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非阿贝尔介质对外源的非线性响应方程*

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摘要 从动力论理论出发,使用弱湍理论方法,有效地展开了夸克-胶子等离子体的动力论方程,从而给出了夸克-胶子等离子体介质对外流的非线性非阿贝尔响应方程.

关键词 夸克-胶子等离子体 动力论理论 涨落

1 引言

介质对外源的响应方程描写的是等离子体宏观场的动力学[1],等离子体的性质反映在介电函数中.外源与等离子体相互作用能够产生丰富的物理效应,例如,荷粒子的能量损失、等离子体回波效应等.特别是荷粒子的能量损失可以作为寻找夸克-胶子等离子体(QGP)的一种信号[2—4].然而研究这些问题需要从响应方程出发.在夸克-胶子等离子体中,虽然涉及介质线性响应方程的问题已有很多讨论,但QGP理论是非阿贝尔、非线性的,线性理论基本上是类阿贝尔的,因而研究非阿贝尔介质的非线性响应方程是非常重要的,况且一些重要的等离子体效应本身就是非线性的。本文就是为了从QGP动力论方程和自洽场方程推导出非阿贝尔等离子体介质对外源的非线性响应方程.后面的内容安排如下:第二节根据弱湍理论,将粒子涨落的分布函数按场量的幂次展开,从而逐级迭代求解Vlasov动力论方程;第三节从自洽场方程出发,导出了QGP对外流的响应方程;第四节做了一个小结.

2 动力论方程的展开和迭代求解

描述QGP介质在自洽场中粒子相互作用的动力论方程可以采用Vlasov动力论方程,如果设夸克、反夸克、胶子的分布函数分别为$f(x,\ p)$、$\bar{f}(x,\ p)$、$G(x,\ p)$,则其表述为[5]

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$$p^{\mu}D_{\mu}f(x,p)+\frac{g}{2}p^{\mu}\partial_{p}^{\nu}\{F_{\mu\nu},f(x,p)\}=0,\tag{1}$$

$$p^{\mu}D_{\mu}\bar{f}(x,p)-\frac{g}{2}p^{\mu}\partial_{p}^{\nu}\{F_{\mu\nu},\bar{f}(x,p)\}=0,\tag{2}$$

$$p^{\mu}\mathscr{D}_{\mu}G(x,p)+\frac{g}{2}p^{\mu}\partial_{p}^{\nu}\{\mathscr{F}_{\mu\nu},G(x,p)\}=0,\tag{3}$$

$F_{\mu\nu}$，$\mathscr{F}_{\mu\nu}$分别是自洽色场在 $SU(3)$的基础表示和伴随表示下的形式，协变导数 $D_{\mu}=\partial_{\mu}+\mathrm{i}gA_{\mu}$，$\mathscr{D}_{\mu}=\partial_{\mu}+\mathrm{i}g\mathscr{A}_{\mu}$，$A_{\mu}=A_{\mu}^{a}\tau^{a}$，$\mathscr{A}_{\mu}=A_{\mu}^{a}T^{a}$，$(T^{a})_{bc}=-\mathrm{i}f_{abc}$，这里$\tau^{a}$，$T^{a}$，$f_{abc}$分别为 $SU(3)$基础表示生成元，伴随表示生成元以及结构常数．

在 QGP 中，粒子密度的涨落同时导致色场的涨落，把分布函数和色场写为

$$f=f^{\mathrm{R}}+f^{\mathrm{T}},\ \bar{f}=\bar{f}^{\mathrm{R}}+\bar{f}^{\mathrm{T}},\quad G=G^{\mathrm{R}}+G^{\mathrm{T}},$$

$$A_{\mu}=A_{\mu}^{\mathrm{R}}+A_{\mu}^{\mathrm{T}},\ F_{\mu\nu}=F_{\mu\nu}^{\mathrm{R}}+F_{\mu\nu}^{\mathrm{T}},$$

