第 18 卷 第 6 期　　高能物理与核物理　　Vol. 18, No. 6
1994 年 6 月　　HIGH ENERGY PHYSICS AND NUCLEAR PHYSICS　　June, 1994

关于夸克凝聚和混合凝聚平移不变性的讨论

卞建国
(中国科学院高能物理研究所　北京　100039)

1993 年 3 月 30 日收到

摘　　要

讨论夸克凝聚 $\langle\Omega|\phi_i(x)\bar{\phi}_j(y)|\Omega\rangle$ 和混合凝聚 $\langle\Omega|\bar{\phi}_i(x)A_\mu(y)\phi_j(z)|\Omega\rangle$ 在 Lorentz 规范下的泰勒展开式，这两个展开式可用来计算算符乘积展开中非微扰修正项的 Wilson 系数. 与目前在 QCD 求和规则方法中所应用的、在固定点规范下的相应展开式相比，新的展开式具有平移不变性的优点.

关键词　QCD 求和规则，夸克凝聚，混合凝聚，平移不变性.

1 引　　言

由 Shifman、Vainshtein 和 Zakharov (1979 年)提出的 QCD 求和规则[1]是处理中能区量子色动力学（QCD）的一种有效方法，它已应用于强子问题的许多领域并取得了定性的成功，例如强子谱[2]、介子衰变常数[3]、形状因子[4]、强子波函数[5,6]和 B 参数[7]等. 然而目前的 QCD 求和规则在技术上存在一个明显的问题，就是平移不变性的破坏. 这可以从三个在 QCD 求和规则中用来计算算符乘积展开非微扰修正项 Wilson 系数的公式看出[8](见附录).

文献[9]讨论了胶子凝聚平移不变性问题. 在 Lorentz 规范

$$\partial_\mu A^{a\mu}(x)=0 \tag{1}$$

下对胶子凝聚 $\langle\Omega|A^a_\mu(x)A^b_\nu(y)|\Omega\rangle$ 作泰勒展开，得到展开式

$$\begin{aligned}&\langle\Omega|A^a_\mu(x)A^b_\nu(y)|\Omega\rangle\\&=g_{\mu\nu}\delta_{ab}\phi_{N_0}+\left(\frac{5}{6}(x-y)^2g_{\mu\nu}-\frac{1}{3}(x-y)_\mu(x-y)_\nu\right)\\&\quad\cdot\delta_{ab}\phi_{N_1}+\cdots\end{aligned}\tag{2}$$

具有明显的平移不变性. 本文将讨论夸克凝聚 $\langle\Omega|\phi_i(x)\bar{\phi}_j(y)|\Omega\rangle$ 和混合凝聚 $\langle\Omega|\bar{\phi}_i(x)A_\mu(y)\phi_j(z)|\Omega\rangle$ 的平移不变性问题. 这三个真空凝聚是 QCD 求和规则中常遇到的.

2 混合凝聚和夸克凝聚展开式

首先讨论混合凝聚 $\langle\Omega|\bar{\phi}_i(x)A_\mu(y)\phi_j(z)|\Omega\rangle$ 在 Lorentz 规范(1)下的泰勒展开式，

这里 i 和 j 是旋量指标，$A_\mu = \frac{\lambda^a}{2} A_\mu^a$。这个凝聚比夸克凝聚 $\langle \Omega | \bar{\psi}_i(x) \psi_j(y) | \Omega \rangle$ 要复杂一些。

将 $\langle \Omega | \bar{\psi}_i(x) A_\mu(y) \psi_j(z) | \Omega \rangle$ 进行泰勒展开，并考虑到由平移不变性，有

$$\partial_\rho \langle \Omega | \bar{\psi}_i(x) A_\mu(x) \psi_j(x) | \Omega \rangle = \partial_\rho \langle \Omega | \bar{\psi}_i(0) A_\mu(0) \psi_j(0) | \Omega \rangle = 0, \tag{3}$$

