Contributions of $\rho(770,1450)\to\omega\pi$ to Cabibbo-favored $D\to h\omega\pi$ decays*

Wen-Fei Wang (王文飞)[1†] Jiao-Yuan Xu (徐姣媛)[1] Si-Hong Zhou (周四红)[2‡] Pan-Pan Shi (石盼盼)[3§]

[1]Institute of Theoretical Physics and State Key Laboratory of Quantum Optics and Quantum Optics Devices, Shanxi University, Taiyuan, Shanxi 030006, China
[2]Mongolia Key Laboratory of Microscale Physics and Atom Innovation, School of Physical Science and Technology, Inner Mongolia University, Hohhot 010021, China
[3]Departamento de Física Teórica and IFIC, Centro Mixto Universidad de Valencia-CSIC Institutos de Investigación de Paterna, Valencia 46071, Spain

Abstract: Recently, the BESIII Collaboration observed the three-body decays $D_s^+\to\eta\omega\pi^+$, $D^+\to K_S^0\pi^+\omega$, and $D^0\to K^-\pi^+\omega$. In this study, we investigate the contributions of the subprocesses $\rho^+\to\omega\pi^+$ in these Cabibbo-favored decays $D\to h\omega\pi$, with $\rho^+=\{\rho(770)^+,\rho(1450)^+,\rho(770)^+\&\rho(1450)^+\}$ and $h=\{\eta,K_S^0,K^-\}$, by introducing these subprocesses into the decay amplitudes of the relevant decay processes via the vector form factor $F_{\omega\pi}$, which has been measured in the related τ and e^+e^- processes. We provide the first theoretical predictions for the branching fractions of the quasi-two-body decays $D_s^+\to\eta[\rho^+\to]\omega\pi^+$, $D^+\to K_S^0[\rho^+\to]\omega\pi^+$, and $D^0\to K^-[\rho^+\to]\omega\pi^+$. Our findings reveal that the contributions from the subprocess $\rho(770)^+\to\omega\pi^+$ are significant in these observed three-body decays $D_s^+\to\eta\omega\pi^+$, $D^+\to K_S^0\omega\pi^+$, and $D^0\to K^-\omega\pi^+$, notwithstanding the contributions originating from the Breit-Wigner tail effect of $\rho(770)^+$. The numerical results of this study suggest that the dominant resonance contributions for the three-body decays $D_s^+\to\eta\omega\pi^+$ and $D^+\to K_S^0\omega\pi^+$ originate from the P-wave intermediate states $\rho(770)^+$, $\rho(1450)^+$ and their interference effects.

Keywords: three-body decays, branching fraction, D meson decays

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I. INTRODUCTION

Recently, the three-body decay $D_s^+\to\eta\omega\pi^+$ was observed for the first time by BESIII Collaboration, with its total branching fraction $\mathcal{B}=(0.54\pm0.12_{\rm stat}\pm0.04_{\rm syst})\%$ [1]. This numerical result aligns with an earlier measurement by the CLEO Collaboration, which reported a branching fraction of $(0.85\pm0.54_{\rm stat}\pm0.06_{\rm syst})\%$ in Ref. [2] for the same decay channel, but the BESIII measurement offers a significantly improved precision. Given the quark structure of the initial and final states in this D_s decay process, the Cabibbo-favored transition $c\to s$ along with $W^+\to u\bar{d}$ will be the dominant process at the quark level. The prospective intermediate states for this process could be the resonances $a_0(980)^+$, $\rho(770)^+$, $b_1(1235)^+$, $\omega(1420)$, etc., and their excited states which will decay into the $\pi^+\eta$, $\omega\pi^+$, and $\omega\eta$ pairs in the final state [3]. The combination of π^+ and $\omega\eta$ with the intermediate resonance $\omega(1420)$ implies a pure annihilation process for this D_s^+ decay. The union of ω and $a_0(980)^+\to\pi^+\eta$ for this decay is very similar to the decay process $D_s^+\to\pi^0[a_0(980)^+\to]\pi^+\eta$, which has been measured by BESIII [4], and the branching fraction was found to be consistent with the triangle rescattering processes via the intermediate processes $D_s^+\to\eta^{(\prime)}\rho^+$, $D_s^+\to K^*\bar{K}$, and $D_s^+\to K\bar{K}^*$ [5−10]. However, the branching fraction for $D_s^+\to\omega a_0(980)^+$ was found to be less than 3.4×10^{-4} in Ref. [10] owing to the cancellation of the rescattering effects and suppressed contribution of the short-distance W annihilation. Consequently, the dominant contributions for the decay $D_s^+\to\eta\omega\pi^+$ are expected to arise from the combination of η and the resonances, which will decay into the $\omega\pi^+$ pair in the final state.

The light meson pair of $\omega\pi$, which originates from the weak current of the matrix element $\langle\omega\pi|V^\mu-A^\mu|0\rangle$ in the three-body hadronic D and B meson decays, is related to

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† E-mail: wfwang@sxu.edu.cn
‡ E-mail: shzhou@imu.edu.cn
§ E-mail: Panpan.Shi@ific.uv.es

the processes $\tau \to \omega\pi\nu_\tau$ and $e^+e^- \to \omega\pi$. The G-parity conservation requires that the $\omega\pi$ pair is determined mainly by the vector part V^μ of the weak current [11], which is associated with the ρ family resonances as the intermediate state via the P-wave transition amplitudes [12]. The axial-vector part A^μ of the weak current for the $\omega\pi$ system corresponds to the resonance $b_1(1235)$ and its excited states. The state $b_1(1235)$, with the quantum numbers $J^{PG} = 1^{++}$, is related to the second-class weak current classified in terms of the parity and G parity [13]. The second-class weak current has been investigated in τ decays in recent years, but no evidence has been observed [14, 15]. The decay amplitude for the $b_1(1235) \to \omega\pi$ subprocess through S- and D-waves in the relevant processes is proportional to the mass difference between the u and d quarks [3], which significantly suppresses the contribution of $\omega\pi$ from the axial-vector resonance $b_1(1235)$ accompanied by the second-class current.

The ordinary decay mode for $\rho(770) \to \omega\pi$ is not allowed because of the pole mass of $\rho(770)$, which is apparently below the threshold of the $\omega\pi$ pair [3]. However, the Breit-Wigner (BW) [16] tail effect of $\rho(770)$, also known as the virtual contribution [17−20], was found indispensable to the production of the $\omega\pi$ pair in the processes of $\tau \to \omega\pi\nu_\tau$ [21−24] and $e^+e^- \to \omega\pi^0$ [25−34]. The excited state $\rho(1450)$ has been observed to decay into the $\omega\pi$ pair in B meson decays by the CLEO, BaBar, and Belle Collaborations [35−37]. This state has been suggested as a $2S$-hybrid mixture in Ref. [38] considering its decay characters [39−41], but its mass is consistent with that in the $2S$ excitation of $\rho(770)$ [42]. The further investigation of the interference between the $\rho(1450)$ and its ground state will provide deeper insights into its nature.