其中上标 R，T 分别为各量的正规部分和扰动部分．利用这些等式和方程(1)—(3)，可得[6]

$$p^{\mu}\partial_{\mu}f^{\mathrm{T}}=-\mathrm{i}gp^{\mu}[A_{\mu}^{\mathrm{T}},f^{\mathrm{R}}]-\frac{g}{2}p^{\mu}\partial_{p}^{\nu}\{F_{\mu\nu}^{\mathrm{T}},\partial_{p}^{\nu}f^{\mathrm{R}}\},\tag{4}$$

$$p^{\mu}\partial_{\mu}\bar{f}^{\mathrm{T}}=-\mathrm{i}gp^{\mu}[A_{\mu}^{\mathrm{T}},\bar{f}^{\mathrm{R}}]+\frac{g}{2}p^{\mu}\partial_{p}^{\nu}\{F_{\mu\nu}^{\mathrm{T}},\bar{f}^{\mathrm{R}}\},\tag{5}$$

$$p^{\mu}\partial_{\mu}G^{\mathrm{T}}=-\mathrm{i}gp^{\mu}[\mathscr{A}_{\mu}^{\mathrm{T}},G^{\mathrm{R}}]-\frac{g}{2}p^{\mu}\partial_{p}^{\nu}\{\mathscr{F}_{\mu\nu}^{\mathrm{T}},G^{\mathrm{R}}\}.\tag{6}$$

现记 $k=(\omega,\boldsymbol{k})$，将方程(4)—(6)作 Fourier 变换，同时根据弱湍理论[7]，扰动部分的分布函数可展开为

$$f(k)=\sum_{i=1}^{\infty}f^{(i)}(k),\tag{7}$$

$f^{(i)}$对应于场量的相应幂次．进而迭代求解可以获得线性结果

$$f^{(1)}(k)=-\mathrm{i}\frac{g}{p_0\omega-\boldsymbol{p}\cdot\boldsymbol{k}}F_{\mu\nu}(k)p^{\mu}\frac{\partial f_0}{\partial p_{\nu}},\tag{8}$$

$$\bar{f}^{(1)}(k)=+\mathrm{i}\frac{g}{p_0\omega-\boldsymbol{p}\cdot\boldsymbol{k}}F_{\mu\nu}(k)p^{\mu}\frac{\partial \bar{f}_0}{\partial p_{\nu}},\tag{9}$$

$$G^{(1)}(k)=-\mathrm{i}\frac{g}{p_0\omega-\boldsymbol{p}\cdot\boldsymbol{k}}\mathscr{F}_{\mu\nu}(k)p^{\mu}\frac{\partial G_0}{\partial p_{\nu}},\tag{10}$$

以及高阶分布函数所满足的迭代关系

$$f^{(n)}(k)=\frac{g}{p_0\omega-\boldsymbol{p}\cdot\boldsymbol{k}}p^{\mu}\int \mathrm{d}k_1\mathrm{d}k_2\delta(k-k_1-k_2)([A_{\mu}(k_1),f^{(n-1)}(k_2)]-$$
$$\langle[A_{\mu}(k_1),f^{(n-1)}(k_2)]\rangle)-\frac{\mathrm{i}}{2}\frac{g}{p_0\omega-\boldsymbol{p}\cdot\boldsymbol{k}}p^{\mu}\int \mathrm{d}k_1\mathrm{d}k_2\delta(k-k_1-k_2)\cdot$$
$$(\{F_{\mu\nu}(k_1),f^{(n-1)}(k_2)\}-\langle\{F_{\mu\nu}(k_1),f^{(n-1)}(k_2)\}\rangle),\tag{11}$$

$$\bar{f}^{(n)}(k)=\frac{g}{p_0\omega-\boldsymbol{p}\cdot\boldsymbol{k}}p^{\mu}\int \mathrm{d}k_1\mathrm{d}k_2\delta(k-k_1-k_2).$$
$$([A_{\mu}(k_1),f^{(n-1)}(k_2)]-\langle[A_{\mu}(k_1),\bar{f}^{(n-1)}(k_2)]\rangle)+$$
$$\frac{\mathrm{i}}{2}\frac{g}{p_0\omega-\boldsymbol{p}\cdot\boldsymbol{k}}p^{\mu}\int \mathrm{d}k_1\mathrm{d}k_2\delta(k-k_1-k_2).$$
$$(\{F_{\mu\nu}(k_1),\bar{f}^{(n-1)}(k_2)\}-\langle\{F_{\mu\nu}(k_1),\bar{f}^{(n-1)}(k_2)\}\rangle),\tag{12}$$

