第24卷 第2期 高能物理与核物理 Vol.24, No.2
2000年2月 HIGH ENERGY PHYSICS AND NUCLEAR PHYSICS Feb.,2000

高阶微商规范不变系统的整体对称性和量子守恒律

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摘要 分别从 Faddeev - Popov (FP)和 Faddeev - Senjanovic (FS)路径积分量子化方法对高阶微商规范不变系统导致的位形空间和相空间生成泛函出发，导出规范系统在量子水平下的守恒律，用于高阶 Maxwell 非 Abel Chern - Simons (CS)理论．得到了高阶 Maxwell 非 Abel CS 理论与标量场耦合系统的量子 BRS 守恒荷和量子守恒角动量，无论从位形空间或相空间的生成泛函出发，其结果是相同的．并对 CS 理论中的分数自旋性质给予了讨论．

关键词 路径积分 对称性和守恒律 规范场 Chern - Simons (CS)理论

1 引言

对称性和守恒律的联系在经典理论中是由 Noether 定理给出的．Noether 定理及其推广，传统的讨论是用位形空间中的变量来表述的．最近正则形式的 Noether 定理已建立[1,2]．由高阶微商描述的动力系统一直被广泛研究[2,3]．拉氏量中含高阶微商项，在量子理论中它能改善费曼图的收敛性．而含 CS 项的系统一直是人们的研究兴趣[4]，特别是 2+1 维的 CS 理论提供一个描述分数自旋和分数统计的理论，它能被用来研究分数量子 Hall 效应[5]．这里我们来研究高阶微商规范不变系统的量子守恒律并将其应用于含高阶微商的 Maxwell 非 Abel Chern - Simons 理论．

在路径积分量子化理论中，出现的是经典的数，这对研究系统的量子对称性质提供了方便．从路径积分导出量子守恒律，通常是由位形空间变量来表述的．它仅适用于相空间路径积分中关于正则动量的积分为 Gauss 型的情形．当系统的约束结构比较复杂时，要作出相空间路径积分中关于动量的积分是十分困难的．甚至是不可能的．因此，从相空间路径积分的量子化方案出发来研究系统的量子正则对称性，就具有更基本的意义．本文第二部分和第三部分分别从位形空间和相空间的生成泛函出发，导出含高阶微商规范不变系统的整体对称性下的守恒律，第四部分将上述结果用于高阶Maxwell非AbelCS

1998 - 12 - 14 收稿

间来分析,其结果是相同的. 这表明 FP 方法对此模型适用. 此时的量子守恒量均需计及鬼粒子的贡献. 最后对理论中的分数自旋性质作了讨论.

2　位形空间中的整体对称性和量子守恒律

设高阶微商规范不变系统的拉氏量为 $\mathscr{L}=\mathscr{L}(x,\psi^a,\psi^a_{,\mu},\cdots\psi^a_{,\mu(m)}\cdots\psi^a_{,\mu(N)})$, ψ^a 的最高阶为 N(本文采用文献[6]的记号),由 Faddeev－Popov 方法,得系统的 Green 函数的生成泛函为

$$Z[J]=\int\mathscr{D}\phi^a\exp\left\{\mathrm{i}\int\mathrm{d}^4x(\mathscr{L}_{\mathrm{eff}}+J_a\phi^a)\right\},\tag{1}$$

其中 ϕ^a 代表所有场量, $\mathscr{L}_{\mathrm{eff}}=\mathscr{L}+\mathscr{L}_{\mathrm{g}}+\mathscr{L}_{\mathrm{gh}}$, $\mathscr{L}_{\mathrm{g}}$ 是与规范条件有关的规范固定项, $\mathscr{L}_{\mathrm{gh}}$ 是鬼粒子项(规范补偿项). 考虑位形空间无穷小整体变换:

$$x^{\mu'}=x^\mu+\Delta x^\mu=x^\mu+\varepsilon_\sigma\tau^{\mu\sigma}(x,\cdots\phi_{,\mu(m)}\cdots),$$
$$\phi^{a'}=\phi^a+\Delta\phi^a=\phi^a+\varepsilon_\sigma\xi^{a\sigma}(x,\cdots\phi_{,\mu(m)}\cdots).$$