In addition to the data for the $D_s^+ \to \eta\omega\pi^+$ decay, the absolute branching fractions of the decays $D^0 \to K^-\pi^+\omega$, $D^0 \to K_S^0\pi^0\omega$, and $D^+ \to K_S^0\pi^+\omega$ were recently determined by the BESIII Collaboration and reported in Ref. [43], with the results as $(3.392 \pm 0.044_{\rm stat} \pm 0.085_{\rm syst})\%$, $(0.848 \pm 0.046_{\rm stat} \pm 0.031_{\rm syst})\%$, and $(0.707 \pm 0.041_{\rm stat} \pm 0.029_{\rm syst})\%$, respectively. To better understand these experimental results, we investigated the Cabibbo-favored three-body decays $D_s^+ \to \eta\omega\pi^+$, $D^+ \to K_S^0\omega\pi^+$, and $D^0 \to K^-\omega\pi^+$ in this study, where the $\omega\pi$ pair is attributed to the decays of the intermediate states $\rho(770)^+$ and $\rho(1450)^+$. The schematic for the quasi-two body decay $D_s^+ \to \eta\rho^+ \to \eta\omega\pi^+$ is shown in Fig. 1. In its rest frame, the state D_s^+ will decay into the intermediate resonance ρ^+ along with the bachelor state η. Subsequently, ρ^+ decays into ω and π^+ via the strong interaction. A similar pattern will arise in the D^+ and D^0 decay processes, with η replaced by K_S^0 and K^-, respectively. The subprocesses $\rho(770,1450)^+ \to \omega\pi^+$ in these decays will be introduced into their decay amplitudes in the isobar formalism [44−46] via the vector form factor $F_{\omega\pi}$. This form factor has been measured in the processes related to τ decay and e^+e^- annihilation.

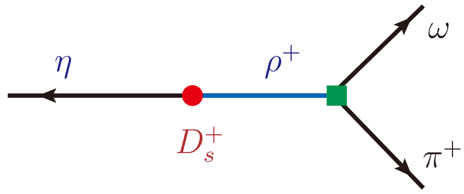

Fig. 1. (color online) Schematic of the cascade decay $D_s^+ \to \eta\rho^+ \to \eta\omega\pi^+$, where ρ^+ denotes the intermediate states $\rho(770,1450)^+$ that decayed into $\omega\pi^+$ in this study.

The contributions of $\rho(770,1450) \to \omega\pi$ to the three-body decays $B \to \bar{D}^{(*)}\omega\pi$ were recently studied in Ref. [47]. The ρ family resonance contributions of the kaon pair have been explored in Refs. [48−54] and in Refs. [55−58] for the three-body B and D meson decays, respectively. In Ref. [25], four resonances, namely, $\rho(770)$, $\rho(1450)$, $\rho(1700)$, and $\rho(2150)$, were employed to parametrize the related transition form factor for $\rho \to \omega\pi$ for the process $e^+e^- \to \omega\pi^0 \to \pi^+\pi^-\pi^0\pi^0$ in the energy range 1.05-2.00 GeV by the SND Collaboration. However, we will examine the contributions of $\rho(1700,2150) \to \omega\pi$ to the relevant decays in future studies considering that the masses of $\rho(1700,2150)$ are very close to or even exceed the masses of the initial $D^{+,0}$ and D_s^+ mesons, the contributions to $\omega\pi$ from $\rho(1700,2150)$ are unimportant when compared with those of $\rho(770)$ and $\rho(1450)$ [25], and in addition, the excited ρ states around 2 GeV are not well understood [34, 59].

Owing to the c-quark mass, the heavy quark expansion tools and factorization approaches, which have been successfully used in hadronic B meson decays for decades, encounter significant challenges when applied to the two-body or three-body hadronic D meson decays. In this context, methods that do not rely on models, such as the topological-diagram approach [60−66] and factorization-assisted topological-amplitude approach [67−70], have been adopted in various D decay studies. In Ref. [55], we constructed a theoretical framework for quasi-two-body D meson decays with the help of electromagnetic form factors, with which we studied the contributions of $\rho(770,1450) \to K\bar{K}$ for the three-body D decays within the flavour $SU(3)$ symmetry. In this study, we adopted the method in [55] and investigated the pertinent decays within the quasi-two-body framework [55, 71, 72], while neglecting the interaction between the $\omega\pi$ system and the corresponding bachelor state in the relevant decay processes.

This paper is organized as follows: in Sec. II, we take $D^0 \to K^-[\rho(770)^+ \to]\omega\pi^+$ as an example to derive the differential branching fractions for the quasi-two-body D meson decay processes. In Sec. III, we present our numerical results for the branching fractions of the quasi-two-body decays $D_s^+ \to \eta[\rho^+ \to]\omega\pi^+$, $D^+ \to K_S^0[\rho^+ \to]\omega\pi^+$, and $D^0\to K^-[\rho^+\to]\omega\pi^+$, along with necessary discussions. Finally, Sec. IV summarizes this study.

II. DIFFERENTIAL BRANCHING FRACTIONS

In this section, we use the decay $D^0\to K^-[\rho(770)^+\to]\omega\pi^+$ as an example to derive the differential branching fraction for the quasi-two-body D meson decay processes. Shrinking the subprocess $\rho(770)^+\to\omega\pi^+$ to the meson $\rho(770)^+$ will yield the two-body decay $D^0\to K^-\rho(770)^+$. The related effective weak Hamiltonian for D decays is written as [73]

$$\mathcal{H}_{\rm eff}=\frac{G_F}{\sqrt{2}}\Bigg[\sum_{q=d,s}\lambda_q(C_1O_1+C_2O_2)-\lambda_b\sum_{i=3}^{6}C_iO_i-\lambda_bC_{8g}O_{8g}\Bigg],\tag{1}$$

where the Fermi coupling constant $G_F=1.1663788(6)\times10^{-5}\,{\rm GeV}^{-2}$ [3]; the product of the Cabbibo-Kobayashi-Maskawa (CKM) [74, 75] matrix elements $\lambda_q=V_{cq}^*V_{uq}$ and $\lambda_b=V_{cb}^*V_{ub}$; C denotes the Wilson coefficients at the scale μ; O_1 and O_2 are the current-current operators; O_3 and O_4 are the QCD penguin operators; and O_{8g} is the chromomagnetic dipole operator. The total decay amplitude for the process of a D meson decays into a pseudoscalar (P) plus a vector (V), which can be described using the typical topological diagram amplitudes $T_{P,V}$, $C_{P,V}, E_{P,V}$, and $A_{P,V}$ according to the diagrams shown in Fig. 2, as well as the additional penguin amplitudes in the factorization-assisted topological-amplitude approach [68] and topological-diagram approach [62−65]. The detailed discussions on these topological diagram amplitudes are available in Refs. [62−65, 68].