$$G^{(n)}(k)=\frac{g}{p_0\omega-\boldsymbol{p}\cdot\boldsymbol{k}}p^{\mu}\int \mathrm{d}k_1\mathrm{d}k_2\delta(k-k_1-k_2).$$
$$([\mathscr{A}_{\mu}(k_1),G^{(n-1)}(k_2)]-\langle[\mathscr{A}_{\mu}(k_1),G^{(n-1)}(k_2)]\rangle)-$$
$$\frac{\mathrm{i}}{2}\frac{g}{p_0\omega-\boldsymbol{p}\cdot\boldsymbol{k}}p^{\mu}\int \mathrm{d}k_1\mathrm{d}k_2\delta(k-k_1-k_2).$$
$$(\{\mathscr{F}_{\mu\nu}(k_1),G^{(n-1)}(k_2)\}-\langle\{\mathscr{F}_{\mu\nu}(k_1),G^{(n-1)}(k_2)\}\rangle).\tag{13}$$

这里〈 〉表示对随机位相作平均．取瞬时规范 $A_a^0=0$，这时色场即为

$$E_a^i(k)=\mathrm{i}\omega A_a^i(k)\tag{14}$$

并利用系统各向同性条件 $\partial_p^i=\frac{p^i}{|\boldsymbol{p}|}\partial_{|p|}$，得到二阶和三阶扰动分布为

$$f^{(2)}(k)=+g^2\frac{1}{p\cdot k+\mathrm{i}0^+}\int \mathrm{d}k_1\mathrm{d}k_2\delta(k-k_1-k_2)\cdot([E_i(k_1),E_j(k_2)]-$$
$$\langle[E_i(k_1),E_j(k_2)]\rangle)\frac{1}{p\cdot k_2+\mathrm{i}0^+}\frac{p^i}{\omega_2}\mathscr{D}_p^j f_0-$$
$$\frac{1}{2}g^2\frac{1}{p\cdot k+\mathrm{i}0^+}\int \mathrm{d}k_1\mathrm{d}k_2\,\delta(k-k_1-k_2)\cdot$$
$$(\{E_i(k_1),E_j(k_2)\}-\langle\{E_i(k_1),E_j(k_2)\}\rangle)\,\mathscr{D}_p^i\frac{1}{p\cdot k_2+\mathrm{i}0^+}\mathscr{D}_p^j f_0\tag{15}$$

$$f^{(3)}(k) = +\mathrm{i}g^3\frac{1}{p\cdot k+\mathrm{i}0^+}\int \mathrm{d}k_1\mathrm{d}k_2\mathrm{d}k_3\,\delta(k-k_1-k_2-k_3)\cdot$$

$$([E_i(k_1),[E_j(k_2),E_k(k_3)]]-\langle[E_i(k_1),[E_j(k_2),E_k(k_3)]]\rangle)\cdot$$

$$\frac{1}{[p\cdot(k_2+k_3)+\mathrm{i}0^+][p\cdot k_3+\mathrm{i}0^+]}\frac{p^ip^j}{\omega_1\omega_2}\mathscr{D}_p^k f_0-$$

$$\frac{\mathrm{i}}{2}g^3\frac{1}{p\cdot k+\mathrm{i}0^+}\int \mathrm{d}k_1\mathrm{d}k_2\mathrm{d}k_3\,\delta(k-k_1-k_2-k_3)\cdot$$

$$([E_i(k_1),\{E_j(k_2),E_k(k_3)\}]-\langle[E_i(k_1),\{E_j(k_2),E_k(k_3)\}]\rangle)\cdot$$

$$\frac{1}{p\cdot(k_2+k_3)+\mathrm{i}0^+}\mathscr{D}_p^j\frac{1}{p\cdot k_3+\mathrm{i}0^+}\frac{p^i}{\omega_1}\mathscr{D}_p^k f_0-$$

$$\frac{\mathrm{i}}{2}g^3\frac{1}{p\cdot k+\mathrm{i}0^+}\int \mathrm{d}k_1\mathrm{d}k_2\mathrm{d}k_3\,\delta(k-k_1-k_2-k_3)\cdot$$

