Sensitivity study of $\bar{K}_1(1270)$ decay dynamics using four $D \to \bar{K}_1(1270)(\to \bar{K}\pi\pi)e^+\nu$ decay channels*

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Abstract: A sensitivity study of $\bar{K}_1(1270)$ decay-mode measurements is performed using semileptonic D-meson decays. The BESIII experiment is used as a case study, in which a simultaneous analysis of $\bar{K}_1(1270)$ decays to the four three-body final states $K^-\pi^+\pi^-$, $K^-\pi^+\pi^0$, $K^0_S\pi^+\pi^-$, and $K^0_S\pi^-\pi^0$ is presented, and a model-independent determination of $\mathcal{B}(\bar{K}_1(1270) \to \bar{K}\pi\pi)$ that does not require detailed knowledge of intermediate resonant contributions is proposed.

Keywords: BESIII, STCF, Charm meson, Axial-vector meson

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I. INTRODUCTION

The strange axial-vector mesons offer interesting possibilities for the study of quantum chromodynamics in the non-perturbative regime. Due to the presence of a strange quark with mass greater than the up and down quark masses, SU(3) symmetry is broken so that the $^3P_1$ and $^1P_1$ states mix with each other to construct the mass eigenstates, $\bar{K}_1(1200)$ and $\bar{K}_1(1400)$, by the mixing angle $\theta_{\bar{K}_1}$ [1]. The mixing angle $\theta_{\bar{K}_1}$ plays a crucial role in determining the theoretical calculations, such as the helicity form factors and branching fractions (BFs) for semileptpnic D-meson decays into strange axial-vector mesons [2−4].

Semileptonic charm decays, induced by the quark-level process $c \to se^+\nu_e$, are predominantly mediated by pseudoscalar (K) and vector ($K^*(892)$) mesons, i.e., contain a kaon and at most one pion in the final-state hadronic systems [5, 6]. However, semileptonic charm decays to higher-multiplicity final states are expected to proceed predominantly via the axial–kaon system [7] and are therefore strongly suppressed. The $D \to \bar{K}\pi\pi e^+\nu_e$ decays provide a unique opportunity to study the properties and interactions of $\bar{K}_1(1270)$ and $\bar{K}_1(1400)$ mesons in a clean environment, without any additional hadrons in the final states. Such studies can lead to a better determination of $\theta_{\bar{K}_1}$, as well as more precise measurements of the masses and widths of the $\bar{K}_1$ mesons, all of which currently carry large uncertainties [8]. Furthermore, by exploiting the measured properties of $D \to \bar{K}_1(1270)\ell^+\nu_\ell$ and $B \to \bar{K}_1(1270)\gamma$ decays, the photon polarization in $b \to s\gamma$ can be determined without considerable theoretical ambiguity [9, 10]. Charge-conjugate decays are implied throughout the paper.

The BFs of $\bar{K}_1(1270)$ decays to different two-body final states of $\bar{K}\rho$, $\bar{K}^*_0(1430)\pi$, $\bar{K}^*(892)\pi$, $\bar{K}\omega$, $\bar{K}f_0(1370)$ reported by the Particle Data Group (PDG) [8] are mostly based on a study of the $K^-\pi^+\pi^-$ system conducted in a $K^-p \to K^-\pi^-\pi^+p$ scattering experiment in 1981 [11], combined with a recent BESIII measurement of the branching ratio $\mathcal{B}(\bar{K}(1270) \to \bar{K}^*(892)\pi)/\mathcal{B}(\bar{K}(1270) \to \bar{K}\rho)$ in the $D^+_s \to K^-K^+\pi^+\pi^0$ decay [12]. All these BFs possesses large uncertainties, that lead to ~20% uncertainties on the $\bar{K}_1(1270) \to \bar{K}\pi\pi$ BFs [13], becoming a bottleneck for precise BF measurements on any decays with $\bar{K}_1(1270)$ as intermediate particles.