The decay amplitude for $D^0\to K^-\rho(770)^+$ is dominated by the color-favored tree amplitude T_P^{2B} with the $D^0\to K^-$ transition, as shown in Fig. 2-(a), where the superscript $2B$ represents a two-body decay process. We have the amplitude T_P^{2B} as [63, 64, 68]

$$\begin{aligned}T_P^{2B}&=\frac{G_F}{\sqrt{2}}V_{cs}^*V_{ud}\left[\frac{C_1}{3}+C_2\right]f_\rho m_\rho F_1^{D\to\bar K}(m_\rho^2)2\epsilon_\rho\cdot p_D\\&=\mathcal{M}_T^{2B}\epsilon_\rho\cdot p_D,\end{aligned}\tag{2}$$

where the subscript ρ denotes the resonance $\rho(770)$; $F_1^{D\to\bar K}(m_\rho^2)$ is the transition form factor for the $D^0\to K^-$ process. Beyond the dominant amplitude T_P^{2B}, a W-exchange nonfactorizable contribution of the amplitude E_V^{2B} in the topological diagram Fig. 2-(c) exists, which can be parametrized as $E_V^{2B}=\mathcal{M}_E^{2B}\epsilon_\rho\cdot p_D$. In addition, we have [68]

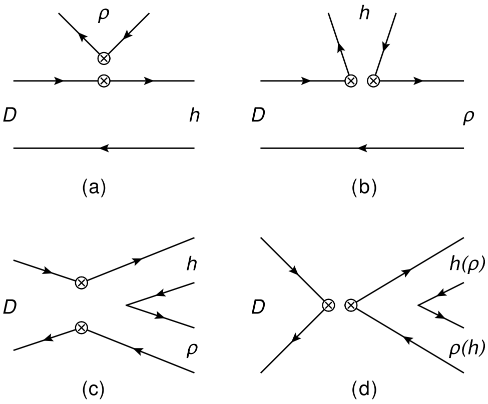

Fig. 2. Typical topological diagrams for the concerned decays at quark level, where ρ stands for the intermediate states $\rho(770,1450)^+$, h for η, K_S^0 or K^-, and the symbol $\otimes$ stands for the weak vertex in this work.

$$\mathcal{M}_E^{2B}=\frac{G_F}{\sqrt{2}}V_{cs}^*V_{ud}C_2\chi_q^Ee^{i\phi_q^E}f_Dm_D\left[f_K/f_\pi\right],\tag{3}$$

in the factorization-assisted topological-amplitude approach, where $\chi_q^E=0.25$ and $\phi_q^E=1.73$ [68] are two parameters that characterize the strength and strong phase of the corresponding amplitude; f_ρ, f_D, f_K, and f_π are the decay constants for $\rho(770)$, D^0, kaon, and pion, respectively. In addition to the amplitudes T_P^{2B} and E_V^{2B}, we need [68]

$$\begin{aligned}C_V^{2B}&=\mathcal{M}_C^{2B}\epsilon_\rho\cdot p_D=\frac{G_F}{\sqrt{2}}V_{cs}^*V_{ud}\left[C_1+C_2(1/3+\chi_V^Ce^{i\phi_V^C})\right]\\&\quad\times f_Km_\rho A_0^{D\to\rho}(m_K^2)2\epsilon_\rho\cdot p_D,\end{aligned}\tag{4}$$

$$\begin{aligned}A^{2B}&=\mathcal{M}_A^{2B}\epsilon_\rho\cdot p_D\\&=\frac{G_F}{\sqrt{2}}V_{cs}^*V_{ud}C_1\chi_q^Ae^{i\phi_q^A}f_Dm_D\left[f_{\eta^{(\prime)}}/f_\pi\right]\epsilon_\rho\cdot p_D,\end{aligned}\tag{5}$$

in the numerical calculations of the decays $D^+\to K_S^0\rho(770)^+$ and $D_s^+\to\eta\rho(770)^+$ according to the topological diagrams Fig. 2-(b) and -(d), respectively. The remaining parameters are set as $\chi_V^C=-0.53$, $\phi_V^C=-0.25$, $\chi_q^A=0.11$, and $\phi_q^A=-0.35$ by referring to [68].

For the two-body $D^0\to K^-\rho(770)^+$ decay, the Lorentz-invariant total decay amplitude is

$$\mathcal{A}^{2B}=\mathcal{M}^{2B}\epsilon_\rho\cdot p_D=(\mathcal{M}_T^{2B}+\mathcal{M}_E^{2B})\epsilon_\rho\cdot p_D.\tag{6}$$

By utilizing the partial decay rate

$$d\Gamma=\frac{1}{32\pi^2}|\mathcal{A}|^2\frac{|\vec{q}|}{m_D^2}d\Omega\tag{7}$$

in *Review of Particle Physics* [3] along with the formula

$$\sum_{\lambda=0,\pm}\varepsilon^{*\mu}(p,\lambda)\varepsilon^{\nu}(p,\lambda)=-g^{\mu\nu}+\frac{p^{\mu}p^{\nu}}{p^2},\tag{8}$$

it is trivial to obtain the partial decay width

$$\Gamma(D^0\to K^-\rho(770)^+)=\frac{|\vec{q}|^3}{8\pi m_\rho^2}|\mathcal{M}^{2B}|^2.\tag{9}$$

The magnitude of the momentum $|\vec{q}|$ for $\rho(770)^+$ or K^- is

$$\left|\vec{q}\right|=\frac{1}{2m_D}\sqrt{\left[m_D^2-(m_\rho+m_K)^2\right]\left[m_D^2-(m_\rho-m_K)^2\right]},\tag{10}$$

in the rest frame of the D^0 meson, and m_i (with $i=\{D^0,\rho(770)^+,K^-\}$) denotes the masses of the relevant particles above.

By connecting the subprocess $\rho(770)^+\to\pi^+\pi^0$ and two-body mode $D^0\to K^-\rho(770)^+$, we obtain the quasi-two-body decay $D^0\to K^-\rho(770)^+\to K^-\pi^+\pi^0$. Its decay amplitude can be written as [55]

$$\mathcal{A}^{Q2B}=\mathcal{M}^{Q2B}\epsilon_\rho\cdot p_D\frac{1}{\mathcal{D}^\rho_{\rm BW}}g_{\rho\pi\pi}\epsilon_\rho\cdot(p_{\pi^+}-p_{\pi^0}).\tag{11}$$

Here, the BW denominator $\mathcal{D}^\rho_{\rm BW}=m_\rho^2-s-im_\rho\Gamma_\rho(s)$ for the $\rho(770)^+$ propagator, and the related s-dependent width is

$$\Gamma_\rho(s)=\Gamma_\rho\frac{m_\rho}{\sqrt{s}}\frac{\left|\vec{q_\pi}\right|^3}{\left|\vec{q_{\pi0}}\right|^3}X^2(\left|\vec{q_\pi}\right|r^\rho_{\rm BW}).\tag{12}$$