得：

$$\begin{aligned} &\langle \Omega | \bar{\psi}_i(x) A_\mu(y) \psi_j(z) | \Omega \rangle \\ &= \langle \Omega | \bar{\psi}_i A_\mu \psi_j | \Omega \rangle + \langle \Omega | \bar{\psi}_i \overleftarrow{\partial}_\rho A_\mu \psi_j | \Omega \rangle x^\rho + \langle \Omega | \bar{\psi}_i \partial_\rho A_\mu \psi_j | \Omega \rangle y^\rho \\ &\quad + \langle \Omega | \bar{\psi}_i A_\mu \partial_\rho \psi_j | \Omega \rangle z^\rho + \langle \Omega | \bar{\psi}_i \overleftarrow{\partial}_\rho \overleftarrow{\partial}_\lambda A_\mu \psi_j | \Omega \rangle \frac{(x-y)^\rho (x-y)^\lambda}{2} \\ &\quad + \langle \Omega | \bar{\psi}_i A_\mu \partial_\rho \partial_\lambda \psi_j | \Omega \rangle \frac{(z-y)^\rho (z-y)^\lambda}{2} + \langle \Omega | \bar{\psi}_i \overleftarrow{\partial}_\rho A_\mu \partial_\lambda \psi_j | \Omega \rangle \\ &\quad \cdot (y-x)^\rho (y-z)^\lambda + \cdots \end{aligned} \tag{4}$$

在 Lorentz 规范下对展开系数进行逐项分析。

1. $\langle \Omega | \bar{\psi}_i A_\mu \psi_j | \Omega \rangle$

令

$$\langle \Omega | \bar{\psi}_i A_\mu \psi_j | \Omega \rangle = A(\gamma_\mu)_{ji}, \tag{5.1}$$

用 γ^μ_{ij} 对上式两边进行收缩，得

$$A = \frac{1}{16} \langle \Omega | \bar{\psi} \not{A} \psi | \Omega \rangle. \tag{5.2}$$

2. $\langle \Omega | \bar{\psi}_i A_\mu \partial_\rho \psi_j | \Omega \rangle$

令

$$\langle \Omega | \bar{\psi}_i A_\mu \partial_\rho \psi_j | \Omega \rangle = (B g_{\mu\rho} + C \gamma_\rho \gamma_\mu)_{ji}, \tag{6.1}$$

用 $(\gamma^\mu \gamma^\rho)_{ij}$ 和 $(g^{\mu\rho})_{ij}$ 分别收缩上式两边，并使用 QCD 运动方程 $[i\not{D}(A) - m]\psi = 0$，得

$$B = \frac{1}{12} \langle \Omega | \bar{\psi} A \cdot \partial \psi | \Omega \rangle - \frac{ig}{48} \langle \Omega | \bar{\psi} \not{A} \not{A} \psi | \Omega \rangle + \frac{im}{48} \langle \Omega | \bar{\psi} \not{A} \psi | \Omega \rangle, \tag{6.2}$$

$$C = -\frac{1}{48} \langle \Omega | \bar{\psi} A \cdot \partial \psi | \Omega \rangle + \frac{ig}{48} \langle \Omega | \bar{\psi} \not{A} \not{A} \psi | \Omega \rangle - \frac{im}{48} \langle \Omega | \bar{\psi} \not{A} \psi | \Omega \rangle, \tag{6.3}$$

同理可得

$$\langle \Omega | \bar{\psi}_i \overleftarrow{\partial}_\rho A_\mu \psi_j | \Omega \rangle = (B' g_{\mu\rho} + C' \gamma_\mu \gamma_\rho)_{ji}, \tag{7.1}$$

其中

$$B' = \frac{1}{12} \langle \Omega | \bar{\psi} \overleftarrow{\partial} \cdot A \psi | \Omega \rangle - \frac{im}{48} \langle \Omega | \bar{\psi} \not{A} \psi | \Omega \rangle + \frac{ig}{48} \langle \Omega | \bar{\psi} \not{A} \not{A} \psi | \Omega \rangle, \tag{7.2}$$

$$C' = -\frac{1}{48} \langle \Omega | \bar{\psi} \overleftarrow{\partial} \cdot A \psi | \Omega \rangle + \frac{im}{48} \langle \Omega | \bar{\psi} \not{A} \psi | \Omega \rangle - \frac{ig}{48} \langle \Omega | \bar{\psi} \not{A} \not{A} \psi | \Omega \rangle, \tag{7.3}$$