Although not used by the PDG for the BF averages, there are still a number of other measurements on the $\bar{K}_1(1270)$ decays. Based on an amplitude analysis of the decay $B^+ \to J/\psi K^+\pi^+\pi^-$, the Belle collaboration found the BFs of $\bar{K}_1(1270) \to \bar{K}\rho, \bar{K}\omega$, and $\bar{K}^*(892)\pi$ to be generally consistent with the PDG averages within two standard de-

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viations, while the measured BF of $\bar{K}_1(1270)\to K_0^*(1430)\pi$ is significantly smaller [14]. Later measurements of the BF ratio $\alpha\equiv\dfrac{\mathcal{B}(\bar{K}_1(1270)\to\bar{K}^*\pi)}{\mathcal{B}(\bar{K}_1(1270)\to\bar{K}\rho)}$, where $\mathcal{B}(\bar{K}_1(1270)\to\bar{K}^*\pi)=\mathcal{B}(\bar{K}_1(1270)\to\bar{K}^*(1430)\pi)$ $+\mathcal{B}(\bar{K}_1(1270)\to\bar{K}^*(892)\pi)$, yield different results depending on the decay channels used [15−18], whereas they are expected to be identical under the narrow width approximation for the $\bar{K}_1(1270)$ meson assuming *CP* conservation in strong decays [19].

The BESIII collaboration, through performing separate studies of the four hadronic systems $K^-\pi^+\pi^-$ [20], $K^-\pi^+\pi^0$ [21], $K_S^0\pi^+\pi^-$ and $K_S^0\pi^-\pi^0$ [22], reported the first observations of semielectronic D-meson decays involving a $\bar{K}_1(1270)$ and measured their BFs based on the assumed $\bar{K}_1(1270)$ decays. In addition, quite recently, an amplitude analysis of the $D^0\to K^-\pi^+\pi^-e^+\nu_e$ and $D^+\to K^-\pi^0\pi^-e^+\nu_e$ decays has been performed [23] with the larger $\psi(3770)$ dataset corresponding to an integrated luminosity of 20.3 fb^{-1}. The measured BFs are summarized in Table 1. In light of these measurements, in this work, a model-independent method is proposed to determine the BFs of $\bar{K}_1(1270)$ decays through a simultaneous analysis of signal yields from the four decay modes $D^0\to K^-\pi^+\pi^-e^+\nu_e$, $D^+\to K^-\pi^+\pi^0e^+\nu_e$, $D^0\to K_S^0\pi^-\pi^0e^+\nu_e$, and $D^+\to K_S^0\pi^+\pi^-e^+\nu_e$. With this method, the feasibility of measuring BFs of $\mathcal{B}(\bar{K}_1(1270)\to\bar{K}^*\pi)$, $\mathcal{B}(\bar{K}_1(1270)\to\bar{K}\rho)$ and $\mathcal{B}(\bar{K}_1(1270)\to\bar{K}\pi\pi)$, based on the current 20.3 fb^{-1} $\psi(3770)$ data sample from BESIII [24], is explored. The projected precisions on the BFs are also evaluated using pseudo-experiments.

II. FORMALISM

Table 1 lists the experimentally measured values of the BF $\mathcal{B}(D\to\bar{K}_1(1270)e^+\nu_e)$, which depend on the assumed decay BFs of the $\bar{K}_1(1270)$. In these measurements [20−23], the BF $\mathcal{B}(D\to\bar{K}\pi\pi e^+\nu_e)$ is expressed as the product of $\mathcal{B}(D\to\bar{K}_1(1270)e^+\nu_e)$ and $\mathcal{B}(\bar{K}_1(1270)\to\bar{K}\pi\pi)$ [25], where $\mathcal{B}(\bar{K}_1(1270)\to\bar{K}\pi\pi)$ denotes the total BF for $\bar{K}_1(1270)$ decays to $K\pi\pi$ final states:

$$\mathcal{B}(\bar{K}_1(1270)\to\bar{K}\pi\pi)=\sum_i f_i\mathcal{B}^i(\bar{K}_1(1270)\to f),\qquad(1)$$

where f_i is the square of the Clebsch–Gordan coefficient corresponding to the $i^{\rm th}$ decay mode of $\bar{K}_1(1270)$: $\bar{K}^*(1430)\pi$, $\bar{K}^*(892)\pi$, $\bar{K}\rho$, $\bar{K}\omega$. The last decay mode is neglected hereafter due to the smallness of the product $\mathcal{B}(\bar{K}_1(1270)\to\bar{K}\omega)\times\mathcal{B}(\omega\to\pi^+\pi^-)$. Regarding the completeness of the $\bar{K}_1(1270)$ decays, we define the sum $\mathcal{B}_{\rm 3body}$ as

$$\begin{aligned}\mathcal{B}_{\rm 3body}&=\mathcal{B}(\bar{K}_1(1270)\to\bar{K}\rho)+\mathcal{B}(\bar{K}_1(1270)\to\bar{K}^*\pi)\\&\quad+\mathcal{B}(\bar{K}_1(1270)\to\bar{K}f_0)\\&=1-\mathcal{B}(\bar{K}_1(1270)\to\bar{K}\omega)=(0.89\pm0.02)\%,\end{aligned}\qquad(2)$$

This is determined by $\mathcal{B}(\bar{K}_1(1270)\to\bar{K}\omega)$ [8].

