

Searching for doubly charmed baryons from $\bar{B}_c$ meson decays*

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Abstract: In this study, we investigate the production of doubly charmed baryons from anti-bottom charmed mesons. Using the effective Lagrangian approach, we discuss triangle diagrams at the hadronic level to access the branching ratios of $\bar{B}_c \rightarrow \mathcal{B}_{ccq} + \mathcal{B}_{c\bar{q}\bar{q}}$. Apparently, the specific process $\bar{B}_c \rightarrow \Xi_{cc}^+ \Xi_c^{\prime 0}$ occupies the largest possibility on the order of 9.1×10^{-5} . In addition, although the production of undiscovered Ω_{cc}^+ is Cabibbo suppressed in $\bar{B}_c \rightarrow \Omega_{cc}^+ \Xi_c^0$, its branching ratio can still reach a level of 10^{-7} . These results are expected to provide valuable support for future experiments.

Keywords: doubly charmed baryon, Bc meson, effective Lagrangian approach

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I. PHENOMENOLOGICAL ANALYSIS

The LHCb collaboration reported experimental evidence of the doubly charmed baryon candidate Ξ_{cc}^{++} in the $\Lambda_c^+ K^- \pi^+ \pi^+$ final state in 2017 [1]. Its mass was determined to be 3621.55 MeV [2], and its lifetime, measured from weakly decays, was 0.256 ps [3]. Prior to this, the doubly charmed baryon Ξ_{cc}^+ with a mass of 3518.7 MeV was first reported by the SELEX collaboration in decay modes $\Xi_{cc}^+ \rightarrow \Lambda_c^+ K^- \pi^+$ and $\Xi_{cc}^+ \rightarrow p D^+ K^-$ [4, 5]; its lifetime was found to be less than 33 fs. However, subsequent experiments, namely FOCUS [6], BaBar [7], Belle [8], and LHCb [9], have not confirmed this state yet. Likewise, for Ω_{cc}^+ baryons, the most recent searches from the LHCb collaboration have not reported a significant detection signal yet. Therefore, further theoretical and experimental efforts to advance the study of doubly heavy baryons are urgently needed.

This is crucial for completing the hadron spectrum and revealing the nature of perturbative and non-perturbative QCD dynamics related to doubly heavy baryons. Various theoretical frameworks, such as lattice QCD [10–14], quark model [15–22], QCD sum rules [23–28], and heavy baryon chiral perturbation theory [29–32], have been applied for research on doubly heavy systems. For ground states of Ξ_{cc} , the majority of theoretical pre-

dictions focus on masses and lifetimes. For instance, the masses are predicted to lie in the range of 3.5–3.7 GeV, which is close to current experimental measurements. Owing to the effect of the destructive Pauli interference of the c quark decay products and valence u quark in the initial state, the lifetime of Ξ_{cc}^{++} is expected to be approximately 2–4 times larger than the one of Ξ_{cc}^+ . Consequently, most theoretical predictions show that the lifetime of Ξ_{cc}^+ is in the range of 40–160 fs. There are also many theoretical predictions for the ground state of Ω_{cc}^+ ; for instance, its mass is predicted to be approximately 3.6–3.9 GeV, and its lifetime is 75–180 fs. We provide some typical results in Table 1.

Here, we present a new study about production of doubly charmed baryons from $\bar{B}_c$ mesons. This production may be achieved by the LHCb and future b-factories. We derive the production processes using the effective Lagrangian method [36, 37]. The complete transition is divided into a weakly decay part associated with the transition $\mathcal{O}^i: \bar{b} \rightarrow \bar{c} d \bar{s}$ and a strongly coupled part related to two strong coupled vertices, *i.e.*, $\mathcal{B}_{cc}\mathcal{B}_c D$ and $\mathcal{B}_c \mathcal{B}_c J/\psi$ ($\mathcal{B}$ is the general baryon). The production can then be described formally by some triangle diagrams at the hadronic level. Under the $SU(3)$ light quark flavor symmetry [38–44], it is convenient to relate different production channels, where the final states can include one

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Table 1. Masses and lifetimes of doubly charmed baryons predicted from several theoretical methods or experiments.