从(7)式和 Lorentz 规范条件，有关系式

$$\langle \Omega | \bar{\psi} \overleftarrow{\partial} \cdot A \psi | \Omega \rangle = -\langle \Omega | \bar{\psi} A \cdot \partial \psi | \Omega \rangle, \tag{8}$$

从上式，可得

$$B = -B',$$

$$C = -C'.$$

3. $\langle \Omega|\bar{\phi}_i\partial_\rho A_\mu\phi_j|\Omega\rangle$

$$\begin{aligned}
&\langle \Omega|\bar{\phi}_i\partial_\rho A_\mu\phi_j|\Omega\rangle \\
&= (Bg_{\mu\rho} + C\gamma_\mu\gamma_\rho)_{ji} - (Bg_{\mu\rho} + C\gamma_\rho\gamma_\mu)_{ji} \\
&= \Big(-\frac{1}{48}\langle \Omega|\bar{\phi}A\cdot\partial\phi|\Omega\rangle + \frac{ig}{48}\langle \Omega|\bar{\phi}\not{A}\not{A}\phi|\Omega\rangle \\
&\quad - \frac{im}{48}\langle \Omega|\bar{\phi}\not{A}\phi|\Omega\rangle\Big)(\gamma_\mu\gamma_\rho - \gamma_\rho\gamma_\mu)_{ji},
\end{aligned} \tag{9}$$

(9)式显然满足规范条件,可以将 $\langle \Omega|\bar{\phi}A\cdot\partial\phi|\Omega\rangle$ 与 $\left\langle \Omega|\bar{\phi}\sigma_{\mu\rho}G_a^{\mu\rho}\frac{\lambda^a}{2}\phi|\Omega\right\rangle$ 联系起来,后者出现在固定点规范下的展开式 (A2-3) 中,从(9)式,可得

$$\begin{aligned}
\langle \Omega|\bar{\phi}_i G_{\mu\rho}\phi_j|\Omega\rangle &= \Big(\frac{1}{24}\langle \Omega|\bar{\phi}A\cdot\partial\phi|\Omega\rangle + \frac{im}{24}\langle \Omega|\bar{\phi}\not{A}\phi|\Omega\rangle \\
&\quad - \frac{ig}{24}\langle \Omega|\bar{\phi}\not{A}\not{A}\phi|\Omega\rangle\Big)(\gamma_\mu\gamma_\rho - \gamma_\rho\gamma_\mu)_{ji} \\
&\quad - ig\langle \Omega|\bar{\phi}_i(A_\mu A_\rho - A_\rho A_\mu)\phi_j|\Omega\rangle,
\end{aligned} \tag{10}$$

这里 $G_{\mu\rho}$ 为胶子场强张量. 用 $[\sigma^{\mu\rho}]_{ij}$ 收缩上式两边,得

$$\begin{aligned}
\langle \Omega|\bar{\phi}A\cdot\partial\phi|\Omega\rangle &= -\frac{i}{4}\langle \Omega|\bar{\phi}\sigma_{\tau\kappa}G^{\tau\kappa}\phi|\Omega\rangle + \frac{i}{2}g\langle \Omega|\bar{\phi}\not{A}\not{A}\phi|\Omega\rangle \\
&\quad + \frac{i}{2}g\langle \Omega|\bar{\phi}A\cdot A\phi|\Omega\rangle - im\langle \Omega|\bar{\phi}\not{A}\phi|\Omega\rangle,
\end{aligned} \tag{11}$$

所以,

$$\begin{aligned}
B &= -\frac{i}{48}\langle \Omega|\bar{\phi}\sigma_{\tau\kappa}G^{\tau\kappa}\phi|\Omega\rangle - \frac{im}{16}\langle \Omega|\bar{\phi}\not{A}\phi|\Omega\rangle \\
&\quad + \frac{ig}{48}\langle \Omega|\bar{\phi}\not{A}\not{A}\phi|\Omega\rangle + \frac{ig}{24}\langle \Omega|\bar{\phi}A\cdot A\phi|\Omega\rangle
\end{aligned} \tag{6.4}$$

$$\begin{aligned}
C &= \frac{i}{192}\langle \Omega|\bar{\phi}\sigma_{\tau\kappa}G^{\tau\kappa}\phi|\Omega\rangle + \frac{ig}{96}\langle \Omega|\bar{\phi}\not{A}\not{A}\phi|\Omega\rangle \\
&\quad - \frac{ig}{96}\langle \Omega|\bar{\phi}A\cdot A\phi|\Omega\rangle.
\end{aligned} \tag{6.5}$$