假设有效作用量 I_{eff} 在(2)式变换下不变,将(2)式定域化,考虑如下的变换

$$\begin{aligned}x^{\mu'}&=x^\mu+\Delta x^\mu=x^\mu+\varepsilon_\sigma(x)\tau^{\mu\sigma}(x,\cdots\phi_{,\mu(m)}\cdots),\\ \phi^{a'}&=\phi^a+\Delta\phi^a=\phi^a+\varepsilon_\sigma(x)\xi^{a\sigma}(x,\cdots\phi_{,\mu(m)}\cdots).\end{aligned}\tag{3}$$

其中 $\varepsilon_\sigma(x)$ 为无穷小任意函数,它们及其各级微商在时空区域边界上为零. 在(3)式变换下有效作用量的变分为[6]

$$\begin{aligned}\Delta I_{\mathrm{eff}}=&\int\mathrm{d}^4x\varepsilon_\sigma(x)\left\{\frac{\delta I_{\mathrm{eff}}}{\delta\phi^a}(\xi^{a\sigma}-\phi^a_{,\mu}\tau^{\mu\sigma})+\partial_\mu\left[\mathscr{L}_{\mathrm{eff}}\tau^{\mu\sigma}+\sum_{n=0}^{N-1}\Pi_a^{\mu\nu(n)}\partial_{\nu(n)}(\xi^{a\sigma}-\phi^a_{,\rho}\tau^{\rho\sigma})\right]\right\}+\\&\int\mathrm{d}^4x\left\{\left[\mathscr{L}_{\mathrm{eff}}\tau^{\mu\sigma}+\sum_{n=0}^{N-1}\Pi_a^{\mu\nu(n)}\partial_{\nu(n)}(\xi^{a\sigma}-\phi^a_{,\mu}\tau^{\mu\sigma})\right]\partial_\mu\varepsilon_\sigma(x)\right\}.\end{aligned}\tag{4}$$

由于假设有效作用量在(2)式不变. 所以第一个积分为零. 根据 $\varepsilon_\sigma(x)$ 的边界条件,(4)式可写为

$$\Delta I_{\mathrm{eff}}=-\int\mathrm{d}^4x\varepsilon_\sigma(x)\partial_\mu\left[\mathscr{L}_{\mathrm{eff}}\tau^{\mu\sigma}+\sum_{n=0}^{N-1}\Pi_a^{\mu\nu(n)}\partial_{\nu(n)}(\xi^{a\sigma}-\phi^a_{,\rho}\tau^{\rho\sigma})\right].\tag{5}$$

设变换(3)式的 Jacobi 行列式为 1,由于生成泛函在(3)式变换下不变,根据 $\varepsilon_\sigma(x)$ 的边界条件,有

$$\begin{aligned}Z[J]=&\int\mathscr{D}\phi^a\left\{1-\mathrm{i}\int\mathrm{d}^4x\varepsilon_\sigma(x)\partial_\mu\left[\mathscr{L}_{\mathrm{eff}}\tau^{\mu\sigma}+\sum_{n=0}^{N-1}\Pi_a^{\mu\nu(n)}\partial_{\nu(n)}(\xi^{a\sigma}-\phi^a_{,\rho}\tau^{\rho\sigma})\right]+\right.\\&\left.\mathrm{i}\int\mathrm{d}^4xJ_a\varepsilon_\sigma(x)(\xi^{a\sigma}-\phi^a_{,\mu}\tau^{\mu\sigma})\right\}\cdot\exp\left\{\mathrm{i}I_{\mathrm{eff}}+\mathrm{i}\int\mathrm{d}^4xJ_a\phi^a\right\},\end{aligned}\tag{6}$$