	LQCD [11]	QM [17]	[1]	QCDSR [24]	NR model [33]	OPE [34, 35]	Exp (mass & lifetime)	
Ξ_{cc}^{++}	3.610 GeV	3.676 GeV	3.627 GeV	3.72 GeV	0.67 ps	0.52 ps	3.622 GeV [2]	0.256 ps [3]
Ξ_{cc}^+	3.610 GeV	3.676 GeV	3.627 GeV	3.72 GeV	0.25 ps	0.19 ps	3.519 GeV [5]	< 33 fs [5]
Ω_{cc}^+	3.738 GeV	3.832 GeV	—	3.73 GeV	0.21 ps	0.22 ps	—	—

doubly charmed baryon $\mathcal{B}_{cc}$ (Ξ_{cc}^{++} , Ξ_{cc}^+ , Ω_{cc}^+) and an anti-charmed triplet baryon $\mathcal{B}_{\bar{c}3}$ or anti-sextet baryon $\mathcal{B}_{\bar{c}6}$,

$$\begin{aligned}
\Gamma(\bar{B}_c \rightarrow \Xi_{cc}^{++} \bar{\Xi}_{\bar{c}}^-) &= \Gamma(\bar{B}_c \rightarrow \Xi_{cc}^+ \bar{\Xi}_{\bar{c}}^0), \\
\Gamma(\bar{B}_c \rightarrow \Xi_{cc}^{++} \bar{\Lambda}_{\bar{c}}^-) &= \Gamma(\bar{B}_c \rightarrow \Xi_{cc}^+ \bar{\Xi}_{\bar{c}}^0), \\
\Gamma(\bar{B}_c \rightarrow \Xi_{cc}^{++} \bar{\Xi}_{\bar{c}}^{\prime -}) &= \Gamma(\bar{B}_c \rightarrow \Xi_{cc}^+ \bar{\Xi}_{\bar{c}}^0), \\
\Gamma(\bar{B}_c \rightarrow \Xi_{cc}^{++} \bar{\Sigma}_{\bar{c}}^-) &= 2\Gamma(\bar{B}_c \rightarrow \Xi_{cc}^+ \bar{\Sigma}_{\bar{c}}^0). \quad (1)
\end{aligned}$$

Furthermore, the rates of decay widths are related to the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements V_{cs} and V_{cd} ,

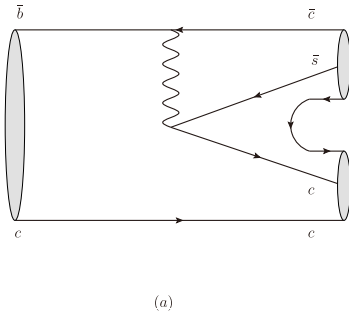
$$\frac{\Gamma(\bar{B}_c \rightarrow \Xi_{cc}^{++} \bar{\Xi}_{\bar{c}}^-)}{\Gamma(\bar{B}_c \rightarrow \Xi_{cc}^{++} \bar{\Lambda}_{\bar{c}}^-)} = \frac{\Gamma(\bar{B}_c \rightarrow \Xi_{cc}^{++} \bar{\Xi}_{\bar{c}}^{\prime -})}{\Gamma(\bar{B}_c \rightarrow \Xi_{cc}^{++} \bar{\Sigma}_{\bar{c}}^-)} = \frac{|V_{cs}|^2}{|V_{cd}|^2}, \quad (2)$$

and we calculate the branching ratio using the effective Lagrangian method. Thus, we can compare the results with the predictions from $SU(3)$ analysis, aiming to provide assistance in understanding and searching for doubly heavy baryons.

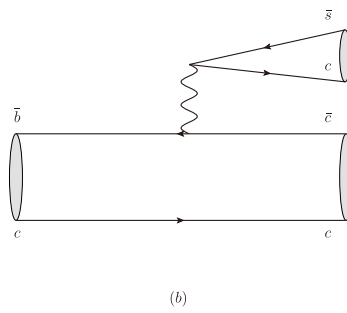
The rest of this paper is organized as follows. In Sec. II, we introduce the calculation framework that describes the production of doubly charmed baryons from $\bar{B}_c$ decays. Several amplitudes of triangle diagrams are obtained using the effective Lagrangian method. In Sec. III, we present numerical analyses and branching ratios of different processes. The final section provides a brief summary.

