 and $\bar{K}^{*0}$ momentum P_3 are taken to be:

$$P_1 = \frac{M_B}{\sqrt{2}}(1,1,0_T),\ P_2 = \frac{M_B}{\sqrt{2}}(1 - r_3^2, r_2^2, 0_T),$$

$$P_3 = \frac{M_B}{\sqrt{2}}(r_3^2, 1 - r_2^2, 0_T), \qquad (2)$$

where $r_2 = M_{D_s^+}/M_B$ and $r_3 = M_{K^{*0}}/M_B$. Denoting the light (anti-) quark momenta in B^+, D_s^+ and $\bar{K}^{*0}$ mesons as k_1, k_2, and k_3, respectively, we can choose

$$k_1 = (x_1 P_1^+, 0, k_{1T}),\ k_2 \quad (x_2 P_2^+, 0, k_{2T}),$$

$$k_3 = (0, x_3 P_3^-, k_{3T}). \qquad (3)$$

The $\bar{K}^{*0}$ meson's longitudinal polarization vector ε and transverse polarization vector ε_T are given by:

$$\varepsilon_L = \frac{M_B}{\sqrt{2} M_{\bar{K}^{*0}}}(-r_3^2, 1 - r_2^2, 0_T),$$

$$\varepsilon_T = \frac{M_B}{\sqrt{2} M_{K^{*0}}}(0,0,1_T). \qquad (4)$$

Then, integration over k_1^-, k_2^-, and k_3^+ in Eq. (1) leads to:

$$\text{Amplitude} \sim \int dx_1 dx_2 dx_3 b_1 db_1 b_2 db_2 b_3 db_3 \times$$
$$Tr[C(t) \Phi_B(x_1, b_1) \Phi_{D_s}(x_2, b_2, \varepsilon) \times$$
$$\Phi_{K^*}(x_3, b_3) H(x_i, b_i, \varepsilon, t) \times$$
$$S_t(x_i) e^{-S(t)}], \qquad (5)$$

where b_i is the conjugate space coordinate of k_{iT}, and t is the largest energy scale in H, as the function in terms of x_i and b_i. The large logarithms ($\ln m_W/t$) coming from QCD radiative corrections to four quark operators are included in the Wilson coefficients $C(t)$. The large double logarithms ($\ln^2 x_i$) on the longitudinal direction are summed by the threshold resummation[9], and they lead to a jet function $S_t(x_i)$ which smears the end-point singularities on x_i. The last factor, $e^{-S(t)}$, contains two kinds of logarithms. One of the large logarithms is due to the renormalization of ultra-violet divergence $\ln tb$, the other is double logarithm $\ln^2 b$ from the overlap of collinear and soft gluon corrections. This Sudakov form factor suppresses the soft dynamics effectively[10]. Thus it makes perturbative calculation of the hard part H reliable.

The heavy B and D_s meson wave functions are restricted by heavy quark symmetry. In the heavy quark limit, we may use only one independent distribution amplitude for each of them[11].

$$\Phi_M(x,b) = \frac{i}{\sqrt{6}}[(\not{P}\gamma_5) \quad M\gamma_5]\phi_M(x,b), \quad (6)$$

where $M = B$, D_s. For the light K^* meson, only the longitudinal wave function for outgoing state is relevant, which is written as:

$$\Phi_{K^*}(x_3, b_3) = \frac{i}{\sqrt{6}}[M_{K^*} \not{\epsilon}_L \phi_{K^*}(x_3, b_3) +$$
$$\not{\epsilon}_L \not{P}_3 \phi_{K^*}^t(x_3, b_3) +$$
$$M_{K^*} I \phi_{K^*}^s(x_3, b_3)]. \qquad (7)$$

Unlike the heavy mesons, there are three different distribution amplitudes.