The Blatt-Weisskopf barrier factor for ρ family resonances is given by [76]

$$X(z)=\sqrt{\frac{1+z_0^2}{1+z^2}},\tag{13}$$

where $\left|\vec{q_\pi}\right|=\frac{1}{2}\sqrt{s-4m_\pi^2}$, $\left|\vec{q_{\pi0}}\right|$ is $\left|\vec{q_\pi}\right|$ at $s=m_\rho^2$, and the barrier radius is $r^\rho_{\rm BW}=1.5$ GeV^{-1} [77]. Note that the amplitude $\mathcal{M}^{Q2B}$ can be obtained from the related amplitude $\mathcal{M}^{2B}$ with the replacement $m_\rho\to\sqrt{s}$. By using Eq. (8), the decay amplitude $\mathcal{A}^{Q2B}$ of Eq. (11) is reduced to

$$\mathcal{A}^{Q2B}=\mathcal{M}^{Q2B}\frac{g_{\rho\pi\pi}}{\mathcal{D}^\rho_{\rm BW}}\left|\vec{p_1}\right|\cdot\left|\vec{p_3}\right|\cos\theta,\tag{14}$$

where

$$\begin{aligned}\left|\vec{p_1}\right|&=\frac{1}{2}\sqrt{s-4m_\pi^2},\\ \left|\vec{p_3}\right|&=\frac{1}{2\sqrt{s}}\sqrt{\left[m_D^2-(\sqrt{s}+m_K)^2\right]\left[m_D^2-(\sqrt{s}-m_K)^2\right]},\end{aligned}\tag{15}$$

are the momenta for the final states π^+ and K^-, respectively, in the rest frame of the pion pair in the $D^0\to K^-\pi^+\pi^0$ decay, and θ is the angle between π^0 and K^- in the same frame for the pion pair. After the integration of $\cos\theta$, it is trivial to obtain the partial decay width [3]

$$\frac{{\rm d}\Gamma}{{\rm d}s}=\frac{\left|\vec{p_1}\right|^3\left|\vec{p_3}\right|^3}{48\pi^3m_D^3}\left|\mathcal{M}^{Q2B}\frac{g_{\rho\pi\pi}}{\mathcal{D}^\rho_{\rm BW}}\right|^2,\tag{16}$$

for the quasi-two-body decay $D^0\to K^-[\rho(770)^+\to]\pi^+\pi^0$. We can further define the pion electromagnetic form factor of the $\rho(770)$ component as [78, 79]

$$F_\pi(s)=\frac{f_\rho g_{\rho\pi\pi}}{\sqrt{2}m_\rho}\frac{m_\rho^2}{\mathcal{D}^\rho_{\rm BW}},\tag{17}$$

and rewrite Eq. (16) as

$$\frac{{\rm d}\Gamma}{{\rm d}s}=\frac{\left|\vec{p_1}\right|^3\left|\vec{p_3}\right|^3}{24\pi^3m_D^3}\left|\mathcal{M}^{Q2B}\right|^2_{f_\rho m_\rho\to F_\pi},\tag{18}$$

which is the same expression as that in Ref. [55].

To calculate the quasi-two-body decay $D^0\to K^-[\rho(770)^+\to]\omega\pi^+$, we introduce the effective Lagrangian [80–82]

$$\mathcal{L}_{\rho\omega\pi}=g_{\rho\omega\pi}\epsilon_{\mu\nu\alpha\beta}\partial^\mu\rho^\nu\partial^\alpha\omega^\beta\pi\tag{19}$$

to describe the ρ and $\omega\pi$ coupling. The related form factor $F_{\omega\pi}(s)$ is expressed as [83–85]

$$\langle\omega(p_a,\lambda)\pi(p_b)|j_\mu(0)|0\rangle=i\epsilon_{\mu\nu\alpha\beta}\varepsilon^\nu(p_a,\lambda)p_b^\alpha p^\beta F_{\omega\pi}(s),\tag{20}$$

where j_μ is the isovector part of the electromagnetic current; λ and ε are the polarization and polarization vector for ω, respectively; $p_a(p_b)$ is the momentum for $\omega(\pi)$, and the momentum $p=p_a+p_b$ for the resonance $\rho(770)^+$. The form factor $F_{\omega\pi}(s)$ in the vector meson dominance model is parametrized as [26, 29, 32, 86]

$$F_{\omega\pi}(s)=\frac{g_{\rho\omega\pi}}{f_\rho}\sum_{\rho_i}\frac{A_i e^{i\phi_i}m_{\rho_i}^2}{D_{\rho_i}(s)},\tag{21}$$

where the summation is over the isovector resonances $\rho_i=\{\rho(770),\rho(1450),\rho(1700),...\}$ in the ρ family, with m_{ρ_i} as their masses. A_i and ϕ_i are the weights and phases for these resonances, respectively, and we can assign $A=1$ and $\phi=0$ for $\rho(770)$. Technically, the contributions from the excitations of $\omega(782)$ should also be included in Eq. (21), but their weights were found to be negligibly small [87].

The decay amplitude for the quasi-two-body process $D^0\to K^-[\rho(770)^+\to]\omega\pi^+$ is primarily written as

$$\mathcal{A}_{\omega\pi}=\mathcal{M}_{\omega\pi}\epsilon_\rho\cdot p_D\frac{1}{\mathcal{D}_{\rm BW}}g_{\rho\omega\pi}\epsilon_{\mu\nu\alpha\beta}\epsilon^\mu_\rho\varepsilon^\nu_\omega p^\alpha_b p^\beta. \tag{22}$$

By using the relation [47]

$$\sum_{\lambda=0,\pm}|\epsilon_{\mu\nu\alpha\beta}p^\mu_3\varepsilon^\nu(p_\omega,\lambda)p^\alpha_\pi p^\beta|^2=s|\overrightarrow{p_\omega}|^2|\overrightarrow{p_3}|^2(1-\cos^2\theta), \tag{23}$$

we obtain the partial decay width [3]

$$\frac{{\rm d}\Gamma}{{\rm d}s}=\frac{s|\overrightarrow{p_\omega}|^3|\overrightarrow{p_3}|^3}{96\pi^3m_D^3}\left|\mathcal{M}_{\omega\pi}\frac{g_{\rho\omega\pi}}{\mathcal{D}_{\rm BW}}\right|^2, \tag{24}$$

for the quasi-two-body decay $D^0\to K^-[\rho(770)^+\to]\omega\pi^+$ after the integration of $\cos\theta$. Then, we can define an auxiliary electromagnetic form factor

$$f_{\omega\pi}=f_\rho^2/m_\rho F_{\omega\pi}=f_\rho m_\rho g_{\rho\omega\pi}/\mathcal{D}_{\rm BW}, \tag{25}$$

and rewrite Eq. (24) as

$$\frac{{\rm d}\Gamma}{{\rm d}s}=\frac{s|\overrightarrow{p_\omega}|^3|\overrightarrow{p_3}|^3}{96\pi^3m_D^3}|\mathcal{M}_{\omega\pi}|^2_{f_\rho m_\rho\to f_{\omega\pi}}. \tag{26}$$

Note that this expression is slightly different from that of the differential branching fraction in Ref. [47] for $B\to\bar{D}^{(*)}\omega\pi$ decays. This discrepancy arises from the difference in the definitions of the quasi-two-body decay amplitudes between the perturbative QCD approach [47] and the present study.