用同样的方式可对(4)式中其它系数进行分析,最后得

$$\begin{aligned}
&\langle \Omega|\bar{\phi}_i(x)A_\mu(y)\phi_j(z)|\Omega\rangle \\
&= A(\gamma_\mu)_{ji} + (Bg_{\mu\rho} + C\gamma_\mu\gamma_\rho)_{ji}(y-x)^\rho + (Bg_{\mu\rho} + C\gamma_\rho\gamma_\mu)_{ji}(z-y)^\rho \\
&\quad + (Dg_{\rho\lambda}\gamma_\mu + E(g_{\rho\mu}\gamma_\lambda + g_{\lambda\mu}\gamma_\rho))_{ji}(x-y)^\rho(x-y)^\lambda/2 \\
&\quad + (D^*g_{\rho\lambda}\gamma_\mu + E^*(g_{\rho\mu}\gamma_\lambda + g_{\lambda\mu}\gamma_\rho))_{ji}(z-y)^\rho(z-y)^\lambda/2 \\
&\quad + (Fg_{\rho\lambda}\gamma_\mu + Gg_{\rho\mu}\gamma_\lambda + Hg_{\mu\lambda}\gamma_\rho + I\gamma_\lambda\gamma_\mu\gamma_\rho)_{ji}(y-x)^\rho(y-z)^\lambda,
\end{aligned} \tag{12.1}$$

其中 A、B、C 由(5.2)、(6.4)和(6.5)式分别给出,

$$D = \frac{1}{288}\Big(\frac{1}{2}m\langle \Omega|\bar{\phi}\sigma_{\tau\kappa}G^{\tau\kappa}\phi|\Omega\rangle - 3m^2\langle \Omega|\bar{\phi}\not{A}\phi|\Omega\rangle$$

$$
\begin{aligned}
&-2mg\langle\Omega|\bar{\psi}\not{A}\not{A}\psi|\Omega\rangle-mg\langle\Omega|\bar{\psi}A\cdot A\psi|\Omega\rangle\\
&-\frac{5}{2}g\langle\Omega|\bar{\psi}\not{A}\sigma_{\tau\kappa}G^{\tau\kappa}\psi|\Omega\rangle+5g^{2}\langle\Omega|\bar{\psi}\not{A}A\cdot A\psi|\Omega\rangle\\
&+i8g\langle\Omega|\bar{\psi}\not{A}A\cdot\partial\psi|\Omega\rangle\Big),
\end{aligned}
\tag{12.2}
$$

$$
\begin{aligned}
E=\frac{1}{288}\Big(&-m\langle\Omega|\bar{\psi}\sigma_{\tau\kappa}G^{\tau\kappa}\psi|\Omega\rangle-3m^{2}\langle\Omega|\bar{\psi}\not{A}\psi|\Omega\rangle+4mg\langle\Omega|\bar{\psi}\not{A}\not{A}\psi|\Omega\rangle\\
&+2mg\langle\Omega|\bar{\psi}A\cdot A\psi|\Omega\rangle+i2g\langle\Omega|\bar{\psi}\not{A}A\cdot\partial\psi|\Omega\rangle+\frac{g}{2}\langle\Omega|\bar{\psi}\not{A}G_{\tau\kappa}\sigma^{\tau\kappa}\psi|\Omega\rangle\\
&-g^{2}\langle\Omega|\bar{\psi}\not{A}A\cdot A\psi|\Omega\rangle\Big),
\end{aligned}
\tag{12.3}
$$