II. BRANCHING RATIOS

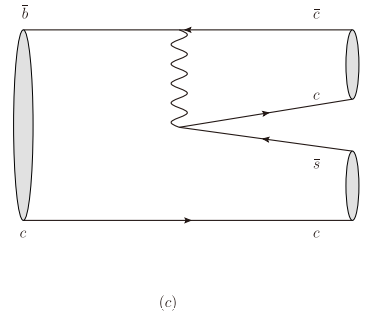
The production of doubly charmed baryons $\mathcal{B}_{cc}$ from



(a)



(b)



(c)

Fig. 1. (a) Production processes of $\bar{B}_c \rightarrow \mathcal{B}_{cc} + \bar{\mathcal{B}}_{\bar{c}}$, which are non-factorizable and color suppressed; (b) and (c) weak decays of $\bar{B}_c \rightarrow D(D^*) + J/\psi(\eta_c)$ in terms of the factorizable external and internal W -emission diagrams, respectively.

$\bar{B}_c$ occurs through the weak decay of the bottom quark $\bar{b} \rightarrow \bar{c} \bar{d} / \bar{s}$, whose topological diagram at the quark level is shown in Fig. 1(a). The non-factorizable color suppressed diagram with two baryons in the final states is unusual and difficult to handle directly using common factorization schemes. In this paper, we consider the process within the framework of the effective Lagrangian approach—a convenient method for studying hadrons at the hadronic level. The total production matrix element can be further divided into a weak transition matrix and a strongly coupled matrix by inserting a complete basis,

$$\langle \mathcal{B}_{cc} \bar{\mathcal{B}}_{\bar{c}} | \mathcal{H}_{\text{eff}} | \bar{B}_c \rangle = \sum_{\lambda} \langle \mathcal{B}_{cc} \bar{\mathcal{B}}_{\bar{c}} | \mathcal{H}_{\lambda} | \lambda \rangle \langle \lambda | \mathcal{H}_{\text{eff}} | \bar{B}_c \rangle. \quad (3)$$

Here, H_{λ} denotes the strong interaction term included in the interaction representation presented later. It is clear that the dominant contribution related to the weak transition matrix corresponds to $\langle D(D^*) \eta_c | \mathcal{H}_{\text{eff}} | \bar{B}_c \rangle$ or $\langle D(D^*) J/\psi | \mathcal{H}_{\text{eff}} | \bar{B}_c \rangle$ (here, we only consider the major contribution $\lambda = D(D^*) J/\psi(\eta_c)$), which is depicted as an accessible emission diagram [45] in Figs. 1(b) and (c). Meanwhile, the remaining one can be decoded using several effective Lagrangians introduced below. Combining them, we obtain productions of spin- $\frac{1}{2}$ doubly charmed baryons $\mathcal{B}_{cc}$ (Ξ_{cc}^{++} , Ξ_{cc}^+ , Ω_{cc}^+) resulting from the triangle diagrams. We proceed with processes under the quark transition $\bar{b} \rightarrow \bar{c} \bar{c} \bar{s}$,

$$\bar{B}_c \rightarrow \Xi_{cc}^{++} + \bar{\Xi}_{\bar{c}}^- (/ \bar{\Xi}_{\bar{c}}^{\prime -}), \quad \bar{B}_c \rightarrow \Xi_{cc}^+ + \bar{\Xi}_{\bar{c}}^0 (/ \bar{\Xi}_{\bar{c}}^{\prime 0}), \quad (4)$$

and under the transition $\bar{b} \rightarrow \bar{c} \bar{d}$,

$$\bar{B}_c \rightarrow \Xi_{cc}^{++} + \bar{\Lambda}_c^- (\bar{\Sigma}_c^-), \quad \bar{B}_c \rightarrow \Xi_{cc}^+ + \bar{\Sigma}_c^0, \quad \bar{B}_c \rightarrow \Omega_{cc}^+ + \bar{\Xi}_c^0. \quad (5)$$

The corresponding triangle diagrams are shown in Fig. 2. Here, we have excluded some unallowable processes in the phase space as well as processes with decoupling vertices.