We need to stress that $\mathcal{D}_{\rm BW}=m_\rho^2-s-{\rm i}\sqrt{s}\Gamma_\rho(s)$ for $\rho(770)^+$ in Eq. (22) is different from $\mathcal{D}^\rho_{\rm BW}$ in Eq. (11). The former has the expression [25, 26]

$$\Gamma_{\rho(770)}(s)=\Gamma_{\rho(770)}\frac{m^2_{\rho(770)}}{s}\left(\frac{|\overrightarrow{p_1}|}{|\overrightarrow{p_1}|_{s=m^2_{\rho(770)}}}\right)^3+\frac{g^2_{\rho\omega\pi}}{12\pi}|\overrightarrow{p_\omega}|^3, \tag{27}$$

for the energy-dependent width of the resonance $\rho(770)$, with

$$|\overrightarrow{p_\omega}|=\frac{1}{2\sqrt{s}}\sqrt{\left[s-(m_\omega+m_\pi)^2\right]\left[s-(m_\omega-m_\pi)^2\right]}. \tag{28}$$

In addition to $\Gamma_{\rho(770)}(s)$, we employ the expression

$$\Gamma_{\rho(1450)}(s)=\Gamma_{\rho(1450)}\Big[\mathcal{B}_{\rho(1450)\to\omega\pi}\Big(\frac{q_\omega(s)}{q_\omega(m^2_{\rho(1450)})}\Big)^3+(1-\mathcal{B}_{\rho(1450)\to\omega\pi})\frac{m^2_{\rho(1450)}}{s}\Big(\frac{q_\pi(s)}{q_\pi(m^2_{\rho(1450)})}\Big)^3\Big] \tag{29}$$

for the energy-dependent width of the excited resonance $\rho(1450)$, as used in Ref. [27] for the process $e^+e^-\to\omega\pi^0\to\pi^0\pi^0\gamma$ by the CMD-2 Collaboration, where $\mathcal{B}_{\rho(1450)\to\omega\pi}$ is the branching ratio of the $\rho(1450)\to\omega\pi$ decay; $\Gamma_{\rho(770)}$ and $\Gamma_{\rho(1450)}$ are the full widths for $\rho(770)$ and $\rho(1450)$, respectively.

III. NUMERICAL RESULTS AND DISCUSSIONS

The key input in this study is the coupling constant $g_{\rho\omega\pi}$ in Eq. (21) for the subprocess $\rho(770)\to\omega\pi$; its value can be estimated from the relation $g_{\rho\omega\pi}\approx 3g^2_{\rho\pi\pi}/(8\pi^2F_\pi)$ [88] with $F_\pi=92$ MeV [3] and can also be calculated from the decay width of $\omega\to\pi^0\gamma$ [3]. In the numerical calculation in this study, we used the value $g_{\rho\omega\pi}=16.0\pm2.0$ GeV^{-1} [47] by considering the corresponding fitted and theoretical values in Refs. [12, 22, 26, 31, 32, 89–91]. Based on the expression for $F_{\omega\pi}(s)$, we have the constraint [47]

$$A_1=\frac{g_{\rho(1450)\omega\pi}f_{\rho(1450)}m_{\rho(770)}}{g_{\rho(770)\omega\pi}f_{\rho(770)}m_{\rho(1450)}}, \tag{30}$$

for the weight A_1 for the subprocess $\rho(1450)\to\omega\pi$ in Eq. (21). With the measured result $f^2_{\rho(1450)}\mathcal{B}(\rho(1450)\to\omega\pi)=0.011\pm0.003$ GeV^2 [35], we have $A_1=0.171\pm0.036$ [47].

In the quasi-two body decay $D_s^+\to\eta[\rho^+\to]\omega\pi$, the mixing between η and η' is considered. The physical η and η' states are related to the quark flavor basis [92, 93]

$$\begin{pmatrix}\eta\\ \eta'\end{pmatrix}=\begin{pmatrix}\cos\phi & -\sin\phi\\ \sin\phi & \cos\phi\end{pmatrix}\begin{pmatrix}\eta_q\\ \eta_s\end{pmatrix}, \tag{31}$$

The meson η was obtained from $\eta_q=(u\bar{u}+d\bar{d})/\sqrt{2}$ and $\eta_s=s\bar{s}$ at the quark level in early studies by using the mixing angle $\phi=39.3^\circ\pm1.0^\circ$ and decay constants $f_{\eta_q}=(1.07\pm0.02)f_\pi$ and $f_{\eta_s}=(1.34\pm0.06)f_\pi$ [92, 93]. Recently, the mixing angle ϕ was measured by the KLOE [94, 95], LHCb [96], and BESIII [97, 98] Collaborations. In this study, we used the angle $\phi=(40.0\pm2.0_{\rm stat}\pm0.6_{\rm syst})^\circ$ that was recently reported by BESIII in Ref. [98] for the η-η' mixing.

The three-parameter fitting formulae for the $D\to\bar{K}$ and $D_s\to\eta_s$ transition form factors are parametrized as

[99]

$$F_1^{D\to\bar{K}}(s)=\frac{0.78}{(1-s/2.11^2)(1-0.24\,s/2.11^2)}\,, \tag{32}$$

$$F_1^{D_s\to\eta_s}(s)=\frac{0.78}{(1-s/2.11^2)(1-0.23\,s/2.11^2)}\,. \tag{33}$$

In addition, we need the form factor [99]

$$A_0^{D\to\rho}(s)=\frac{0.66}{(1-s/1.87^2)(1-0.36\,s/1.87^2)} \tag{34}$$

for the $D^+\to K_S^0\rho^+$ decay. The result $F_1^{D\to\bar{K}}(0)=0.78$ in [99] agrees well with the lattice determination $F_1^{D\to\bar{K}}(0)=0.765(31)$ [100]. The form factor for $D_s\to\eta$ was recently measured by BESIII, and the results were $f_{+,0}^{\eta}(0)|V_{cs}|=0.452\pm0.010_{\rm stat}\pm0.007_{\rm syst}$ [101] and $f_1^{\eta}(0)|V_{cs}|=0.4519\pm0.0071_{\rm stat}\pm0.0065_{\rm syst}$ [98] in the $D_s^+\to\eta^{(\prime)}\mu^+\nu_\mu$ and $D_s^+\to\eta^{(\prime)}e^+\nu_e$ decays, respectively. In Ref. [102], the corresponding form factors were determined to be $f_+^{\eta}(0)=0.442\pm0.022_{\rm stat}\pm0.017_{\rm syst}$. Given the mixing angle ϕ [98] and $|V_{cs}|$ [3], the result $F_1^{D_s\to\eta_s}(0)=0.78$ in Eq. (33) is consistent with the measurements in Refs. [98, 101, 102] presented by BESIII Collaboration.