$$
\begin{aligned}
F=&\frac{m^{2}}{96}\langle\Omega|\bar{\psi}\not{A}\psi|\Omega\rangle+\frac{g^{2}}{96}\langle\Omega|\bar{\psi}\not{A}\not{A}\not{A}\psi|\Omega\rangle-\frac{mg}{144}\langle\Omega|\bar{\psi}\not{A}\not{A}\psi|\Omega\rangle\\
&+\frac{g}{144}\langle\Omega|\bar{\psi}G_{\tau\kappa}\sigma^{\tau\kappa}\not{A}\psi|\Omega\rangle+\frac{g}{144}\langle\Omega|\bar{\psi}\not{A}G_{\tau\kappa}\sigma^{\tau\kappa}\psi|\Omega\rangle\\
&-\frac{g^{2}}{72}\langle\Omega|\bar{\psi}A\cdot A\not{A}\psi|\Omega\rangle-\frac{g^{2}}{72}\langle\Omega|\bar{\psi}\not{A}A\cdot A\psi|\Omega\rangle\\
&+\frac{ig}{36}\langle\Omega|\bar{\psi}\overleftarrow{\partial}\cdot A\not{A}\psi|\Omega\rangle+\frac{ig}{72}\langle\Omega|\bar{\psi}A_{\mu}\not{A}\partial^{\mu}\psi|\Omega\rangle\\
&-\frac{ig}{36}\langle\Omega|\bar{\psi}\not{A}A\cdot\partial\psi|\Omega\rangle-\frac{ig}{72}\langle\Omega|\bar{\psi}\overleftarrow{\partial}_{\mu}\not{A}A^{\mu}|\Omega\rangle\\
&+\frac{1}{72}\Big\langle\Omega|\bar{\psi}\frac{\lambda^{a}}{2}\gamma^{\nu}\psi\sum_{f}\bar{\psi}_{f}\frac{\lambda^{a}}{2}\gamma_{\nu}\psi_{f}|\Omega\Big\rangle+\frac{1}{36}\langle\Omega|\bar{\psi}(AG\gamma+\gamma GA)\psi|\Omega\rangle\\
&-\frac{m}{144}\langle\Omega|\bar{\psi}\sigma_{\tau\kappa}G^{\tau\kappa}\psi|\Omega\rangle+\frac{mg}{72}\langle\Omega|\bar{\psi}A\cdot A\psi|\Omega\rangle
\end{aligned}
\tag{12.4}
$$

$$
\begin{aligned}
G=&H^{*}\\
=&\frac{m^{2}}{96}\langle\Omega|\bar{\psi}\not{A}\psi|\Omega\rangle-\frac{g^{2}}{96}\langle\Omega|\bar{\psi}\not{A}\not{A}\not{A}\psi|\Omega\rangle+\frac{1}{288}mg\langle\Omega|\bar{\psi}\not{A}\not{A}\psi|\Omega\rangle\\
&-\frac{g}{288}\langle\Omega|\bar{\psi}\sigma_{\tau\kappa}G^{\tau\kappa}\not{A}\psi|\Omega\rangle-\frac{g}{288}\langle\Omega|\bar{\psi}\not{A}G^{\tau\kappa}\sigma_{\tau\kappa}\psi|\Omega\rangle\\
&+\frac{1}{144}g^{2}\langle\Omega|\bar{\psi}A\cdot A\not{A}\psi|\Omega\rangle+\frac{g^{2}}{144}\langle\Omega|\bar{\psi}\not{A}A\cdot A\psi|\Omega\rangle\\
&+\frac{i}{144}g\langle\Omega|\bar{\psi}\overleftarrow{\partial}\cdot A\not{A}\psi|\Omega\rangle+\frac{ig}{144}\langle\Omega|\bar{\psi}\overleftarrow{\partial}_{\mu}\not{A}A^{\mu}\psi|\Omega\rangle\\
&+\frac{ig}{72}\langle\Omega|\bar{\psi}\not{A}A\cdot\partial\psi|\Omega\rangle-\frac{ig}{144}\langle\Omega|\bar{\psi}A_{\mu}\not{A}\partial^{\mu}\psi|\Omega\rangle\\
&-\frac{1}{144}\Big\langle\Omega|\bar{\psi}\frac{\lambda^{a}}{2}\gamma^{\nu}\psi\sum_{f}\bar{\psi}_{f}\frac{\lambda^{a}}{2}\gamma_{\nu}\psi_{f}|\Omega\Big\rangle-\frac{1}{72}\langle\Omega|\bar{\psi}(AG\gamma+\gamma GA)\psi|\Omega\rangle\\
&+\frac{5}{576}m\langle\Omega|\bar{\psi}\sigma_{\tau\kappa}G^{\tau\kappa}\psi|\Omega\rangle-\frac{5}{288}mg\langle\Omega|\bar{\psi}A\cdot A\psi|\Omega\rangle,
\end{aligned}
\tag{12.5}
$$