$$(\{E_i(k_1),[E_j(k_2),E_k(k_3)]\}-\langle\{E_i(k_1),[E_j(k_2),E_k(k_3)]\}\rangle)\cdot$$

$$\mathscr{D}_p^i\frac{1}{[p\cdot(k_2+k_3)+\mathrm{i}0^+][p\cdot k_3+\mathrm{i}0^+]}\frac{p^j}{\omega_2}\mathscr{D}_p^k f_0+$$

$$\frac{\mathrm{i}}{4}g^3\frac{1}{p\cdot k+\mathrm{i}0^+}\int \mathrm{d}k_1\mathrm{d}k_2\mathrm{d}k_3\,\delta(k-k_1-k_2-k_3)\cdot$$

$$(\{E_i(k_1),\{E_j(k_2),E_k(k_3)\}\}-\langle\{E_i(k_1),\{E_j(k_2),E_k(k_3)\}\}\rangle)\cdot$$

$$\mathscr{D}_p^i\frac{1}{p\cdot(k_2+k_3)+i0^+}\mathscr{D}^k\frac{1}{p\cdot k_3+i0^+}\mathscr{D}_p^k f_0 \tag{16}$$

其中 $\mathscr{D}_p^i=p^0\partial_p^i-p^i\partial^0$. 由于公式的复杂性,这里只写出了有关夸克的结果,反夸克和胶子有类似的结果,读者很容易得到它们.

3　介质对外流的响应方程

现在从 Fourier 变换下的自洽场方程

$$-\mathrm{i}k_\mu F^{\mu\nu}(k)+\mathrm{i}g\int \mathrm{d}k_1\mathrm{d}k_2\,\delta(k-k_1-k_2)[A_\mu(k_1),F^{\mu\nu}(k_2)]=j^\nu(k), \tag{17}$$

着手讨论介质对外流的响应问题．该方程中的色流由外源 j_0^ν 和涨落扰动所激发的色流 $\sum_{i=1}^{\infty}j^{\nu(i)}$ 构成，后者可表述为[5]

$$j^{\nu(i)}(k) = -\frac{g}{2}\int \frac{\mathrm{d}^3 p}{(2\pi)^3 p^0} p^\nu (N_f(f^{(i)} - \bar{f}^{(i)}) + 2\mathrm{i}\tau_a f^{abc} G^{(i)}_{bc}), \tag{18}$$

为简单起见，考虑纵场情形，方程(16)改写如下

$$-\omega E(k) - \mathrm{i}g\int \mathrm{d}k_1 \mathrm{d}k_2 \delta(k-k_1-k_2) \frac{\boldsymbol{k}\cdot\boldsymbol{k}_1 \boldsymbol{k}\cdot\boldsymbol{k}_2}{|\boldsymbol{k}||\boldsymbol{k}_1||\boldsymbol{k}_2|} \frac{1}{\omega_1\omega_2}[E(k_1), E(k_2)] -$$

$$g^2\int \mathrm{d}k_1\mathrm{d}k_2\mathrm{d}k_3 \delta(k-k_1-k_2-k_3) \frac{\boldsymbol{k}\cdot\boldsymbol{k}_2 \boldsymbol{k}\cdot\boldsymbol{k}_3}{|\boldsymbol{k}||\boldsymbol{k}_1||\boldsymbol{k}_2||\boldsymbol{k}_3|} \frac{1}{\omega_1\omega_2\omega_3}[E(k_1), [E(k_2), E(k_3)]] =$$

$$\mathrm{i}\frac{\boldsymbol{k}}{|\boldsymbol{k}|}\cdot\left(\sum_{i=1}^{\infty} \boldsymbol{j}^{(i)}(k) + \boldsymbol{j}_0(k)\right), \tag{19}$$

现记$\boldsymbol{j}^{(i)}(k) = \frac{\boldsymbol{k}}{|\boldsymbol{k}|}\cdot\boldsymbol{j}^{(i)}$，当 i 仅取 1 时，就是通常所研究的线性近似，从方程(19)可得熟识的结果

$$-\omega\varepsilon^l(k)E(k) = \mathrm{i}\frac{\boldsymbol{k}}{|\boldsymbol{k}|}\cdot\boldsymbol{j}_0, \tag{20}$$