A transition variable, which is directly related to the BFs for different signal reconstruction modes and can be determined experimentally in a straightforward manner, is defined as:

$$\beta^{-1}\equiv1-\frac{\mathcal{B}(K_1^-(1270)\to K^-\pi^+\pi^-)}{\mathcal{B}(\bar{K}_1^0(1270)\to K^-\pi^+\pi^0)}.\qquad(3)$$

Substituting Eq. (3) into the expression for α and defining the ratio $\delta_\alpha\equiv\dfrac{\mathcal{B}(\bar{K}_1(1270)\to\bar{K}f_0)}{\mathcal{B}(\bar{K}_1(1270)\to\bar{K}\rho)}$, the BF ratio α can be expressed as:

$$\alpha=\frac{3}{4}[\beta(1-3\delta_\alpha)-2].\qquad(4)$$

The BFs for $\bar{K}_1(1270)$ decays can then be expressed as:

Table 1. Summary of the measured BFs and the corresponding integrated luminosities ($\int\mathcal{L}dt$) for four D-meson semileptonic decay modes reported in Refs. [20−23]. The first and second uncertainties are statistical and systematic, respectively. For the BFs of $D\to\bar{K}_1(1270)e^+\nu_e$ decays, a third uncertainty arises from the assumed BFs of $\bar{K}_1(1270)$ decays [8].

Decay mode	Signal yield	$\mathcal{B}(D\to\bar{K}\pi\pi e^+\nu)\times10^{-4}$	$\mathcal{B}(D\to\bar{K}_1(1270)e^+\nu)\times10^{-3}$	$\int\mathcal{L}dt$
$D^0\to K^-\pi^+\pi^-e^+\nu_e$	109 ± 13	$(3.95\pm0.41^{+0.31}_{-0.52})$	$(1.09\pm0.13^{+0.09}_{-0.16}\pm0.12)$	2.93fb^{-1}
	731 ± 35	$(3.20\pm0.20\pm0.20)$	$(1.02\pm0.06\pm0.06\pm0.03)$	20.3fb^{-1}
$D^+\to K^-\pi^+\pi^0e^+\nu_e$	120 ± 13	$(10.6\pm1.2\pm0.8)$	$(2.30\pm0.26^{+0.18}_{-0.21}\pm0.25)$	2.93fb^{-1}
	1270 ± 56	$(12.70\pm0.60\pm0.40)$	$(2.27\pm0.11\pm0.07\pm0.07)$	20.3fb^{-1}
$D^0\to K_S^0\pi^-\pi^0e^+\nu_e$	17 ± 5	$(1.69^{+0.53}_{-0.46}\pm0.15)$	$(1.05^{+0.33}_{-0.28}\pm0.12\pm0.12)$	2.93fb^{-1}
$D^+\to K_S^0\pi^+\pi^-e^+\nu_e$	20 ± 6	$(1.47^{+0.45}_{-0.40}\pm0.14)$	$(1.29^{+0.40}_{-0.35}\pm0.18\pm0.15)$	2.93fb^{-1}

$$\mathcal{B}(K_1^-(1270)\to K^-\pi^+\pi^-)=\mathcal{B}_{3\text{body}}\cdot\frac{3+4\alpha+9\delta_\alpha}{9(1+\alpha+\delta_\alpha)}$$
$$=\frac{4}{3}\mathcal{B}_{3\text{body}}\cdot\frac{\beta-3\delta_\alpha\beta+3\delta_\alpha-1}{(3\beta-9\delta_\alpha\beta+4\delta_\alpha-2)},\quad(5)$$

$$\mathcal{B}(\bar{K}_1^0(1270)\to K^-\pi^+\pi^0)=\mathcal{B}_{3\text{body}}\cdot\frac{6+4\alpha}{9(1+\alpha+\delta_\alpha)}$$
$$=\frac{4}{3}\mathcal{B}_{3\text{body}}\cdot\frac{\beta-3\delta_\alpha\beta}{(3\beta-9\delta_\alpha\beta+4\delta_\alpha-2)}.\quad(6)$$