In the numerical calculation, we applied the mean lives $\tau_{D^\pm}=(1033\pm5)\times10^{-15}$ s, $\tau_{D^0}=(410.3\pm1.0)\times10^{-15}$ s, and $\tau_{D_s^\pm}=(501.2\pm2.2)\times10^{-15}$ s for the $D_{(s)}$ mesons [3]. The decay constants for $\rho(770)$ and its excited state $\rho(1450)$ used in this study are $f_{\rho(770)}=0.216\pm0.003$ GeV [103] and $f_{\rho(1450)}=0.185^{+0.030}_{-0.035}$ GeV [104, 105]. The masses of the particles in the decays of interest, decay constants for kaon and pion, full widths of the resonances $\rho(770)$ and $\rho(1450)$ (in units of GeV), CKM matrix elements $|V_{ud}|$ and $|V_{cs}|$ [3], and decay constants f_D and f_{D_s} for $D_{(s)}$ [106, 107] are presented in Table 1.

To verify the reliability of the parameters used in this study, we calculated the branching ratios for the quasi-two-body decays, involving $D_s^+\to\eta[\rho(770)^+\to]\pi^+\pi^0$, $D^+\to K_S^0[\rho(770)^+\to]\pi^+\pi^0$, and $D^0\to K^-[\rho(770)^+\to]\pi^+\pi^0$, and compared them with the experimental results in Ref. [3]. By utilizing the differential branching fraction of Eq. (18), obtaining the branching fractions for the quasi-two-body decays $D_s^+\to\eta[\rho(770)^+\to]\pi^+\pi^0$, $D^+\to K_S^0[\rho(770)^+\to]\pi^+\pi^0$, and $D^0\to K^-[\rho(770)^+\to]\pi^+\pi^0$, shown in Table 2, is a trivial problem. The table also shows the corresponding two-body data from *Review of Particle Physics* [3]. In our numerical results of the relevant quasi-two-body in Table 2, the first source of the error corresponds to the uncertainties in the decay constant $f_{\rho(770)}=0.216\pm0.003$ GeV [103], whereas the uncertainties in the CKM matrix elements $|V_{ud}|$ and $|V_{cs}|$ in Table 1 contribute to the second source of error. It can be seen that these errors are much smaller than their corresponding central values. For the D_s decay, the uncertainty in the mixing angle ϕ contributes the third error. We neglected the errors arising from the uncertainties in the other parameters because of their very small contributions. Considering $\mathcal{B}(\rho^+\to\pi^+\pi^0)\approx$ 100% [3], our results in Table 2 for the decays $D_s^+\to\eta[\rho(770)^+\to]\pi^+\pi^0$ and $D^0\to K^-[\rho(770)^+\to]\pi^+\pi^0$ are consistent with the data. However, for the decay $D^+\to K_S^0[\rho(770)^+\to]\pi^+\pi^0$, our result deviates from the experimental measurement by more than 4σ. Note that our results for the D^+ decay agree with those in Ref. [68]. This indicates that further investigations are required to understand the branching ratio for $D^+\to K_S^0\pi^+\pi^0$.

Next, we investigated the contributions of

Table 1. Masses for the relevant states, decay constants for pion and kaon, full widths of $\rho(770)$ and $\rho(1450)$ (in units of GeV), and CKM matrix elements $|V_{ud}|$ and $|V_{cs}|$ in [3], along with the decay constants f_D and f_{D_s} from [106, 107].

$m_{D^\pm}=1.870$	$m_{D^0}=1.865$	$m_{D_s^\pm}=1.968$	$m_{\pi^\pm}=0.140$
$m_\omega=0.783$	$m_{K^\pm}=0.494$	$m_{K^0}=0.498$	$m_\eta=0.548$
$f_\pi=0.130$	$f_K=0.156$	$f_D=0.212$	$f_{D_s}=0.250$
$m_{\rho(770)}=0.775$			$\Gamma_{\rho(770)}=0.147$
$m_{\rho(1450)}=1.465\pm0.025$			$\Gamma_{\rho(1450)}=0.400\pm0.060$
$\lvert V_{ud}\rvert=0.97367\pm0.00032$			$\lvert V_{cs}\rvert=0.975\pm0.006$

Table 2. Branching fractions for the concerned quasi-two-body $D_{(s)}$ decays with the subprocess $\rho(770)^+\to\pi^+\pi^0$, and the corresponding two-body data from *Review of Particle Physics* [3].

Decay mode	$\mathcal{B}$	Data [3]
$D_s^+\to\eta[\rho(770)^+\to]\pi^+\pi^0$	$(7.11\pm0.20\pm0.09\pm0.47)\%$	$(8.9\pm0.8)\%$
$D^+\to K_S^0[\rho(770)^+\to]\pi^+\pi^0$	$(3.67\pm0.10\pm0.05)\%$	$(6.14^{+0.60}_{-0.35})\%$
$D^0\to K^-[\rho(770)^+\to]\pi^+\pi^0$	$(9.12\pm0.25\pm0.11)\%$	$(11.2\pm0.7)\%$

Table 3. Predicted branching fractions for the relevant quasi-two-body $D_{(s)}$ decays with the subprocesses $\rho(770,1450)^+\to\omega\pi^+$ along with the experimental measurements in [3] for the related three-body decay processes.

Decay mode	$\mathcal{B}$
$D_s^+\to\eta[\rho(770)^+\to]\omega\pi^+$	$(1.90\pm0.35\pm0.05\pm0.14)\times10^{-3}$
$D_s^+\to\eta[\rho(1450)^+\to]\omega\pi^+$	$(0.39\pm0.16\pm0.14\pm0.03)\times10^{-3}$
$D_s^+\to\eta[\rho(770\&1450)^+\to]\omega\pi^+$	$(2.89\pm0.63\pm0.29\pm0.20)\times10^{-3}$
$D^+\to K_S^0[\rho(770)^+\to]\omega\pi^+$	$(1.03\pm0.20\pm0.03)\times10^{-3}$
$D^+\to K_S^0[\rho(1450)^+\to]\omega\pi^+$	$(0.16\pm0.07\pm0.06)\times10^{-3}$
$D^+\to K_S^0[\rho(770\&1450)^+\to]\omega\pi^+$	$(1.53\pm0.34\pm0.15)\times10^{-3}$
$D^0\to K^-[\rho(770)^+\to]\omega\pi^+$	$(1.96\pm0.38\pm0.05)\times10^{-3}$
$D^0\to K^-[\rho(1450)^+\to]\omega\pi^+$	$(0.28\pm0.12\pm0.10)\times10^{-3}$
$D^0\to K^-[\rho(770\&1450)^+\to]\omega\pi^+$	$(2.86\pm0.63\pm0.31)\times10^{-3}$
Three-body mode	Data [3]
$D_s^+\to\eta\omega\pi^+$	$(5.4\pm1.3)\times10^{-3}$
$D^+\to K_S^0\omega\pi^+$	$(7.1\pm0.5)\times10^{-3}$
$D^0\to K^-\omega\pi^+$	$(3.39\pm0.10)\times10^{-2}$