其中

$$\varepsilon^l(k) = 1 + \frac{m^2}{k}\left[1 + \frac{\omega}{2|\boldsymbol{k}|}\ln\left|\frac{\omega-|\boldsymbol{k}|}{\omega+|\boldsymbol{k}|}\right| + \frac{\mathrm{i}\pi\omega}{2|\boldsymbol{k}|}\Theta(|\boldsymbol{k}|-\omega)\right] \tag{21}$$

为线性介电函数．(20)式是介质对外流的线性响应方程，它完全类同于 Abelian 等离子体[8,9]，很难反映 QGP 的非 Abelian 特性．计及涨落激发的高阶流，研究非线性响应，将发现情形有所不同．将 i 取到 3，立即得到 2 阶和 3 阶非线性流为：

$$j^{(2)}_{\mathrm{quark}}(k) = -\frac{1}{2}g^3\gamma_{k,k_1,k_2}\frac{1}{\omega_2}\int p^2\frac{\mathrm{d}|\boldsymbol{p}|}{2\pi^2}\partial_{|\boldsymbol{p}|}(f_0+\bar{f}_0)([E(k_1), E(k_2)] - \langle[E(k_1), E(k_2)]\rangle) +$$

$$\frac{3}{4}g^3\gamma_{k,k_1,k_2}\int p^2\frac{\mathrm{d}|\boldsymbol{p}|}{2\pi^2}\partial^2_{|\boldsymbol{p}|}(f_0-\bar{f}_0)(\{E(k_1), E(k_2)\} - \langle\{E(k_1), E(k_2)\}\rangle) \tag{22}$$

$$J^{(2)}_{\mathrm{gluon}}(k) = -\frac{1}{2}g^3\gamma_{k,k_1,k_2}\frac{1}{\omega_2}\int p^2\frac{\mathrm{d}|\boldsymbol{p}|}{2\pi^2}\partial_{|\boldsymbol{p}|}G_0([\varepsilon(k_1), \varepsilon(k_2)] - \langle[\varepsilon(k_1), \varepsilon(k_2)]\rangle) +$$

$$\frac{3}{4}g^3\gamma_{k,k_1,k_2}\int p^2\frac{\mathrm{d}|\boldsymbol{p}|}{2\pi^2}\partial^2_{|\boldsymbol{p}|}G_0(\{\varepsilon(k_1), \varepsilon(k_2)\} - \langle\{\varepsilon(k_1), \varepsilon(k_2)\}\rangle), \tag{23}$$

$$j^{(3)}_{\mathrm{quark}}(k) = -\frac{\mathrm{i}}{2}g^4\Gamma_{k,k_1,k_2,k_3}\frac{1}{\omega_1\omega_2}\int p^2\frac{\mathrm{d}|\boldsymbol{p}|}{2\pi^2}\partial_{|\boldsymbol{p}|}(f_0+\bar{f}_0)([E(k_1), [E(k_2), E(k_3)]] -$$

$$[E(k_1),\ \langle[E(k_2),\ E(k_3)]\rangle] - \langle[E(k_1),\ [E(k_2),\ E(k_3)]]\rangle) +$$

$$\frac{i}{4}g^4\Gamma_{k,\,k_1,k_2,k_3}\frac{1}{\omega_1}\int p^2\frac{d|\boldsymbol{p}|}{2\pi^2}\,\partial^2_{|\boldsymbol{p}|}(f_0-\bar{f}_0)([E(k_1),\ \{E(k_2),\ E(k_3)\}] -$$

$$[E(k_1),\ \langle\{E(k_2),\ E(k_3)\}\rangle] - \langle[E(k_1),\ \{E(k_2),\ E(k_3)\}]\rangle) +$$

$$\frac{i}{4}g^4\Gamma_{k,\,k_1,k_2,k_3}\frac{1}{\omega_2}\int p^2\frac{d|\boldsymbol{p}|}{2\pi^2}\,\partial^2_{|\boldsymbol{p}|}(f_0-\bar{f}_0)(\{E(k_1),\ [E(k_2),\ E(k_3)]\} -$$

$$\{E(k_1),\ \langle[E(k_2),\ E(k_3)]\rangle\} - \langle\{E(k_1),\ [E(k_2),\ E(k_3)]\}\rangle) -$$