由将(6)式对 $\varepsilon_\sigma(x)$ 求泛函微商,得

$$\begin{aligned}&\int\mathscr{D}\phi^a\left\{\partial_\mu\left[\mathscr{L}_{\mathrm{eff}}\tau^{\mu\sigma}+\sum_{n=0}^{N-1}\Pi_a^{\mu\nu(n)}\partial_{\nu(n)}(\xi^{a\sigma}-\phi^a_{,\rho}\tau^{\rho\sigma})\right]-J_a(\xi^{a\sigma}-\phi^a_{,\mu}\tau^{\mu\sigma})\right\}\times\\&\exp\left\{\mathrm{i}I_{\mathrm{eff}}+\mathrm{i}\int\mathrm{d}^4xJ_a\phi^a\right\}=0.\end{aligned}\tag{7}$$

将(7)式关于$J(x_j)$求n次泛函微商,可得

$$\int \mathscr{D}\phi^a\Big(\Big\{\partial_\mu\Big[\mathscr{L}_{\rm eff}\tau^{\mu\sigma}+\sum_{n=0}^{N-1}\Pi_a^{\mu\nu(n)}\partial_{\nu(n)}(\xi^{a\sigma}-\phi^a_{,\rho}\tau^{\rho\sigma})\Big]-J_a(\xi^{a\sigma}-\phi^a_{,\mu}\tau^{\mu\sigma})\Big\}\cdot\phi^a(x_1)\cdots\phi^a(x_n)+(-\mathrm{i})\sum_j\phi^a(x_1)\cdots\phi^a(x_{j-1})\phi^a(x_{j+1})\cdots\phi^a(x_n)N^\sigma\delta(x-x_j)\Big)\cdot\exp\Big\{\mathrm{i}I_{\rm eff}+\mathrm{i}\int \mathrm{d}^4xJ_a\phi^a\Big\}=0, \tag{8}$$

其中$N^\sigma=-(\xi^{a\sigma}-\phi^a_{,\mu}\tau^{\mu\sigma})$,让(8)式中外源$J_a=0$,得

$$\langle 0|\mathrm{T}^*\Big\{\partial_\mu\Big[\mathscr{L}_{\rm eff}\tau^{\mu\sigma}+\sum_{n=0}^{N-1}\Pi_a^{\mu\nu(n)}\partial_{\nu(n)}(\xi^{a\sigma}-\phi^a_{,\rho}\tau^{\rho\sigma})\Big]\Big\}\cdot\phi^a(x_1)\cdots\phi^a(x_n)|0\rangle = \mathrm{i}\sum_j\langle 0|\mathrm{T}^*[\phi^a(x_1)\cdots\phi^a(x_{j-1})\phi^a(x_{j+1})\cdots\phi^a(x_n)N^\sigma|0\rangle\cdot\delta(x-x_j). \tag{9}$$

固定t,让$t_1\cdots t_m\to+\infty, t_{m+1}\cdots t_n\to-\infty$. 可得[7]

$$\langle \mathrm{out}, m|\Big\{\partial_\mu\Big[\mathscr{L}_{\rm eff}\tau^{\mu\sigma}+\sum_{n=0}^{N-1}\Pi_a^{\mu\nu(n)}\partial_{\nu(n)}(\xi^{a\sigma}-\phi^a_{,\rho}\tau^{\rho\sigma})\Big]\Big\}|n-m,\mathrm{in}\rangle=0, \tag{10}$$

m和n任意,因而有

$$\partial_\mu\Big[\mathscr{L}_{\rm eff}\tau^{\mu\sigma}+\sum_{n=0}^{N-1}\Pi_a^{\mu\nu(n)}\partial_{\nu(n)}(\xi^{a\sigma}-\phi^a_{,\rho}\tau^{\rho\sigma})\Big]=0. \tag{11}$$

对(11)式在场所在的区域积分,利用Gauss定理及场在区域的边界上为零的性质,可得守恒量(算符)为

$$Q^\sigma=\int \mathrm{d}^3x\Big[\mathscr{L}_{\rm eff}\tau^{0\sigma}+\sum_{n=0}^{N-1}\Pi^{o\nu(n)}\partial_{\nu(n)}(\xi^{a\sigma}-\phi^a_{,\rho}\tau^{\rho\sigma})\Big]. \tag{12}$$

(12)式与经典Noether定理的区别在于,在经典情形,整体变换是保持原始作用量$I=\int\mathscr{L}\mathrm{d}^4x$不变,这里在(2)式变换下,有效拉氏量$I_{\rm eff}$不变. 因此,(12)式中出现的是$\mathscr{L}_{\rm eff}$而不是$\mathscr{L}$. 此外,在量子水平上,变换的行列式必须为1,才能保证守恒量(12)式存在. 可见,经典理论中对称性与守恒量的联系,在量子理论中不一定再保持.