$$
\begin{aligned}
I = & \frac{g^2}{96}\langle\Omega|\bar{\psi}AAA\psi|\Omega\rangle - \frac{mg}{96}\langle\Omega|\bar{\psi}AA\psi|\Omega\rangle + \frac{g}{384}\langle\Omega|\bar{\psi}\sigma_{\tau\kappa}G^{\tau\kappa}A\psi|\Omega\rangle \\
& + \frac{1}{384}\langle\Omega|\bar{\psi}A\sigma_{\tau\kappa}G^{\tau\kappa}\psi|\Omega\rangle + \frac{ig}{192}\langle\Omega|\bar{\psi}\overleftarrow{\partial}\cdot AA\psi|\Omega\rangle \\
& + \frac{ig}{192}\langle\Omega|\bar{\psi}A_\mu A\partial^\mu\psi|\Omega\rangle - \frac{g^2}{192}\langle\Omega|\bar{\psi}AA\cdot A\psi|\Omega\rangle \\
& - \frac{g^2}{192}\langle\Omega|\bar{\psi}A\cdot AA\psi|\Omega\rangle - \frac{ig}{192}\langle\Omega|\bar{\psi}AA\cdot\partial\psi|\Omega\rangle \\
& - \frac{ig}{192}\langle\Omega|\bar{\psi}\partial^\mu AA_\mu\psi|\Omega\rangle + \frac{1}{192}\left\langle\Omega\left|\bar{\psi}\frac{\lambda^a}{2}\gamma^\nu\psi\sum_f\bar{\psi}_f\frac{\lambda^a}{2}\gamma_\nu\psi_f\right|\Omega\right\rangle \\
& - \frac{m}{192}\langle\Omega|\bar{\psi}\sigma_{\tau\kappa}G^{\tau\kappa}\psi|\Omega\rangle + \frac{1}{96}\langle\Omega|\bar{\psi}(AG\gamma+\gamma GA)\psi|\Omega\rangle \\
& + \frac{mg}{96}\langle\Omega|\bar{\psi}A\cdot A\psi|\Omega\rangle,
\end{aligned}
\tag{12.6}
$$

这里 $AG\gamma = A_\mu G^{\mu\rho}\gamma_\rho$.

用类似的方式可求出夸克凝聚 $\langle\Omega|\bar{\psi}_i(x)\psi_j(y)|\Omega\rangle$ 的展开式,

$$
\begin{aligned}
& \langle\Omega|\bar{\psi}_i(x)\psi_j(y)|\Omega\rangle \\
& = \frac{1}{4}\delta_{ij}\langle\Omega|\bar{\psi}\psi|\Omega\rangle + (y-x)^\mu(\gamma_\mu)_{ji}\frac{i}{16}(-m\langle\Omega|\bar{\psi}\psi|\Omega\rangle \\
& + g\langle\Omega|\bar{\psi}A\psi|\Omega\rangle) + (y-x)^2(y-x)^\mu(\gamma_\mu)_{ji}\frac{i}{32}(m^2\langle\Omega|\bar{\psi}\psi|\Omega\rangle \\
& - 3m^2g^2\langle\Omega|\bar{\psi}A\psi|\Omega\rangle + 3mg^2\langle\Omega|\bar{\psi}AA\psi|\Omega\rangle - \frac{g}{2}\langle\Omega|\bar{\psi}AG_{\tau\kappa}\sigma^{\tau\kappa}\psi|\Omega\rangle \\
& + g^3\langle\Omega|\bar{\psi}AA\cdot A\psi|\Omega\rangle + 2ig^2\langle\Omega|\bar{\psi}AA\cdot\partial\psi|\Omega\rangle) + \cdots.
\end{aligned}
\tag{13}
$$

3 讨 论

上几节介绍了夸克凝聚和混合凝聚在 Lorentz 规范下的展开式,仅算到六维算符,这二个展开式具有明显的平移不变性. 这个性质是这二个真空凝聚在固定点规范下的展开式所不具有的.