$$\frac{i}{8}g^4\Gamma_{k,\,k_1,k_2,k_3}\int p^2\frac{d|\boldsymbol{p}|}{2\pi^2}\,\partial^3_{|\boldsymbol{p}|}(f_0+\bar{f}_0)(\{E(k_1),\ \{E(k_2),\ E(k_3)\}\} -$$

$$[E(k_1),\ \langle\{E(k_2),\ E(k_3)\}\rangle\} - \langle\{E(k_1),\ \{E(k_2),\ E(k_3)\}\}\rangle), \tag{24}$$

$$J^{(3)}_{\text{gluon}}(k) = -\frac{i}{2}g^4\Gamma_{k,\,k_1,k_2,k_3}\frac{1}{\omega_1\omega_2}\int p^2\frac{d|\boldsymbol{p}|}{2\pi^2}\,\partial_{|\boldsymbol{p}|}G_0([\varepsilon(k_1),\ [\varepsilon(k_2),\ \varepsilon(k_3)]] -$$

$$[\varepsilon(k_1),\ \langle[\varepsilon(k_2),\ \varepsilon(k_3)]\rangle] - \langle[\varepsilon(k_1),\ [\varepsilon(k_2),\ \varepsilon(k_3)]]\rangle) +$$

$$\frac{i}{4}g^4\Gamma_{k,\,k_1,k_2,k_3}\frac{1}{\omega_1}\int p^2\frac{d|\boldsymbol{p}|}{2\pi^2}\,\partial^2_{|\boldsymbol{p}|}G_0([\varepsilon(k_1),\ \{\varepsilon(k_2),\ \varepsilon(k_3)\}] -$$

$$[\varepsilon(k_1),\ \langle\{\varepsilon(k_2),\ \varepsilon(k_3)\}\rangle] - \langle[\varepsilon(k_1),\ \{\varepsilon(k_2),\ \varepsilon(k_3)\}]\rangle) +$$

$$\frac{i}{4}g^4\Gamma_{k,\,k_1,k_2,k_3}\frac{1}{\omega_2}\int p^2\frac{d|\boldsymbol{p}|}{2\pi^2}\,\partial^2_{|\boldsymbol{p}|}G_0(\{\varepsilon(k_1),\ [\varepsilon(k_2),\ \varepsilon(k_3)]\} -$$

$$\{\varepsilon(k_1),\ \langle[\varepsilon(k_2),\ \varepsilon(k_3)]\rangle\} - \langle\{\varepsilon(k_1),\ [\varepsilon(k_2),\ \varepsilon(k_3)]\}\rangle) -$$

$$\frac{i}{8}g^4\Gamma_{k_1,k_2,k_3}\int p^2\frac{d|\boldsymbol{p}|}{2\pi^2}\,\partial^3_{|\boldsymbol{p}|}G_0(\{\varepsilon(k_1),\ \{\varepsilon(k_2),\varepsilon(k_3)\}\} -$$

$$\{\varepsilon(k_1),\ \langle\{\varepsilon(k_2),\ \varepsilon(k_3)\}\rangle\} - \langle\{\varepsilon(k_1),\ \{\varepsilon(k_2),\varepsilon(k_3)\}\}\rangle), \tag{25}$$

其中

$$j_{\text{gluon}} = 2\tau_a\text{tr}(T_a J), \tag{26}$$

$$\gamma_{k,k_1,\,k_2} = \int dk_1 dk_2\,\delta(k-k_1-k_2)\,\frac{d\Omega}{d\pi}\,\frac{\boldsymbol{v}\boldsymbol{k}}{|\boldsymbol{k}|}\,\frac{\boldsymbol{v}\boldsymbol{k}_1}{|\boldsymbol{k}_1|}\,\frac{\boldsymbol{v}\boldsymbol{k}_2}{|\boldsymbol{k}_2|}\,\frac{1}{vk+i0^+}\,\frac{1}{vk_1+i0^+}, \tag{27}$$