Since δ_α depends on $\mathcal{B}(\bar{K}_1(1270)\to\bar{K}\rho)$, it is necessary to remove this dependence to make the measurement model-independent. To better quantify the associated uncertainty, we introduce the shorthand $r_{f_0}=\frac{\mathcal{B}(\bar{K}_1(1270)\to\bar{K}f_0)}{\mathcal{B}_{3\text{body}}-\mathcal{B}(\bar{K}_1(1270)\to\bar{K}f_0)}=(3.5\pm2.3)\%$ [8] into Eq. (4):

$$\alpha=\frac{3}{4}[\beta(1-r_{f_0}(1+\alpha)-2]$$
$$=\frac{3(\beta-3r_{f_0}\beta-2)}{9r_{f_0}\beta+4}=\frac{3\beta-2}{9r_{f_0}\beta+4}-1,\quad(7)$$

This leads to an updated expression for δ_α:

$$\delta_\alpha=r_{f_0}(1+\alpha)=\frac{r_{f_0}(3\beta-2)}{9r_{f_0}\beta+4}.\quad(8)$$

By eliminating α in favor of δ_α, the BFs $\mathcal{B}(K_1^-(1270)\to K^-\pi^+\pi^-)$ and $\mathcal{B}(\bar{K}_1^0(1270)\to K^-\pi^+\pi^0)$ in Eqs. (5-6) can be expressed as:

$$\mathcal{B}(K_1^-(1270)\to K^-\pi^+\pi^-)$$
$$=\frac{4}{3}\mathcal{B}_{3\text{body}}\cdot\frac{(\beta-1)(1-3\delta_\alpha)}{3\beta(1-3\delta_\alpha)+4\delta_\alpha-2}$$
$$=\frac{4}{3}\mathcal{B}_{3\text{body}}\cdot\frac{(\beta-1)\cdot(4+6r_{f_0})}{4(3\beta-2)(r_{f_0}+1)},\quad(9)$$

$$\mathcal{B}(\bar{K}_1^0(1270)\to K^-\pi^+\pi^0)$$
$$=\frac{4}{3}\mathcal{B}_{3\text{body}}\cdot\frac{\beta(1-3\delta_\alpha)}{3\beta(1-3\delta_\alpha)+4\delta_\alpha-2}$$
$$=\frac{4}{3}\mathcal{B}_{3\text{body}}\cdot\frac{\beta\cdot(4+6r_{f_0})}{4(3\beta-2)(r_{f_0}+1)}.\quad(10)$$

In Eqs. 9 and 10, the parameter β is the sole free parameter in the formulation, whereas $\mathcal{B}_{3\text{body}}$ and r_{f_0} depend on external inputs, $\mathcal{B}(\bar{K}_1(1270)\to\bar{K}\omega)$ and $\mathcal{B}(\bar{K}_1(1270)\to\bar{K}f_0(1370))$, from the PDG. The value of β can be determined directly from experimental data by fitting the corresponding signal yields. Once β is obtained, all other physical observables—including the BFs and related quantities—can be derived from it, as they are explicit functions of β. This framework thus provides a consistent and model-independent approach in which all derived parameters are fully constrained by the experimentally determined value of β.

III. EXPERIMENTAL POTENTIALS

The BESIII collaboration has individually measured $D^0\to K^-\pi^+\pi^-e^+\nu_e$, $D^+\to K^-\pi^0\pi^-e^+\nu_e$, $D^0\to K_S^0\pi^0\pi^-e^+\nu_e$ and $D^+\to K_S^0\pi^+\pi^-e^+\nu_e$ decays [20–22], with the double-tag method [26, 27] and the $\psi(3770)$ dataset corresponding to an integrated luminosity of 2.93 fb^{-1}. A combined analysis of the $D^0\to K^-\pi^+\pi^-e^+\nu_e$ and $D^+\to K^-\pi^0\pi^-e^+\nu_e$ decays has been performed [23] with the larger $\psi(3770)$ dataset corresponding to an integrated luminosity of 20.3 fb^{-1}. In this section, a sensitivity study is performed by simultaneously fitting across the four decay modes, to determine the BFs of $\bar{K}_1(1270)$ decays, and the BF ratio α, at the same time.