In the decay $B^+ \to D_s^+ \bar{K}^{*0}$, the effective Hamiltonian at the scale lower than M_W is the same as $B^+ \to D_s^+ \bar{K}^0$ decay

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{ub}^* V_{cd}[C_1(\mu) O_1(\mu) + C_2(\mu) O_2(\mu)], \qquad (8)$$

$$O_1 = (b\gamma_\mu P_L d)(\bar{c}\gamma^\mu P_L u),$$

$$O_2 = (\bar{b}\gamma_\mu P_L u)(\bar{c}\gamma^\mu P_L d), \qquad (9)$$

where the projection operator is defined by $P_L = (1 - \gamma_5)/2$. $V_{ub}^* V_{cd}$ are the products of the CKM matrix elements, and $C_{1,2}(\mu)$ are the Wilson coefficients. Accord-

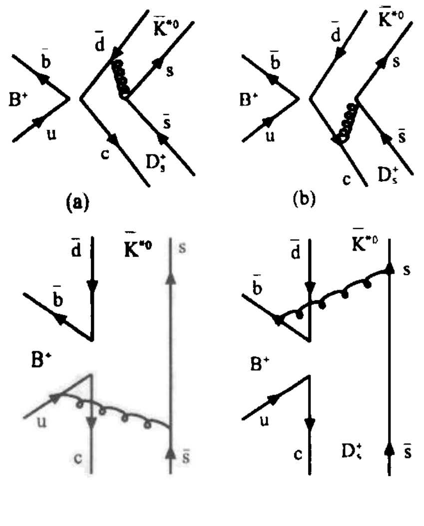

Fig. 1.
factorizable

ing to the effective Hamiltonian, the lowest order diagrams of $B^+ \to D_s^+ \bar{K}^{*0}$ are drawn in Fig. 1. As stated above, this decay only has annihilation diagrams.

After perturbative QCD calculations, we get the factorizable contribution F_a, and the nonfactorizable contribution M_a illustrated by Fig. 1(a)(b) and Fig. 1 (c)(d), respectively.

$$\begin{aligned} F_a = & 16\pi C_F M_B^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \phi_{D_s}(x_2, b_2) \times \\ & [\{(1 - x_3 - 3r_2^2 + 2x_3 r_2^2)\phi_{K^*}(x_3, b_3) + \\ & r_2(1 - 2x_3) r_3 \phi_{K^*}^t(x_3, b_3) - \\ & r_2(3 - 2x_3) r_3 \phi_{K^*}^s(x_3, b_3)\} E_f(t_a^1) \times \\ & h_a(x_2, x_3, b_2, b_3) - \{(1 - r_2^2) x_2 \phi_{K^*}(x_3, b_3) - \\ & 2r_2(1 + x_2) r_3 \phi_{K^*}^s(x_3, b_3)\} E_f(t_a^2) \times \\ & h_a(1 - x_3, 1 - x_2, b_3, b_2)], \end{aligned} \quad (10)$$

$$\begin{aligned} M_a = & \frac{1}{\sqrt{2N_c}} 64\pi C_F M_B^2 \int_0^1 dx_1 dx_2 dx_3 \times \\ & \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \phi_{D_s}(x_2, b_2) \times \\ & [\{x_2(1 - 2r_2^2)\phi_{K^*}(x_3, b_2) + \\ & r_2(x_2 + x_3 - 1) r_3 \phi_{K^*}^t(x_3, b_2) + \\ & r_2(x_2 - x_3 + 1) r_3 \phi_{K^*}^s(x_3, b_2)\} \times \\ & E'_m(t_m^1) h_a^{(1)}(x_1, x_2, x_3, b_1, b_2) - \\ & \{(-(x_2 + 2x_3 - 2) r_2^2 + x_3)\phi_{K^*}(x_3, b_2) + \\ & r_2(1 - x_2 - x_3) r_3 \phi_{K^*}^t(x_3, b_2) + \\ & r_2(3 + x_2 - x_3) r_3 \phi_{K^*}^s(x_3, b_2)\} \times \\ & E'_m(t_m^2) h_a^{(2)}(x_1, x_2, x_3, b_1, b_2)]. \end{aligned} \quad (11)$$

In our work, r_3^2, r_2^4 and x_1 in numerators are neglected. In Eqs. (10), (11), $C_F = 4/3$ is the group factor of $SU(3)_c$ gauge group, and the functions E_f, $t_a^{1,2}$, h_a are given in the appendix of Ref. [5].

Comparing with the previously calculated $B^0 \to D_s^{(*)-} \bar{K}^+$ decay[5], we find that the leading twist contribution,

ours. However the subleading twist contribution, which is proportional to $r_2 r_3$ in Eqs. (10) and (11), is quite different.