$$\Gamma_{k,k_1,k_2,k_3} = \int \mathrm{d}k_1 \mathrm{d}k_2 \mathrm{d}k_3\, \delta(k - k_1 - k_2 - k_3)\, \frac{\mathrm{d}\Omega}{4\pi}\, \frac{\boldsymbol{v}\boldsymbol{k}}{|\boldsymbol{k}|}\, \frac{\boldsymbol{v}\boldsymbol{k}_1}{|\boldsymbol{k}_1|}\, \frac{\boldsymbol{v}\boldsymbol{k}_2}{|\boldsymbol{k}_2|}\, \frac{\boldsymbol{v}\boldsymbol{k}_3}{|\boldsymbol{k}_3|} \cdot$$

$$\frac{1}{vk + \mathrm{i}0^+}\, \frac{1}{v(k_2 + k_3) + i0^+}\, \frac{1}{vk_3 + i0^+}\,. \tag{28}$$

将这些代入(19)式,在其两边乘以 $E(k')$ 并作积分 $\int \mathrm{d}k'$,忽略三点关联的贡献[1,7],保留领头项的贡献,可得

$$-\varepsilon^l(k) \int \mathrm{d}k'\, \omega \langle E(k) E(k')\rangle =$$

$$\frac{1}{2} g^4 m_{\mathrm{D}}^2 \Gamma_{k,k_1,k_2,k_3} [\tau_a,\, [\tau_b,\, \tau_c]] + \mathrm{i}\int \mathrm{d}k' \frac{\boldsymbol{k}}{|\boldsymbol{k}|} \cdot \langle j_{\exp}(k) E(k')\rangle, \tag{29}$$

这里 m_{D}代表 Debye 质量. 在作上面的运算时,用到了 $SU(3)$生成元代数[10]:

$$\tau_d tr(\tau_d\{T_a,\, \{T_b,\, T_c\}\}) = N\{\tau_a,\, \{\tau_b,\, \tau_c\}\} + 2\delta_{bc}\tau_a + 2\delta_{ab}\tau_c + 2\delta_{ac} - d_{bcd}, \tag{30}$$

$$\mathrm{tr}(\tau_d[\tau_a,\, [\tau_b,\, \tau_c]]) = -\frac{1}{2} f_{bce} f_{aed}, \tag{31}$$

$$\mathrm{tr}(\tau_d\{\tau_a,\, \{\tau_b,\, \tau_c\}\}) = \frac{1}{N} \delta_{ad}\delta_{bc} + \frac{1}{2} d_{bce} d_{aed}, \tag{32}$$

$$f_{abb} = 0, \tag{33}$$

$$f_{acd} f_{bcd} = N\delta_{ab}, \tag{34}$$

$$d_{acd} d_{bcd} = \frac{N-1}{N} \delta_{ab}\,. \tag{35}$$

既然场是来自外流和系统内涨落两部分的贡献,那么可将 $E(k)$分解成规则部分 $\tilde{E}$和随机部分 E^{T},且考虑

$$\langle E^{\mathrm{T}}(k)\rangle = 0 \tag{36}$$

$$\begin{aligned}&\langle E^{\mathrm{T}}(k_1)\, E^{\mathrm{T}}(k_2) E^{\mathrm{T}}(k_3) E^{\mathrm{T}}(k_4)\rangle = \\ &\langle E^{\mathrm{T}}(k_1) E^{\mathrm{T}}(k_2)\rangle\langle E^{\mathrm{T}}(k_3) E^{\mathrm{T}}(k_4)\rangle + \\ &\langle E^{\mathrm{T}}(k_1) E^{\mathrm{T}}(k_3)\rangle\langle E^{\mathrm{T}}(k_2) E^{\mathrm{T}}(k_4)\rangle + \\ &\langle E^{\mathrm{T}}(k_1) E^{\mathrm{T}}(k_4)\rangle\langle E^{\mathrm{T}}(k_2) E^{\mathrm{T}}(k_3)\rangle,\end{aligned} \tag{37}$$

$$\langle E^{\mathrm{T}}(k)\, E^{\mathrm{T}}(k')\rangle = -\langle (E^{\mathrm{T}}(k))^2\rangle\, \delta(k - k') \tag{38}$$

这样方程(29)变为

$$-\omega(\varepsilon^{l}\delta_{ab}+\varepsilon^{n}_{ab})\langle E_b(k)E_c(k)\rangle)=\mathrm{i}\frac{\boldsymbol{k}}{|\boldsymbol{k}|}\cdot\langle j_0(k)E(k)\rangle, \tag{39}$$