One-dimensional pseudo-datasets of M^2_{miss} are generated for the decay modes of $D^0\to K^-\pi^+\pi^-e^+\nu_e$ and $D^{0,+}\to K_S^0\pi^-\pi^{0,+}e^+\nu_e$. Here M^2_{miss} is the missing mass square $M^2_{\text{miss}}\equiv E^2_{\text{miss}}/c^4-|\vec{p}_{\text{miss}}|^2/c^2$, with E_{miss} and $\vec{p}_{\text{miss}}$ being the total energy and momentum of all missing particles in the event, respectively. For the decay mode of $D^+\to K^-\pi^+\pi^0e^+\nu_e$, as the distribution of $U_{\text{miss}}\equiv E_{\text{miss}}-|\vec{p}_{\text{miss}}|c$ was used instead for signal yield extraction in Ref [20], the signal and background shapes of M^2_{miss} from the mode of $D^0\to K_S^0\pi^-\pi^0e^+\nu_e$ are used as approximations. The expected signal yields of the decays $D^0\to K^-\pi^+\pi^-e^+\nu_e$, $D^+\to K^-\pi^0\pi^-e^+\nu_e$, $D^0\to K_S^0\pi^0\pi^-e^+\nu_e$ and $D^+\to K_S^0\pi^+\pi^-e^+\nu_e$, based on the 20.3 fb^{-1} $\psi(3770)$ data, are estimated with

$$N(D\to\bar{K}\pi\pi e^+\nu_e)=2N(D\bar{D})\times\sum_i\mathcal{B}_i^{\text{ST}}\varepsilon^i_{\text{DT}}\times\mathcal{B}(D\to\bar{K}\pi\pi e^+\nu_e)\quad(11)$$

where $N(D\bar{D})$ denotes the total number of produced $D\bar{D}$ pairs [24], $\mathcal{B}_i^{\text{ST}}$ is the BF of the i^{th} tag mode, and $\varepsilon_i^{\text{DT}}$ is the double-tag efficiency. The sum runs over the same tag modes as in Refs. [20–22], and the values of $\varepsilon_i^{\text{DT}}$ are assumed to be identical to those in Refs. [20–22].

The background events are generated using background probability density functions previously determined from Monte Carlo (MC) simulations in Refs. [21, 22]. The estimated yields of the combinatorial and $D\to\bar{K}\pi\pi$ peaking backgrounds are scaled by a factor of seven to account for the smaller datasets used in Refs. [20–22].

To extract the parameters of interest, a simultaneous unbinned maximum-likelihood fit is performed on the four pseudo-datasets. The probability density functions that model the signal and background components are adopted from Refs. [21, 22], where they were determined from MC simulations. During the fit, the signal and combinatorial background yields are allowed to float, while the peaking-background yields are fixed to their generated values.

To minimize systematic effects from common sources such as luminosity, tagging, and tracking efficiencies, the parameter β is expressed in terms of ratios of signal yields:

$$\beta_{D^0}^{-1} = 1 - \frac{\mathcal{N}(K^-\pi^+\pi^-e^+\nu_e)}{\mathcal{N}(K^-\pi^+\pi^0e^+\nu_e)} = 1 - \frac{\mathcal{N}(K^-\pi^+\pi^-e^+\nu_e)}{\dfrac{\mathcal{N}(K_S^0\pi^-\pi^0e^+\nu_e)}{\mathcal{B}(K_S^0\to\pi^+\pi^-)/2}}, \tag{12}$$

$$\beta_{D^+}^{-1} = 1 - \frac{\mathcal{N}(K^-\pi^+\pi^-e^+\nu_e)}{\mathcal{N}(K^-\pi^+\pi^0e^+\nu_e)} = 1 - \frac{\dfrac{\mathcal{N}(K_S^0\pi^+\pi^-e^+\nu_e)}{\mathcal{B}(K_S^0\to\pi^+\pi^-)/2}}{\mathcal{N}(K^-\pi^+\pi^0e^+\nu_e)}. \tag{13}$$

where $\mathcal{N}$ denotes the efficiency-corrected signal yield for each decay mode. Assuming $\beta_{D^0}=\beta_{D^+}$, the average value of β is extracted from a simultaneous fit to the pseudo-datasets for the four decay modes. The remaining observables are then determined using Eqs. 9 and 10. The one-dimensional fit projections onto the $M^2_{\rm miss}$ distributions for the four decays are shown in Fig. 1 and the fit results are summarized in Table 2.