The total decay width for $B^+ \to D_s^+ \bar{K}^{*0}$ decay is given as

$$\Gamma(B^+ \to D_s^+ \bar{K}^{*0}) = \frac{G_F^2 M_B^3}{128\pi}(1 - r_2^2) \times |V_{ub}^* V_{cd}(f_B F_a + M_a)|^2. \quad (12)$$

The decay width for CP conjugated mode, $B^- \to D_s^- \bar{K}^{*0}$, is the same value as $B^+ \to D_s^+ \bar{K}^{*0}$, just replacing $V_{ub}^* V_{cd}$ by $V_{ub} V_{cd}^*$. Since there is only one kind of CKM phase involved in this decay, there is no CP violation in the standard model.

3 Numerical Results

We use the same B and D_s meson wave functions as before[5]

$$\phi_B(x, b) = N_B x^2(1 - x)^2 \exp\left[-\frac{M_B^2 x^2}{2\omega_b^2} - \frac{1}{2}(\omega_b b)^2\right],$$

$$\phi_{D_s}(x, b) = \frac{3}{\sqrt{2N_c}} f_{D_s} x(1 - x) \times \{1 + a_{D_s}(1 - 2x)\} \exp\left[-\frac{1}{2}(\omega_{D_s} b)^2\right].$$

The K^* meson's distribution amplitudes are given by light cone QCD sum rules[12]:

$$\begin{aligned} \phi_{K^*}(x) = & \frac{f_{K^*}}{2\sqrt{2N_c}} 6x(1 - x)\{1 + 0.57 \times \\ & (1 - 2x) + 0.07 \cdot C_2^{3/2}(1 - 2x)\}, \\ \phi_{K^*}^t(x) = & \frac{f_{K^*}^T}{2\sqrt{2N_c}}\{0.3(1 - 2x)[3(1 - 2x)^2 \\ & 10(1 - 2x) - 1] + 1.68 C_4^{1/2}(1 - 2x) + \\ & 0.06(1 - 2x)^2[5(1 - 2x)^2 - 3] + \\ & 0.36[1 - 2(1 - 2x)(1 + \ln(1 - x))]\}, \\ \phi_{K^*}^s(x) = & \frac{f_{K^*}^T}{2\sqrt{2N_c}}\{3(1 - 2x)[1 + 0.2(1 - 2x) + \\ & 0.6(10x^2 - 10x + 1)] - 0.12x(1 - x) + \\ & 0.36[1 - 6x - 2\ln(1 - x)]\}, \end{aligned} \quad (17)$$

with the Gegenbauer polynomials,

$$C_2^{1/2}(\xi) = \frac{1}{2}(3\xi^2 - 1),$$

$$C_4^{1/2}(\xi) = \frac{1}{8}(35\xi^4 - 30\xi^2 + 3). \quad (18)$$

In addition, we use the following input parameters, for meson decay constants and the CKM matrix elements and the lifetime of B^+[13],

$$f_B = 190\text{MeV}, f_{D_s} = 220\text{MeV}, f_{K^*}^{(T)} = 200\text{MeV}, \quad (19)$$

$$|V_{ub}| = 3.6 \times 10^{-3}, \ |V_{cd}| = 0.224, \quad \tau_{B^+} = 1.67 \times 10^{-12} \mathrm{s}. \tag{20}$$

The branching ratio obtained from the analytic formulas may be sensitive to many parameters especially those in the meson wave functions[5,6]. Similar to the $B \to D_S^{(*)} K$ decays[5,6], we found that the branching ratio is not sensitive to the variety of the parametevs in the K^* meson wave function. But they are sensitive to the heavy B and D_s meson wave function parameters. For illustration of the uncertainties of the branching ratios, we choose the following B and D_s meson wave function parameters

$$0.35\mathrm{GeV} \leqslant \omega_b \leqslant 0.45\mathrm{GeV}, \tag{21}$$

$$0.21\mathrm{GeV} \leqslant \omega_{D_s} \leqslant 0.30\mathrm{GeV}, \tag{22}$$

$$0.21\mathrm{GeV} \leqslant a_{D_s} \leqslant 0.30\mathrm{GeV}. \tag{23}$$

In above parameter range, the branching ratio normalized by the decay constants and the CKM matrix elements is:

$$Br(B^+ \to D_s^+ K^{*0}) = (1.5 \pm 0.3) \times 10^{-8} \times \left(\frac{f_B f_{D_s}}{190\mathrm{MeV} \cdot 220\mathrm{MeV}}\right)^2 \left(\frac{|V_{ub}^* V_{cd}|}{0.0036 \cdot 0.224}\right)^2. \tag{24}$$