$\rho(770,1450)\to\omega\pi$ to the relevant quasi-two-body $D_{(s)}$ decays.ByusingEq.(26),weobtainthebranchingfractionsforthe decays $D_s^+\to\eta[\rho^+\to]\omega\pi^+$, $D^+\to K_S^0[\rho^+\to]\omega\pi^+$, and $D^0\to K^-[\rho^+\to]\omega\pi^+$ in Table 3, with the intermediate $\rho^+\in\{\rho(770)^+,\rho(1450)^+,\rho(770)^+\&\rho(1450)^+\}$. For these predicted branching fractions in Table 3, the first source of the error relates to the uncertainties in $g_{\rho\omega\pi}=16.0\pm2.0$ GeV^{-1} and $A_1=0.171\pm0.036$. The second error originates from the uncertainties in the decay constants for $\rho(770)$ and $\rho(1450)$, mass $m_{\rho(1450)}=1.465\pm0.025$ GeV, and full width $\Gamma_{\rho(1450)}=0.400\pm0.060$ GeV for $\rho(1450)$. For these three decay channels with the meson η in their final states in Table 3, the uncertainty in the mixing angle ϕ will contribute the third error. The other errors in the predictions, originating from the uncertainties in the masses and decay constants of the initial and final states, uncertainties in the CKM matrix elements, etc., are very small and have been neglected.

The parameters $\chi_q^{E,A}$, $\phi_q^{E,A}$, χ_V^C, and ϕ_V^C in Eqs. (3)−(5) were fitted for the two-body D decays, with a light pseudoscalar and ground vector meson as their final states. In principle, these parameters are not really appropriate to the decays with $\rho(1450)$ as the intermediate state. However, note that these parameters do not exist in the dominant decay amplitudes for the relevant decay processes in this study. To identify the effects of these parameters on the branching fractions with the intermediate state $\rho(1450)$, we take $D_s^+\to\eta[\rho(1450)^+\to]\omega\pi^+$ as an example. When we vary the related parameters with 50% uncertainties in their values for this decay channel, we obtain the branching fraction as $\mathcal{B}=(0.39\pm0.03)\times10^{-3}$. This means the effects of these parameters would be very small when compared with those of the corresponding errors in Table 3 for the $D_s^+\to\eta[\rho(1450)^+\to]\omega\pi^+$ decay. The form factor $A_0^{D\to\rho}$ in Eq. (34) is for the $D\to\rho(770)$ transition. In addition, for the decay $D^+\to K_S^0[\rho(1450)^+\to]\omega\pi^+$, the form factor A_0 for the $D\to\rho(1450)$ transition is needed. According to the discussions in Ref. [104] for the same intermediate state $\rho(1450)$ in quasi-two-body B meson decays, we used $A_0^{D\to\rho(1450)}\approx f_{\rho(1450)}/f_{\rho(770)}A_0^{D\to\rho(770)}$ considering the lack of information regarding the form factor $A_0^{D\to\rho(1450)}$ in literature. By applying this form factor with a 20% uncertainty of its central value, we obtain the branching fraction $(0.16\pm0.03)\times10^{-3}$ for the $D^+\to K_S^0[\rho(1450)^+\to]\omega\pi^+$ decay. Apparently, the error is not large.

In Ref. [1], the branching fraction for the three-body decay $D_s^+\to\eta\omega\pi^+$ was measured to be $(0.54\pm0.12_{\rm stat}\pm 0.04_{\rm syst})\%$; by comparing this value with the corresponding prediction $(2.89\pm0.72)\times10^{-3}$ in Table 3 for $D_s^+\to\eta[\rho(770\&1450)^+\to]\omega\pi^+$, we conclude that the three-body decay $D_s^+\to\eta\omega\pi^+$ is dominated by the contribution from the subprocess $\rho(770\&1450)^+\to\omega\pi^+$, roughly contributing to half of the total branching fraction when employing the phase difference $\phi_1=\pi$ between the intermediate states $\rho(770)$ and $\rho(1450)$ [22, 26, 27, 29, 31, 32]. However, in Ref. [47], with $\phi_1=\pi$, the shapes of the predicted differential branching fractions for $B^0\to D^{*-}\rho^+\to D^{*-}\omega\pi^+$ did not show a good agreement with the distribution of $\omega\pi$ measured by the Belle Collaboration in [37] for the $B^0\to D^{*-}\omega\pi^+$ decay. The interference between the subprocesses $\rho(770)^+\to\omega\pi^+$ and $\rho(1450)^+\to\omega\pi^+$ is seriously affected by the phase difference $\phi_1=\pi$ for the $D_s^+\to\eta\omega\pi^+$ decay. We switched the phase difference ϕ_1 from zero to 2π and found that we could obtain the maximum branching fraction $\mathcal{B}=3.67\times10^{-3}$ as the central value for $D_s^+\to\eta[\rho(770\&1450)^+\to]\omega\pi^+$ by selecting $\phi_1 = 1.35\pi$. The prediction in this case is roughly 68% of the measured total branching fraction for $D_s^+ \to \eta\omega\pi^+$ presented by BESIII in [1].

One could argue that as a virtual bound state [17, 18], the intermediate state $\rho(770)^+$ cannot completely exhibit its properties in the quasi-two-body decay $D_s^+ \to \eta[\rho(770)^+ \to]\omega\pi^+$. It is true that the invariant masses of the $\omega\pi$ pair exclude the region around the $\rho(770)$ pole mass. However, as shown in Fig. 3, the width of $\rho(770)$ renders its contribution quite sizable in the energy region of interest of three-body $D_{(s)}$ decays. It is important to explicitly consider the subthreshold resonances in amplitude analysis in experimental studies, even if they contribute via the tail of their energy distribution. The differential branching fractions shown in Fig. 3 for the quasi-two-body decay $D_s^+ \to \eta[\rho^+ \to]\omega\pi^+$ with $\rho^+ = \{\rho(770)^+, \rho(1450)^+, \rho(770)^+ + \rho(1450)^+\}$ are not typical curves such as those in $\rho(770, 1450) \to \pi\pi$ in three-body B meson decays [104]. The peaks of the dashed line for $\rho(770)^+ \to \omega\pi^+$ and dotted line for $\rho(1450)^+ \to \omega\pi^+$ in Fig. 3 arise from the BW expressions for the involved resonances and mainly from the phase space factor in Eq. (26).