3 相空间中的整体对称性和量子守恒律

规范不变的系统,其相空间描述必含约束[6],此时,相空间Green函数的生成泛函为[8]

$$Z[J]=\int\mathscr{D}\psi^a_{(s)}\mathscr{D}\pi_a^{(s)}\mathscr{D}\lambda_m\mathscr{D}C_l\mathscr{D}\bar{C}_1\cdot\exp\Big\{\mathrm{i}\Big(I^{\rm p}_{\rm eff}+\int \mathrm{d}^4xJ_a^{(s)}\psi^a_{(s)}\Big)\Big\}, \tag{13}$$

其中$I^{\rm p}_{\rm eff}=\int\mathrm{d}^4x\mathscr{L}^{\rm p}_{eff}=\int\mathrm{d}^4x\Big[\pi_a^{(s)}\dot\psi^a_{(s+1)}-\mathscr{H}_c+\lambda_m\Phi_m+\frac{1}{2}\int\mathrm{d}^4x\bar{C}_k\{\Phi_k(x),\Phi_l(y)\}C_1(y)\Big]$,$\mathscr{H}_c$是关于场$\psi^a(x)$的正则哈氏量密度. $\{\Phi_m\}$是所有约束(对仅含第二类约束的系统)或约束和规范条件的总体(对含第一类约束的系统),$\lambda_m(x)$是拉氏乘子. 为了简便,让$\phi_{(s)}=(\psi^a_{(s)},\lambda_m,C_l,\bar{C}_k)$,$J^{(s)}=(J_a^{(s)},\eta_m,\xi_k,\bar{\xi}_l)$,$\eta_m,\xi_k,\bar{\xi}_l$分别是$\lambda_m,\bar{C}_k,C_l$的外源,$\pi_{(s)}=(\pi^a_{(s)})$. 则上式为

$$Z[J]=\int\mathscr{D}\phi_{(s)}\mathscr{D}\pi^{(s)}\exp\Big\{\mathrm{i}\Big(I^{\rm p}_{\rm eff}+\int\mathrm{d}^4xJ^{(s)}\phi_{(s)}\Big)\Big\}, \tag{14}$$

动力系统的正则整体对称性在经典的情况下产生守恒律[2] 现在让我们考虑一个和整体变换相联系的定域变换

$$
\begin{aligned}
x^{\mu'} &= x^\mu + \Delta x^\mu = x^\mu + \varepsilon_\sigma(x)\tau^{\mu\sigma}(x,\phi_{(s)},\pi^{(s)}),\\
\phi'_{(s)}(x') &= \phi_{(s)}(x) + \Delta\phi_{(s)} = \phi_{(s)}(x) + \varepsilon_\sigma(x)\xi^\sigma_{(s)}(x,\phi_{(s)},\pi^{(s)}),\\
\pi^{(s)'}(x') &= \pi^{(s)} + \Delta\pi^{(s)} = \pi^{(s)} + \varepsilon_\sigma(x)\eta^{(s)\sigma}(x,\phi_{(s)},\pi^{(s)}).
\end{aligned}
\tag{15}
$$

$\varepsilon_\sigma(x)(\sigma=1,2,\cdots r)$是无穷小任意函数,他们的值和导数在时空区域的边界上为零. 假设有效正则作用量在整体变换(即上式中的 $\varepsilon_\sigma(x)$用参数 ε_σ 代替)下是不变的,则在定域变换下[9],有

$$
\delta I^{\mathrm{P}}_{\mathrm{eff}} = -\int \mathrm{d}^4 x\varepsilon_\sigma(x)\{\partial_\mu[(\pi^{(s)}\phi_{(s+1)} - \mathscr{H}_{\mathrm{eff}})\tau^{\mu\sigma}] + \mathrm{D}[\pi^{(s)}(\xi^\sigma_{(s)} - \phi_{(s),\mu}\tau^{\mu\sigma})]\}. \tag{16}
$$