Considering the uncertainty of f_B, f_{D_s} and the CKM matrix elements, the branching ratio of the $B^+ \to D_s^+ \bar{K}^{*0}$ decay is at the order of 10^{-8}. This is still far from the current experimental upper limit[13],

$$Br(B^+ \to D_s^+ \bar{K}^{*0}) < 5 \times 10^{-4}. \tag{25}$$

Table 1. Decay branching ratios calculated in PQCD approach using the same parameters for B and D_s wave functions, and $f_{D_s} = f_{D_s^*} = 220$MeV.

Decay channel	$D_s^+ \bar{K}^0$	$D_s^{*+} K^0$	$D_s^+ \bar{K}^{*0}$
$Br(10^{-8})$	1.48 ± 0.24	2.48 ± 0.42	1.52 ± 0.27

For comparison, we list the branching ratios of $B^+ \to D_s^{(*)+} K^0$ and $B^+ \to D_s^+ \bar{K}^{*0}$ calculated in PQCD approach, using the same parameters for B and $D_s^{(*)}$ wave functions. For simplicity, we set $f_{D_s} = f_{D_s^*} = 220$MeV. From the above result, we can see that the branching ratio of $B^+ \to D_s^+ K^{*0}$ is a little bit larger than that of $B^+ \to D_s^+ K^0$[5]. From equation (10, 11) and formulas in Ref.[5], we find the coefficients of sub-leading twist become negative, but they are all proportional to $r_2 r_3$ which is a little bit small. Because of the suppression of CKM matrix elements, the branching ratio of $B^+ \to D_s^+ \bar{K}^{*0}$ is much smaller than that of the neutral decay $B^0 \to D_s^- \bar{K}^{*+}$.

Since pure annihilation type B decays are suppressed compared to spectator diagram decays, the soft final state interactions may be important[14]. In our case, B^+ meson can decay into D^{*0} and π^+, which then scatter into D_s^+, $\bar{K}^{*0}$ through final state interaction by exchanging a K^+ meson. This picture is depicted in Fig.2, which is difficult to calculate accurately, since final state interaction is purely non-perturbative[14]. In Ref.[5], the results from PQCD approach for $B^0 \to D_s^- K^+$ decay were consistent with the experiment, which shows that the soft final state interaction may not be important.

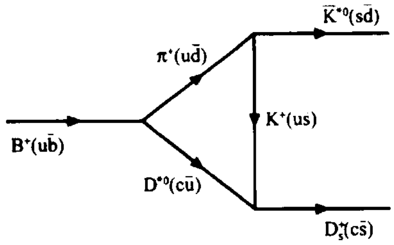

Fig.2. dominant diagram for final state interaction.

4 Conclusion

In hadronic two-body B meson decays, the energy release is larger than 1GeV. The final state mesons move very fast. Therefore the soft final state interaction may not be important in the two-body B meson decays. The PQCD approach based on k_T factorization theorem is applicable to the calculation of B meson decays.

In this work, we calculate the $B^+ \to D_s^+ \bar{K}^{*0}$ decay in the PQCD approach. Since neither of quarks in B^+ meson appear in the final state mesons, this process occurs only via annihilation type diagrams. We find that, the branching ratio of $B^+ \to D_s^+ \bar{K}^{*0}$ is small in SM, which is around 10^{-8}. This branching ratio will be measured in the LHCb in future. This branching ratio predicted in the SM is small. Therefore, one has to measure it carefully to find the new physics contribution.

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微扰QCD方法计算稀有衰变 $B^+\to D_s^+\bar{K}^{*0}$ *

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摘要 在标准模型中,稀有衰变道 $B^+\to D_s^+\bar{K}^{*0}$ 只有通过纯湮没图才可以发生. 这样这个衰变道的分支比很小. 利用基于 k_T 因子化的微扰方法给出分支比的预测,发现它在 10^{-8} 的量级上. 这个衰变道估计在将来的LHC上得到测量,对检验标准模型以及探寻新物理有着重要的意义.

关键词 微扰QCD 湮没图 稀有衰变 波函数

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