The absolute branching fraction for the three-body decay $D^+ \to K_S^0\pi^+\omega$ was determined to be $(0.707 \pm 0.041_{\rm stat} \pm 0.029_{\rm syst})\%$ by BESIII in Ref. [43], which is 4.6 times larger than the prediction $(1.53 \pm 0.37) \times 10^{-3}$ in Table 3 for the quasi-two-body decay $D^+ \to K_S^0[\rho(770\&1450)^+ \to]\omega\pi^+$. We need to stress that the contribution weight of $D^+ \to K_S^0[\rho(770\&1450)^+ \to]\omega\pi^+$ in the corresponding three-body decay could be enhanced by improving the calculation method employed in this study, as our prediction for $D^+ \to K_S^0[\rho(770)^+ \to]\pi^+\pi^0$ in Table 2 is just approximately half of the value in [3]. This also means the branching fraction for $D^+ \to K_S^0[\rho(770\&1450)^+ \to]\omega\pi^+$ could be twice the corresponding prediction in Table 3. In addition, the $D^+ \to b_1(1235)$ transition with an emitted K_S^0 could generate the $\omega\pi^+$ pair from $b_1(1235)$ for the $D^+ \to K_S^0\pi^+\omega$ decay. The $\omega\pi^+$ pair generated from the $D^+ \to b_1(1235)$ transition is unlike that from the matrix element $\langle\omega\pi|V^\mu - A^\mu|0\rangle$ in $D_s^+ \to \eta\omega\pi^+$ decay, where the amplitude is proportional to the mass difference between the u and d quarks. The measurement of $\mathcal{B}(D^+ \to b_1(1235)^0 e^+\nu_e) \times \mathcal{B}(b_1(1235)^0 \to \omega\pi^0) = (1.16 \pm 0.44_{\rm stat} \pm 0.16_{\rm syst}) \times 10^{-4}$ by BESIII [108] suggests a possible contribution of $b_1(1235)^+ \to \omega\pi^+$ to the $D^+ \to K_S^0\pi^+\omega$ decay. However, we know that the contribution would not be large because we have the theoretical estimation 1.7×10^{-3} as the branching fraction for $D^+ \to \bar{K}^0 b_1(1235)^+$ [109].

The input parameters for the quasi-two-body D meson decays were updated very recently in Ref. [70]. With the updated inputs in [70], the quasi-two-body decay branching fractions for $D_s^+ \to \eta\rho(770)^+$ and $D^0 \to K^-\rho(770)^+$ agree well with the corresponding results in Tables 2−3 with the subprocesses $\rho(770)^+ \to \pi^+\pi^0$ and $\rho(770)^+ \to \omega\pi^+$, respectively. In addition, the branching fractions for the quasi-two-body decays $D^+ \to K_S^0[\rho(770)^+ \to]\pi^+\pi^0$ and $D^+ \to K_S^0[\rho(770\&1450)^+ \to]\omega\pi^+$ will increase to $(4.45 \pm 0.14)\%$ and $(2.63 \pm 0.64) \times 10^{-3}$, respectively. The latter numerical value is approximately 37% of the total branching fraction of $D^+ \to K_S^0\pi^+\omega$ in [43]. For this three-body decay $D^+ \to K_S^0\pi^+\omega$, the contribution of $D^+ \to \bar{K}^{*0}\pi^+$ with the subprocess $\bar{K}^{*0} \to K_S^0\omega$ must also be considered. When we apply the branching fraction $(1.04 \pm 0.12)\%$ to $D^+ \to \bar{K}^{*0}\pi^+$ along with $\bar{K}^{*0} \to K^-\pi^+$ [3] and the discussions for the virtual contributions of K^* in [110], we cannot expect a large contribution of $D^+ \to \bar{K}^{*0}\pi^+ \to K_S^0\omega\pi^+$ to this three-body decay process. In addition, the cascade decay $D^+ \to K^*(892)^+\omega \to K_S^0\pi^+\omega$ is a doubly Cabibbo suppressed model. This establishes the quasi-two-body process $D^+ \to K_S^0[\rho(770\&1450)^+ \to]\omega\pi^+$ as the most important process for the three-body decay $D^+ \to K_S^0\pi^+\omega$.

The branching fraction for the decay $D^0 \to K^-[\rho(770\&1450)^+ \to]\omega\pi^+$ was predicted to be $(2.86 \pm 0.70) \times 10^{-3}$ in this study, which is less than a tenth of the measurement $(3.392 \pm 0.044_{\rm stat} \pm 0.085_{\rm syst})\%$ for the three-body decay $D^0 \to K^-\pi^+\omega$ in Ref. [43]. The value $(6.5 \pm 3.0) \times 10^{-3}$ for $D^0 \to \bar{K}^*(892)^0\omega$ with the subprocesses $\bar{K}^*(892)^0 \to K^-\pi^+$ and $\omega \to \pi^+\pi^-\pi^0$ is approximately twice our result for $D^0 \to K^-[\rho(770\&1450)^+ \to]\omega\pi^+$ in this study. Actually, the three-body decay $D^0 \to K^-\pi^+\omega$ is very different from the process $D_s^+ \to \eta\omega\pi^+$, as the former has very rich intermediate states. The resonances $\bar{K}^*(892, 1410, 1680)^0$, $\bar{K}^*_{0,2}(1430)^0$, $K^*(892, 1410, 1680)^-$, $K_1(1270)^-$, and $\rho(770, 1450)^+$ decay into $K^-\pi^+$, $K^-\omega$, and $\pi^+\omega$, respectively, in this three-body decay $D^0 \to K^-\pi^+\omega$. The analyses of the complete resonance contributions for the decay process $D^0 \to K^-\pi^+\omega$ are beyond the scope of this study and will be addressed in future studies.

IV. SUMMARY

The Cabibbo-favored three-body decays $D_s^+ \to \eta\omega\pi^+$, $D^+ \to K_S^0\pi^+\omega$, and $D^0 \to K^-\pi^+\omega$ were recently observed by the BESIII Collaboration, but an amplitude analysis was not conducted. To better understand the relevant experimental measurements, we studied the contributions of the subprocesses $\rho(770, 1450)^+ \to \omega\pi^+$ in these three-body D meson decays by introducing them into the decay amplitudes of the relevant decay channels via the vector form factor $F_{\omega\pi}(s)$, which has been measured in the related processes of τ decay and e^+e^- annihilation.

With the parameters $g_{\rho\omega\pi} = 16.0 \pm 2.0$ GeV^{-1} and $A_1 = 0.171 \pm 0.036$ for the vector form factor $F_{\omega\pi}$, we predicted the branching fractions for the first time for the quasi-two-body decays $D_s^+ \to \eta[\rho^+ \to]\omega\pi^+$, $D^+ \to K_S^0[\rho^+ \to]\omega\pi^+$, and $D^0 \to K^-[\rho^+ \to]\omega\pi^+$ with the intermediate $\rho^+ = \{\rho(770)^+, \rho(1450)^+, \rho(770)^+ + \rho(1450)^+\}$. By comparing our predictions with the experimental data, we found that the contributions of the subprocess $\rho(770)^+ \to \omega\pi^+$ are significant in the three-body decays $D_s^+ \to \eta\omega\pi^+$,

$D^+\to K_S^0\omega\pi^+$, and $D^0\to K^-\omega\pi^+$, notwithstanding the contributions originating from the Breit-Wigner tail effect of $\rho(770)^+$. The interference effects between the resonances $\rho(770)^+$ and $\rho(1450)^+$ enhance the P-wave resonance contributions for the $\omega\pi^+$ pair in these three-body decays. The numerical results of this study suggest that the dominant resonance contributions for the three-body decays $D_s^+\to\eta\omega\pi^+$ and $D^+\to K_S^0\omega\pi^+$ originate from the P-wave intermediate states $\rho(770)^+$, $\rho(1450)^+$ decaying into the $\omega\pi^+$ pair and their interference effects.

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