应注意到固定点规范下的展开式 (A1—3) 中的系数已被唯象地定出,它们是[1,6,10]

$$
\begin{aligned}
& \langle\Omega|u\bar{u}|\Omega\rangle = \langle\Omega|d\bar{d}|\Omega\rangle = (250\mathrm{MeV})^3, \\
& \langle\Omega|s\bar{s}|\Omega\rangle \approx \langle\Omega|u\bar{u}|\Omega\rangle/1.3, \\
& \frac{\alpha_s}{\pi}\langle\Omega|G^2|\Omega\rangle = (447\mathrm{MeV})^4,
\end{aligned}
\tag{14}
$$

$$\langle\Omega|\bar{\psi}\sigma_{\tau\kappa}G^{\tau\kappa}\psi|\Omega\rangle = 2M_0^2\langle\Omega|\bar{\psi}\psi|\Omega\rangle,\quad M_0^2 = 0.2\mathrm{GeV},$$

对四夸克凝聚取真空中间整插入近似,即

$$\langle\Omega|\bar{\psi}\Gamma_1\psi\bar{\psi}\Gamma_2\psi|\Omega\rangle \approx \frac{1}{(4N_c)^2}(\mathrm{tr}\,\Gamma_1\mathrm{tr}\,\Gamma_2 - \mathrm{tr}\,\Gamma_1\Gamma_2)\langle\Omega|\bar{\psi}\psi|\Omega\rangle^2,$$

所以 (A1—3) 式是实用的.

(14)式中夸克凝聚 $\langle\Omega|u\bar{u}|\Omega\rangle$ 和 $\langle\Omega|d\bar{d}|\Omega\rangle$ 的值可在新的展开式中继续使用，因为它们来自于著名的 PCAC 关系 $(m_u+m_d)\langle\Omega|\bar{u}u+\bar{d}d|\Omega\rangle=-m_\pi^2f_\pi$；别的值需重新计算，从表面上看，它们本身是不依赖于规范的，但实际上计算这些值的唯象方法应用了固定点规范下的展开式 (A1—3). 对于那些新的展开式(2)、(12)和(13)特有的系数项，也需讨论它们的值.

感谢黄涛研究员、戴元本先生、何祚庥先生和赵光达教授的指导和有益讨论.

附　　录

QCD 求和规则常用的三个公式[1]是

$$\langle\Omega|A_\mu^a(x)A_\nu^b(y)|\Omega\rangle=\frac{1}{4}x^\lambda y^\rho\langle\Omega|G_{\lambda\mu}^aG_{\rho\nu}^b|\Omega\rangle+\cdots$$

$$=\frac{1}{384}x^\lambda y^\rho(g_{\lambda\rho}g_{\mu\nu}-g_{\lambda\nu}g_{\mu\rho})\delta^{ab}\langle\Omega|G^2|\Omega\rangle+\cdots,\tag{A1}$$

$$\begin{aligned}\langle\Omega|\bar{\psi}_i^\alpha(x)\psi_j^\beta(y)|\Omega\rangle\Big|_{y=0}=\frac{1}{12}\delta_{\alpha\beta}\Big\{&\Big[\delta_{ij}+\frac{i}{4}mx^\mu(\gamma_\mu)_{ij}\Big]\langle\Omega|\bar{\psi}\psi|\Omega\rangle\\&-\frac{1}{16}x^2\Big[\delta_{ij}+\frac{i}{6}mx^\mu(\gamma_\mu)_{ij}\Big]g\langle\Omega|\bar{\psi}\sigma_{\mu\nu}\frac{\lambda^a}{2}G_a^{\mu\nu}\psi|\Omega\rangle\\&+\frac{i}{288}x^2x^\mu(\gamma_\mu)_{ij}g^2\langle\Omega|\bar{\psi}\gamma^\rho\frac{\lambda^a}{2}\psi\sum_f\bar{\psi}_f\gamma_\rho\frac{\lambda^a}{2}\psi_f|\Omega\rangle\Big\},\end{aligned}\tag{A2}$$