A total of 2000 pseudo-experiments are performed to assess potential biases introduced by the fit model. The resulting distribution of the pulls, defined as $\frac{\alpha_{\rm fit}-\alpha_{\rm nominal}}{\sigma_{\rm fit}}$, where $\alpha_{\rm fit}$ and $\sigma_{\rm fit}$ denote, respectively, the fitted α central value and its uncertainty in each pseudo-experiment, is shown in Fig. 2 and is consistent with a normal distribution, indicating that the fit model is unbiased in determining α.

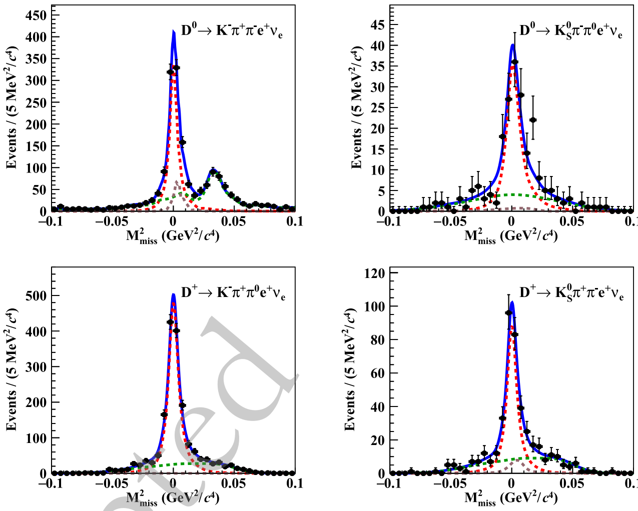

Fig. 1. (color online) Simultaneous fit to the $M^2_{\rm miss}$ distributions of the pseudo-datasets. Points with error bars show the pseudo-data; red dashed lines represent the signal shapes, green dashed lines the combinatorial background shapes, and brown dashed lines the peaking backgrounds from $D\to\bar{K}\pi\pi\pi$ decays.

Regarding potential sources of systematic uncertainty in the measurement, the double-tag method ensures that most tag-side uncertainties cancel. The uncertainties associated with the tracking and particle-identification efficiencies for e^+ and charged pions largely cancel in the ratios defined in Eqs. 12 and 13. The uncertainties on the π^0 and K_S^0 reconstruction efficiencies are 1% [13, 28]. This systematic uncertainty is evaluated by applying Gaussian constraints to the efficiency parameters during the fit, yielding a relative uncertainty of 3.8%. Similarly, the un-

Table 2. The measured values were obtained from a simultaneous fit to a single pseudo-dataset statistically matched to the 20.3 fb^{-1} $\psi(3770)$ dataset collected by BESIII. For each result ("Output") reported here, the first uncertainty is statistical, the second is systematic, and the third arises from the external input on $\mathcal{B}(\bar{K}_1(1270)\to\bar{K}\omega)$ [8]. The BESIII results [23] are listed for comparison.

Parameters	Input	Output	BESIII results
α [%]	20.3	$22.7\pm15.0\pm1.0\pm0.6$	$20.3\pm2.1\pm8.7$
$\mathcal{B}(\bar{K}_1(1270)\to\bar{K}^*(892)\pi)$ [%]	15.0	$16.5\pm9.0\pm0.7\pm3.5$	$19.5\pm1.9\pm5.2$ ★
			$10.9\pm1.2\pm3.0$ †
$\mathcal{B}(\bar{K}_1(1270)\to\bar{K}\rho)$ [%]	74.0	$72.5\pm9.0\pm0.7\pm3.5$	$71.8\pm2.3\pm23.9$ ★
			$79.3\pm2.0\pm25.7$ †
$\mathcal{B}(K_1^-(1270)\to K^-\pi^+\pi^-)$ [%]	31.3	$31.5\pm1.1\pm0.7\pm0.4$	31.3 ± 0.9
$\mathcal{B}(\bar{K}_1^0(1270)\to K^-\pi^-\pi^0)$ [%]	56.0	$55.7\pm2.1\pm1.3\pm0.8$	56.0 ± 2.7
$\mathcal{B}(D^0\to K_1^-(1270)e^+\nu_e)$ [$\times10^3$]	1.02	$1.01\pm0.05\pm0.02\pm0.01$	$1.02\pm0.06\pm0.06\pm0.03$
$\mathcal{B}(D^+\to \bar{K}_1^0(1270)e^+\nu_e)$ [$\times10^3$]	2.27	$2.29\pm0.10\pm0.05\pm0.01$	$2.27\pm0.11\pm0.07\pm0.07$

★ From the channel of $D^0\to K^-\pi^+\pi^-e^+\nu_e$; † From the channel of $D^+\to K^-\pi^+\pi^0e^+\nu_e$.