假设变换(15)的行列式为 1,相空间的生成泛函(14)在定域变换(15)下是不变的. 即 $\delta Z[J]/\delta\varepsilon_\sigma(x)|_{\varepsilon_\sigma=0}=0$. 将(15)和(16)代入(14)并对 $\varepsilon_\sigma(x)$求泛函微商,可得

$$
\begin{aligned}
&\int\mathscr{D}\phi_{(s)}\mathscr{D}\pi^{(s)}\{\partial_\mu[(\pi^{(s)}\phi_{(s+1)} - \mathscr{H}_{\mathrm{c}})\tau^{\mu\sigma}] + \mathrm{D}[\pi^{(s)}(\xi^\sigma_{(s)} - \phi_{(s),\mu}\tau^{\mu\sigma})]\}\times\\
&\qquad \exp\{\mathrm{i}\int \mathrm{d}^4 x(\pi^{(s)}\phi_{(s+1)} - \mathscr{H}_{\mathrm{c}} + J^{(s)}\phi_{(s)})\} = 0.
\end{aligned}
\tag{17}
$$

对外源求微商,同上节一样的作法,最后得系统的守恒量为

$$
Q^\sigma = \int \mathrm{d}^3 x[\pi^{(s)}(\xi^\sigma_{(s)} - \phi_{(s),k}\tau^{k\sigma}) - \mathscr{H}_{\mathrm{eff}}\tau^{0\sigma}], \tag{18}
$$

其中 $\mathscr{H}_{\mathrm{eff}}$是与 $\mathscr{L}_{\mathrm{eff}}$相应的有效正则作用量.

4 高阶 Maxwell 非 Abel Chern - Simons 系统的守恒律

A.Foussats 等人曾研究过高阶 Maxwell 非 Abel 场与旋量场的耦合的 Feynman 规则[10],现在进一步研究规范场 A^a_μ 与标量场 ϕ 的耦合的对称性和守恒律. 系统的拉氏量密度为

$$
\mathscr{L} = (\mathrm{D}_\mu\phi)^+(\mathrm{D}^\mu\phi) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{\kappa}{4\pi}\varepsilon^{\mu\nu\rho}(\partial_\mu A^a_\nu A^a_\rho + \frac{1}{3}f^{abc}A^a_\mu A^b_\nu A^c_\rho) - \frac{c^2}{4\pi}\mathrm{D}_\rho F^a_{\mu\nu}\mathrm{D}^\rho F^{a\mu\nu}, \tag{19}
$$

其中 T^a 为规范群的生成元,$\mathrm{D}_\mu\phi^a=\partial_\mu\phi^a+gA^\gamma_\mu T^\gamma_{\alpha\beta}\phi^\beta$, $F^a_{\mu\nu}=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu+f^{abc}A^b_\mu A^c_\nu$,规范不变性要求 $\kappa=\frac{n}{4\pi}(n\in Z)$[11].

利用 Faddeev - Popov 量子化方法,取 Coulomb 规范,从 Green 函数的生成泛函导出系统的有效拉氏量为

$$
\begin{aligned}
\mathscr{L}_{\mathrm{eff}} &= \mathscr{L} + \mathscr{L}_{\mathrm{gh}} - \frac{1}{2\alpha}(\partial^i A^{ia})^2,\\
\mathscr{L}_{\mathrm{gh}} &= -\partial^i\overline{C^a}\mathrm{D}^{ab}_i C^b,
\end{aligned}
\tag{20}
$$

其中$\overline{C^a}$和 C^a 为鬼粒子场,α 为参数. 在下列 BRS 变换

$$
\delta\phi = -\mathrm{i}\tau T^a C^a\phi,\qquad \delta\phi^+ = \mathrm{i}\tau\phi^+ T^a C^a
$$

$$\delta A_\mu^a = -\tau D_{b\mu}^a C^b, \quad \delta C^a = \frac{1}{2}\tau f^{abc}C_b C_c, \quad \delta \bar{C}^a = -\frac{\tau}{\alpha}\partial^\mu A_\mu^a \tag{21}$$

下有效作用量 $\mathscr{L}_{\rm eff}$是不变的,其中 τ 为 Grassmann 参量,由(12)式可求出系统的量子 BRS 守恒荷. $\mathscr{L}_{\rm eff}$在空间转动变换下保持不变且变换的行列式为 1,由(12)式可求出系统量子水平的守恒角动量. 其结果与经典 Noether 定理不同的是在于需计及鬼粒子对角动量的贡献.