$$\begin{aligned}&\langle\Omega|\bar{\psi}_i^\alpha(x)A_\rho^a(z)\psi_j^\beta(y)|\Omega\rangle\Big|_{y=0}\\&=\frac{1}{4}\Big(\frac{\lambda^a}{2}\Big)_{\beta\alpha}\langle\Omega|\bar{\psi}_i(x)A_\rho(z)\psi_j(y)|\Omega\rangle\\&=\frac{1}{4}\Big(\frac{\lambda^a}{2}\Big)_{\beta\alpha}\frac{z^\mu}{96}\Big\{\Big[\sigma_{\mu\rho}-\frac{m}{2}(x_\mu\gamma_\rho-x_\rho\gamma_\mu)+\frac{i}{2}mx^\nu\gamma_\nu\sigma_{\mu\rho}\Big]_{ji}\\&\cdot\langle\Omega|\bar{\psi}\sigma_{\tau\kappa}G_a^{\tau\kappa}\frac{\lambda^a}{2}\psi|\Omega\rangle+\Big[\Big(-\frac{2}{3}x_\mu\gamma_\rho+\frac{2}{3}x_\rho\gamma_\mu\Big)+\frac{i}{2}x^\nu\gamma_\nu\sigma_{\mu\rho}\Big]_{ji}\\&\cdot g\langle\Omega|\bar{\psi}\gamma^\lambda\frac{\lambda^a}{2}\psi\sum_f\bar{\psi}_f\gamma_\lambda\frac{\lambda^a}{2}\psi_f|\Omega\rangle,\end{aligned}\tag{A3}$$

这三个公式是在固定点规范 $x^\mu A_\mu^a(x)=0$ 下推导出来的.

(A1—3) 式都不具有平移不变性. (A1) 式左边是 $x-y$ 的函数，而右边却是 xy 的函数；(A2) 和 (A3) 式仅当 $y=0$ 时成立，否则两式右边会出现与 $A(y)$ 有关的项，这些项由于固定点规范的原因在 $y=0$ 时消失，这些都破坏了展开式的平移不变性.

参　考　文　献

[1] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, *Nucl. Phys.*, **B147**(1979)385; 448.
[2] L.J. Reinder et al., *Nucl. Phys.*, **B196**(1982)125;
M. Shifman and T.M. Aliev, *Phys. Lett.*, **B112**(1982) 401;
B.L. Ioffe, *Nucl. Phys.*, **B118**(1981)317; **B191**(1981)591(E). Also see ref.(1).
[3] C.A. Dominguez, *Phys. Lett.*, **B197**(1987) 423.
[4] B.L. Ioffe and A.V. Radyushkin, *Phys. Lett.*, **B115**(1982) 410; Sov. J., *Nucl. Phys.*, **39**(1984) 811; Ya.I. Kogan and V.M. Belyaev, *Phys. Lett.*, **B136**(1984)109.
[5] V.L. Chernyak and A.R. Zhitnitsky, *Nucl. Phys.*, **B201**(1982)492; **B246**(1984)52.

[6] V.L. Chernyak and A.R. Zhitnitsks, *Phys. Rep.*, **112**(1984)173.
[7] K.G. Chetyrkin et al., *Phys. Lett.*, **B174**(1986) 104; R. Deckon, *Nucl. Phys.*, **B277**(1986)661.
[8] S. Narison, CERN-TH. 4624/86.
[9] 卞建国,高能物理与核物理,**15**(1991)973.
[10] Y. Chung, H.G. Dosch, M. Kremer and D. Schall., *Z. Phys.*, **C25**(1984)151; B.L. Ioffe *Nucl. Phys.*, **B118**(1981) 317; **B191** (1981)591.

Discussion on the Translational Invariance of the Quark and Mixed Condensates

Bian Jianguo

(*Institute of High Energy Physics, Acadamia Sinica, Beijing* 100039)

Received on March 30, 1993

Abstract

The Taylor expansions of the quark and mixed condensates are presented Lorents gauge, which as well as the expansion of gluon condensate can be used to calculate the Wilson coefficients of the nonperturbative correction terms in the operator product expansion. In contrast to those current expansions in the QCD sum rules in the fixed gauge, the new expansion has the advantage of being translational invariant.

Key words QCD sum rules, quark condensate, mixed condensate, translational invariance.