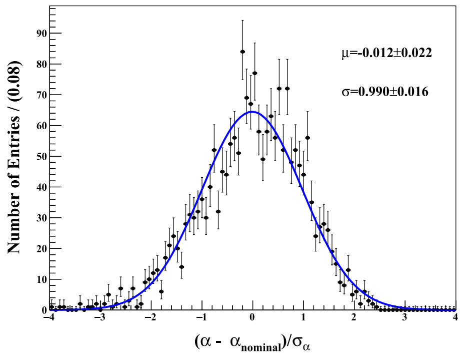

Fig. 2. (color online) The α pull distribution is defined as the difference between the reconstructed value α' and the input value $\alpha_{\rm nominal}$, normalized by the estimated uncertainty σ_α.

certainty originating from the assumed input branching fractions ($\mathcal{B}_{3\,\rm body}$ and r_{f_0}) is estimated by applying Gaussian constraints to these input parameters, contributing an additional 2.7%. Adding these independent sources in quadrature results in a conservative total estimate of 5%.

Compared with BESIII's amplitude analysis based on the 20.3 fb^{-1} $\psi(3770)$ dataset [23], the expected statistical uncertainties on α and on the BFs for $\bar{K}_1(1270)\to\bar{K}\rho,\bar{K}^*\pi$ in this work are larger, because this analysis does not exploit the full kinematic information of the $D\to\bar{K}\pi\pi e^+\nu_e$ decay (e.g., angular and q^2 distributions). Nevertheless, when systematic uncertainties are taken into account, this method achieves a significantly improved precision for $\mathcal{B}(\bar{K}_1(1270)\to\bar{K}\rho,\bar{K}^*\pi)$. With this method, the expected precision on the BFs for $\bar{K}_1(1270)\to K^-\pi^{+,0}\pi^-$ is comparable to the BESIII results, while the input uncertainties on $\mathcal{B}(D\to\bar{K}_1(1270)e^+\nu_e)$ are considerably reduced.

IV. SUMMARY

In this work, a sensitivity study is performed to evaluate the feasibility of measuring the absolute branching fraction (BF) $\mathcal{B}(\bar{K}_1(1270)\to\bar{K}\pi\pi)$ and the ratio $\alpha=\mathcal{B}(\bar{K}_1(1270)\to\bar{K}^*\pi)/\mathcal{B}(\bar{K}_1(1270)\to\bar{K}\rho)$. A model-independent approach to studying $\bar{K}_1(1270)$ decays is proposed, in which signal yields are simultaneously extracted from the four $K\pi\pi$ final states through a combined fit. The study demonstrates that a systematic uncertainty of approximately 5% can be achieved with the current $\psi(3770)$ data sample (20.3 fb^{-1}) from the BESIII experiment, providing a significant improvement over previous results [23].

By not relying on specific signal decay models, the combined analysis yields substantially lower systematic uncertainties for $\mathcal{B}(\bar{K}_1(1270)\to\bar{K}^*\pi)$, $\mathcal{B}(\bar{K}_1(1270)\to\bar{K}\rho)$, and their ratio α, while providing a robust, model-independent validation of existing amplitude analysis results. Furthermore, these results lay the groundwork for high-precision probes of axial-vector meson structure and decay dynamics and will become increasingly advantageous with larger datasets from the Super Tau-Charm Factory, where the statistical uncertainties are expected to be reduced by at least one order of magnitude [29, 30].