这里

$$\varepsilon^{n}_{ab}=\frac{g^4m_{\mathrm{D}}^2}{2\omega^2}f_{dbe}f_{cae}\int\mathrm{d}k'\,\mathrm{d}k\mathrm{d}k_1\mathrm{d}k_2\mathrm{d}k_3\langle E_c^{\mathrm{T}}(k_1)\,E_d^{\mathrm{T}}(k_1)\rangle$$

$$\left(\frac{\omega}{\omega_1^2}\,\delta(k_3-k')\delta(k_2+k_1)\Gamma_{k,k_1,k_2,k_3}+\frac{1}{\omega_1}\,\delta(k_2-k')\delta(k_3+k_1)\,\Gamma_{k,k_1,k_2,-k_1}\right) \tag{40}$$

如果考虑相同色指标关联的贡献，方程(39)进一步可简化为

$$-\omega[\varepsilon^{l}(k)+\varepsilon^{n}(k)][\tilde{E}^2(k)+I(k)]=\mathrm{i}\frac{\boldsymbol{k}}{k}\cdot\boldsymbol{j}_0(k)\tilde{E}(k), \tag{41}$$

其中

$$\varepsilon^{n}=-\frac{Ng^2m_{\mathrm{D}}^2}{4\omega^2}\int\mathrm{d}k'\,\mathrm{d}k_1\mathrm{d}k_2\mathrm{d}k_3\delta(k-k_1-k_2-k_3)\,I(\boldsymbol{k}_1)\frac{\pi}{\omega_1^2}\,\delta(\omega_1-\omega_1(\boldsymbol{k}_1))\cdot$$

$$(\omega\delta(k_3-k')\delta(k+k_2)\Gamma_{k,k_1,k_2,k_3}+\omega_1\delta(k_2-k')\,\delta(k_3+k_1)\Gamma_{k,k_1,k_2,k_3}) \tag{42}$$

并定义了涨落场强度

$$I(k)=-\int\mathrm{d}\omega'\,\mathrm{d}k'\,\langle E^{\mathrm{T}}(k)E^{\mathrm{T}}(k')\rangle. \tag{43}$$

既然涨落场强度是平方场量的量纲，那么方程(41)明显地是关于场量 $\tilde{E}$ 的非线性方程．不仅如此，由于涨落引起的非线性介电响应通过非线性介电函数已贡献到这个方程里．很容易看出，非线性介电函数的领头项(42)是非 Abelian 的．另外，如果涨落消失，非线性响应方程(41)将回到线性响应方程(20)．

为了计算的简化，考虑条件 $I\ll\tilde{E}^2$，可将非线性方程(41)线性化，从而获得

$$-\omega[\varepsilon^{l}(\omega,k)+\varepsilon^{n}(\omega,k)]\tilde{E}(\omega,k)=\mathrm{i}\frac{\boldsymbol{k}}{k}\cdot\boldsymbol{j}_{\mathrm{ext}}, \tag{44}$$

虽然这一方程对于场量 $\tilde{E}$ 是线性的，但其与线性响应方程(20)相比较，它含有由于系统涨落引起的非线性介电函数，是一个包含了非线性响应效应的线性化方程并且是在一定条件下的近似结果．方程(41)或(44)可给出外流在 QGP 介质中激发的场，这是计算 QGP 系统的非线性效应所必须的．

4 小结

1. 根据弱湍理论，可以展开 QGP 的动力论方程，并能逐级求解．

2. 通过求解各级分布函数给出涨落激发的各阶流，进而推导出 QGP 介质对外流的非线性响应方程．

3. 非线性方程在一定程度上反应了 QGP 的非 Abelian 特性，而线性响应方程却完全类同于 Abelian 等离子体的情形．

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Nonlinear Response Equation of Non-Abelian Medium to External Source *

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Abstract The kinetic equations for a quark-gluon plasma (QGP) are effectively expanded in terms of fluctuating field using weak turbulent theory. The nonlinear response equation of QGP is derived.

Key words quark-gluon plasma, Kinetic theory, Fluctuation

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