下面在相空间中来分析,通过 Ostrogradsky 变换,A_μ,$B_\mu = \dot{A}_\mu$,ϕ,ϕ^+ 正则动量如下

$$P^{a\mu} = F^{a\mu 0} + \frac{\kappa}{4\pi}\varepsilon^{0\mu\nu}A_\nu^a - \frac{c^2}{\pi}\mathrm{D}_i\mathrm{D}^i F^{a\mu\nu} - \mathrm{D}_0 Q^{a\mu} - \frac{c^2}{\pi}\mathrm{D}_i\mathrm{D}_0 F^{a\mu i} + f^{abc}A_0^c Q^{b\mu},$$

$$Q^{a\mu} = \frac{c^2}{\pi}\mathrm{D}_0 F^{a\mu 0}, \pi = \frac{\partial \mathscr{L}}{\partial \dot{\phi}} = (\mathrm{D}_0\phi)^+, \pi^+ = \frac{\partial \mathscr{L}}{\partial \dot{\phi}^+} = \mathrm{D}_0\phi, \tag{22}$$

初级约束为

$$\Lambda^{(0)a} = Q^{a0} \approx 0. \tag{23}$$

由约束的自洽性条件可得所有的次级约束如下

$$\begin{aligned} &\Lambda^{(1)a} = -P^{a0} + \mathrm{D}_i Q^{ai} \approx 0, \\ &\Lambda^{(2)a} = -\mathrm{D}_i P^{ai} - \frac{\kappa}{4\pi}\partial_i A_j^a \varepsilon^{ij} - f^{abc}B_i^b Q^{ci} - f^{abc}A_0^b \mathrm{D}_i Q^{ci} \approx 0. \end{aligned} \tag{24}$$

不难验证 $\Lambda^{(0)}$,$\Lambda^{(1)a}$,$\Lambda^{(2)a}$都是第一类约束,对于这 3 个第一类约束分别取规范条件

$$\Omega_0^a = B_0^a \approx 0, \Omega_1^a = \partial_i B^{ai} \approx 0, \ \Omega_2^a = \partial_i A^{ai} \approx 0, \tag{25}$$

由 Faddeev - Senjavonic 量子化方法[10],可得 Green 函数的生成泛函为

$$Z[J] = \int \mathscr{D}A_\mu^a \mathscr{D}P^{a\mu}\mathscr{D}B_\mu^a \mathscr{D}Q^{a\mu}\mathscr{D}\phi\mathscr{D}\pi\mathscr{D}\phi^+ \mathscr{D}\pi^+ \prod_l \delta(\Lambda_l)\delta(\Omega_l)\det\{\Lambda,\Omega\} \cdot \exp$$

$$\{\mathrm{i}\int \mathrm{d}^3 x(B_\mu^a P^{a\mu} + \dot{B}_\mu^a Q^{a\mu} + \pi\dot{\phi} + \pi^+\dot{\phi}^+ - \mathscr{H}_c + J_{1a}^\mu A_\mu^a + J_{2a}^\mu B_\mu^a + J_1\phi + J_1^+\phi^+)\}, \tag{26}$$