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References

[1] H.-Y. Cheng and X.-W. Kang, Eur. Phys. J. C 77: 587 (2017), [Erratum: Eur.Phys.J.C 77, 863 (2017)], arXiv: 1707.02851 [hep-ph].
[2] G.-Y. Wang, L. Roca, E. Wang, W.-H. Liang, and E. Oset, Eur. Phys. J. C **80**, 388 (2020), arXiv: 2002.07610[hep-ph]
[3] S. Momeni, Eur. Phys. J. C **80**, 553 (2020), arXiv: 2004.02522[hep-ph]
[4] S. Momeni and R. Khosravi, J. Phys. G **46**, 105006 (2019), arXiv: 1903.00860[hep-ph]
[5] N. Isgur, D. Scora, B. Grinstein, and M. B. Wise, Phys. Rev. D **39**, 799 (1989)
[6] D. Scora and N. Isgur, Phys. Rev. D **52**, 2783 (1995), arXiv: hep-ph/9503486
[7] H. Hatanaka and K.-C. Yang, Phys. Rev. D **78**, 074007 (2008), arXiv: 0808.3731[hep-ph]
[8] S. Navas *et al.* (Particle Data Group), Phys. Rev. D **110**, 030001 (2024)
[9] W. Wang, F.-S. Yu, and Z.-X. Zhao, Phys. Rev. Lett. **125**, 051802 (2020), arXiv: 1909.13083[hep-ph]
[10] L. Bian, L. Sun, and W. Wang, Phys. Rev. D **104**, 053003 (2021), arXiv: 2105.06207[hep-ph]
[11] C. Daum *et al.* (ACCMOR), Nucl. Phys. B **187**, 1 (1981)
[12] M. Ablikim *et al.* (BESIII), Phys. Rev. D **104**, 032011 (2021), arXiv: 2103.02482[hep-ex]
[13] M. Ablikim *et al.* (BESIII), JHEP **10**, 199 (2024), arXiv: 2408.04422[hep-ex]
[14] H. Guler *et al.* (Belle), Phys. Rev. D **83**, 032005 (2011), arXiv: 1009.5256[hep-ex]
[15] M. Artuso *et al.* (CLEO), Phys. Rev. D **85**, 122002 (2012), arXiv: 1201.5716[hep-ex]
[16] M. Ablikim *et al.* (BESIII), Phys. Rev. D **95**, 072010 (2017), arXiv: 1701.08591[hep-ex]
[17] R. Aaij *et al.* (LHCb), Eur. Phys. J. C **78**, 443 (2018), arXiv: 1712.08609[hep-ex]
[18] P. d'Argent, N. Skidmore, J. Benton, J. Dalseno, E. Gersabeck, S. Harnew, P. Naik, C. Prouve, and J. Rademacker, JHEP **05**, 143 (2017), arXiv: 1703.08505[hep-ex]
[19] P.-F. Guo, D. Wang, and F.-S. Yu, Nucl. Phys. Rev. **36**, 125 (2019), arXiv: 1801.09582[hep-ph]
[20] M. Ablikim *et al.* (BESIII), Phys. Rev. Lett. **123**, 231801 (2019), arXiv: 1907.11370[hep-ex]

[21] M. Ablikim *et al.* (BESIII), Phys. Rev. Lett. **127**, 131801 (2021), arXiv: 2102.10850[hep-ex]

[22] M. Ablikim *et al.* (BESIII), JHEP **09**, 089 (2024), arXiv: 2403.19091[hep-ex]

[23] M. Ablikim *et al.* (BESIII), Phys. Rev. Lett. **135**, 091801 (2025)

[24] M. Ablikim *et al.* ((BESIII),, BESIII), Chin. Phys. C **48**, 123001 (2024), arXiv: 2406.05827[hep-ex]

[25] $\mathcal{B}\left(K_1^-(1270) \to K^-\pi^+\pi^-\right) = \frac{1}{3}\mathcal{B}_{K_1\to K\rho} + \frac{4}{9}\mathcal{B}_{K_1\to K^*\pi} + \mathcal{B}_{K_1\to K\omega} \times \mathcal{B}_{\omega\to\pi+\pi^-} + \mathcal{B}_{K_1\to K f_0}, \mathcal{B}\left(\bar{K}_1^0(1270) \to K^-\pi^+\pi^0\right) = \frac{2}{3}\mathcal{B}_{K_1\to K\rho} + \frac{4}{9}\mathcal{B}_{K_1\to K^*\pi}$.

[26] R. M. Baltrusaitis *et al.* (MARK-III), Phys. Rev. Lett. **56**, 2140 (1986)

[27] J. Adler *et al.* (MARK-III), : 153 (1987).

[28] M. Ablikim *et al.* (BESIII), Phys. Rev. D **110**, L111102 (2024), arXiv: 2404.09219[hep-ex]

[29] M. Achasov *et al.*, Front. Phys. (Beijing) **19**, 14701 (2024), arXiv: 2303.15790[hep-ex]

[30] Y.-L. Fan, X.-D. Shi, X.-R. Zhou, and L. Sun, Eur. Phys. J. C **81**, 1068 (2021), arXiv: 2107.06118[hep-ex]