其中 $\det\{\Lambda,\Omega\} = \det A \cdot \det M^{ab} \cdot \det M^{ab}$,$A = -\delta^{ab}\delta(x-y)$,记 $M_c = (M^{ab})$,因子 $\det M_c \delta(\partial_i A^{ai})$可用 $\det M_{il}\delta(\partial_\mu A^{a\mu})$来代替[10],$M_l = (\delta^{ab}\partial_\mu\partial^\mu - f^{abc}A_\mu^c\partial^\mu)\delta(x-y)$,行列式通过 Grassmann 变量 $C^a(x)$,$\bar{C}^a(y)$的积分来表达,由理论的规范无关性,将规范条件因子写在指数上(利用 δ 函数的性质). 于是生成泛函为

$$Z[J] = \int \mathscr{D}A_\mu^a \mathscr{D}P^{a\mu}\mathscr{D}B_\mu^a \mathscr{D}Q^{a\mu}\mathscr{D}\phi\mathscr{D}\pi\mathscr{D}\phi^+ \mathscr{D}\pi^+ \mathscr{D}^+ \lambda\mathscr{D}\bar{\lambda}\mathscr{D}C^a\mathscr{D}\bar{C}^a \times$$

$$\exp\{\mathrm{i}\int \mathrm{d}^3 x[\mathscr{L}_{\rm eff} + J_{1a}^\mu A_\mu^a + J_{2a}^\mu B_\mu^a + J_1\phi + J_1^+\phi^+ + \bar{J}_{3a}C^a + \bar{C}^a J_{3a}]\}, \tag{27}$$

其中

$$\begin{aligned} &\mathscr{L}_{\rm eff}^{\rm p} = \mathscr{L}^{\rm p} + \mathscr{L}_{\rm g} + \mathscr{L}_{\rm gh} + \mathscr{L}_{\rm m}, \\ &\mathscr{L}^{\rm p} = B_\mu^a P^{a\mu} + \dot{B}_\mu^a Q^{a\mu} + \dot{\phi}\,\pi + \dot{\phi}^+\,\pi^+ - \mathscr{H}_c, \\ &\mathscr{L}_{\rm g} = -\frac{1}{2\alpha_2}(\Omega_2^a)^2 = -\frac{1}{2\alpha_2}(\partial_\mu A^{a\mu})^2, \\ &\mathscr{L}_{\rm gh} = -\partial^\mu \bar{C}^a \mathrm{D}_{b\mu}^a C_b, \\ &\mathscr{L}_{\rm m} = \lambda_0^a \Lambda^{(0)a} + \lambda_1^a \Lambda^{(1)a} + \lambda_2^a \Lambda^{(2)a} - \frac{1}{2\alpha_0}(\Omega_0^a)^2 - \frac{1}{2\alpha_1}(\Omega_1^a)^2. \end{aligned} \tag{28}$$

在上面的生成泛函中对鬼场 $\bar{C}^a(x)$ 和 $C^a(x)$ 分别引入了外源 $J_{3a},\bar{J}_{3a}$,考虑相空间中场的 BRS 变换

$$
\begin{aligned}
&\delta\phi = -\mathrm{i}\tau T^a C^a\phi, && \delta\pi = \mathrm{i}\tau\pi T^a C^a, \\
&\delta\phi^+ = \mathrm{i}\tau\phi^+ T^a C^a, && \delta\pi^+ = -\mathrm{i}\tau T^a C^a\pi^+, \\
&\delta A_\mu^a = -\tau D_{b\mu}^a C^b, && \delta P^{a\mu} = f_{bc}^a P^{c\mu} C^b\tau - f_{bc}^a Q^{c\mu}\dot{C}^b, \\
&\delta B_\mu^a = \partial_0(-\tau D_{b\mu}^a C^b), && \delta Q^{a\mu} = f_{bc}^a Q^{c\mu} C^b\tau, \\
&\delta C^a = \frac{1}{2} f^{abc} C_b C_c, && \delta\bar{C}^a = -\frac{1}{\alpha_2}\partial^\mu A_\mu^a,
\end{aligned}
\tag{29}
$$

$\mathscr{L}_{\mathrm{eff}}^p$中的前三项之和的 BRS 变换下不变,由第一类约束产生的规范变换仍保持第一类约束不离开约束超曲面[6]. 即是说 $\delta\mathscr{L}_{\mathrm{eff}}^p\approx 0$,且 BRS 变换的 Jacobi 行列式为 1,这样由(9)式就得系统在量子水平下的 BRS 守恒荷

$$
Q = \int \mathrm{d}^2 x (P^{a\mu}\delta A_\mu^a + Q^{a\mu}\delta B_\mu^a + \pi\delta\phi + \delta\phi^+\pi^+ + \bar{R}_a\delta C^a + \delta\bar{C}^a R_a). \tag{30}
$$

其中 $\bar{R}_a$ 和 R_a 分别为鬼场 C^a 和 $\bar{C}^a$ 的正则共轭动量,将(22)式代入上式,可见其结果与前面位形空间中导致的结果相同.

有效正则拉氏量 $\mathscr{L}_{\mathrm{eff}}^p$量在$(x_1,x_2)$平面内的空间转动下上不变的. 且变换的行列式为 1,又 $\tau^{0\sigma}=0$. 因此系统在量子水平下的守恒角动量为

$$
\begin{aligned}
J_{12} = \int \mathrm{d}^2 x \Big\{ & P^{a\mu}\left(x_2\frac{\partial A_\mu^a}{\partial x_1} - x_1\frac{\partial A_\mu^a}{\partial x_2}\right) + Q^{a\mu}\left(x_2\frac{\partial B_\mu^a}{\partial x_1} - x_1\frac{\partial B_\mu^a}{\partial x_2}\right) + \\
& P^{a\mu}\Big(\sum_{12}\Big)_{\mu\nu} A_\nu^a + Q^{a\mu}\Big(\sum_{12}\Big)_{\mu\nu} B_\nu^a + \pi\left(x_2\frac{\partial\phi}{\partial x_1} - x_1\frac{\partial\phi}{\partial x_2}\right) + \left(x_2\frac{\partial\phi^+}{\partial x_1} - x_1\frac{\partial\phi^+}{\partial x_2}\right)\pi^+ + \\
& \bar{R}_a\left(x_2\frac{\partial C^a}{\partial x_1} - x_1\frac{\partial C^a}{\partial x_2}\right) + \left(x_2\frac{\partial\bar{C}^a}{\partial x_1} - x_1\frac{\partial\bar{C}^a}{\partial x_2}\right) R_a \Big\},
\end{aligned}
\tag{31}
$$

其中 $\Big(\sum_{jk}\Big)_{\mu\nu} = g_{j\mu}g_{k\nu} - g_{j\nu}g_{k\mu}$. 因此,量子水平下的角动量要包括鬼场的贡献. 将(22)式代入上式,其结果也与前面位形空间所得结果相同. 以上讨论均未考虑重正化效应,对非 Abel Chern-Simons 理论,重正化后 κ 值要产生移动.

从上面我们得出的结论,从位形空间生成泛函和从相空间生成泛函得到的结果相同. 这表明 F-P 方法对这个 Maxwell 非 Abel CS 场和复标量耦合模型是适用的. 但相空间中的路径积分比位形空间中的路径积分更基本. 因为,在相空间中,我们可以避免考虑动量是否可积问题. 同时,也可以看出,我们得到的量子守恒角动量与由未量子化的原始拉氏量按经典 Noether 定理导出的结果不同之处在于还必须计及鬼粒子场对系统角动量的贡献,因而,经典非 Abel CS 理论中可能呈现出的分数自旋性质[12],在量子水平上,该性质是否保持,有待进一步研究

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Global Symmetry and Quantal Conservation Laws for a Gauge - Invariant System With a Higher - Order Lagrangian

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Abstract Based on the configuration-space generating functional obtained by using the Faddeev-Popov trick for a gauge-invariant system and phase-space generating functional obtained by using the Faddeev-Senjanovic method for a constrained Hamiltonian system with a singular higher-order Lagrangian respectively, the conservation laws at the quantum level were derived. A preliminary application of the present formulation to the higher-order Maxwell non-Abelian Chern-Sinons(CS) theory is given, the quantal BRS conserved charge and quantal angular momentum for higher-order Maxwell non-Abelian CS term coupled to scalar fields were obtained. The results arising from configuration-space generating functional coincide with the result deriving from phase-space generating functional, and the quantal conserved angular momentum differ from the classical one in that one needs to take into account the contribution of the the angular momentum of ghost fields in Maxwell non-Abelian CS theories. The fractional spin property for CS theories is discussed.

Key words path integral, symmetries and conservation laws, gauge fields, Chern-Simons theories

Received 14 December 1998