A. Effective Lagrangians

We adopt the effective Lagrangian to study the production of doubly charmed baryons. The process can be regarded as $\bar{B}_c$ mesons first decaying into $D(D^*)$ and $J/\psi(\eta_c)$ mesons and then exchanging with charm baryons $\mathcal{B}_c$ to produce anti-charmed baryons $\bar{\mathcal{B}}_c$ and doubly charmed baryons $\mathcal{B}_{cc}$, $\bar{B}_c \rightarrow M_1 M_2 \xrightarrow{\mathcal{B}_c} \bar{\mathcal{B}}_c \mathcal{B}_{cc}$ ($M_1 M_2$ can be $J/\psi D$, $\eta_c D$, $J/\psi D^*$, or ηD^*). The weak decay $\bar{B}_c \rightarrow J/\psi(\eta_c) + D(D^*)$ can occur via the factorizable W -emission process shown in Fig. 1(b). It typically provides the largest contribution according to the topological classification of weak decays. The effective Hamiltonian is

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{cs} (C_1(\mu) \mathcal{O}_1(\mu) + C_2(\mu) \mathcal{O}_2(\mu)) + h.c., \quad (6)$$

where G_F is the Fermi constant, $C_{1,2}(\mu)$ denotes the Wilson coefficients, and $\mathcal{O}_{1,2}(\mu)$ denotes the tree level fermion operators, with $\mathcal{O}_1 = (\bar{b}_\alpha c_\beta)_{V-A} (\bar{c}_\beta s_\alpha)_{V-A}$ and $\mathcal{O}_2 =$

$(\bar{b}_\alpha c_\alpha)_{V-A} (\bar{c}_\beta s_\beta)_{V-A}$. The combinations of the effective Wilson coefficient $a_{1,2}$ are $a_1 = C_1 + C_2/N_c$ and $a_2 = C_2 + C_1/N_c$, where N_c denotes the number of colors.

The amplitude of the weak process can be expressed as the product of two current hadronic matrix elements, accompanied by form factors and decay constants,

$$\begin{aligned} \mathcal{M}(\bar{B}_c \rightarrow J/\psi D(D^*)) \\ = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{cs} a_1 \langle J/\psi | (\bar{b}c)_{V-A} | \bar{B}_c \rangle \langle D(D^*) | (\bar{c}s)_{V-A} | 0 \rangle \\ + \frac{G_F}{\sqrt{2}} V_{cb}^* V_{cs} a_2 \langle J/\psi | (\bar{b}c)_{V-A} | \bar{B}_c \rangle \langle D(D^*) | (\bar{c}s)_{V-A} | 0 \rangle, \end{aligned} \quad (7)$$

$$\begin{aligned} \mathcal{M}(\bar{B}_c \rightarrow \eta_c D(D^*)) \\ = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{cs} a_1 \langle \eta_c | (\bar{b}c)_{V-A} | \bar{B}_c \rangle \langle D(D^*) | (\bar{c}s)_{V-A} | 0 \rangle \\ + \frac{G_F}{\sqrt{2}} V_{cb}^* V_{cs} a_2 \langle \eta_c | (\bar{b}c)_{V-A} | \bar{B}_c \rangle \langle D(D^*) | (\bar{c}s)_{V-A} | 0 \rangle. \end{aligned} \quad (8)$$

The matrix elements between pseudoscalar or vector mesons and vacuum have the following formations [46]:

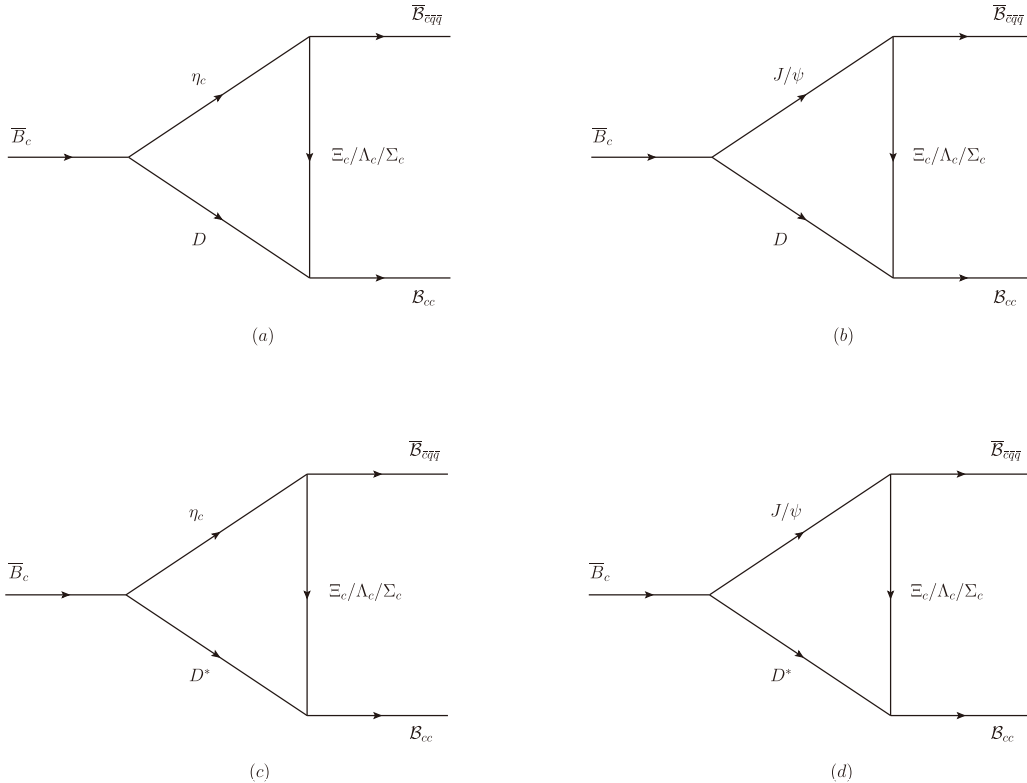


Fig. 2. Triangle diagrams attached to the production of doubly charmed baryons, mediated by the exchange of charm baryons $\Xi_c, \Lambda_c, \Sigma_c$.

$$\langle P | (\bar{c}s)_{V-A} | 0 \rangle = i f_P p_\mu, \quad \langle V | (\bar{c}s)_{V-A} | 0 \rangle = m f_V \varepsilon_\mu^*. \quad (9)$$

where P and V stand for pseudoscalar and vector mesons, respectively; f_P and f_V are the pseudoscalar and vector meson decay constants, respectively; and ε_μ^* represents the polarization vector of the vector meson.

We follow the parameterized form of the transition $\bar{B}_c \rightarrow J/\psi D(D^*)$ [47],

$$\begin{aligned} & \langle J/\psi(P_V) | (\bar{b}c)_{V-A} | \bar{B}_c(P_{B_c}) \rangle \\ &= \frac{2}{m_{B_c} + m_V} \varepsilon_{\mu\nu\rho\sigma} \varepsilon^{*\nu} P_{B_c}^\rho P_V^\sigma V(q^2) + i \varepsilon_\mu^* (m_{B_c} + m_V) A_1(q^2) \\ & - i \frac{\varepsilon^* \cdot q}{m_{B_c} + m_V} (P_{B_c} + P_V)_\mu A_2(q^2) - i \frac{\varepsilon^* \cdot q}{q^2} 2m_V q_\mu A_3(q^2) \\ & + i \frac{\varepsilon^* \cdot q}{q^2} 2m_V q_\mu A_0(q^2), \end{aligned} \quad (10)$$

$$\begin{aligned} & \langle \eta(P_P) | (\bar{b}c)_{V-A} | \bar{B}_c(P_{B_c}) \rangle \\ &= (P_{B_c} + P_P - \frac{m_{B_c}^2 - m_P^2}{q^2} q)_\mu F_1(q^2) + \frac{m_{B_c}^2 - m_P^2}{q^2} q_\mu F_0(q^2). \end{aligned} \quad (11)$$

where ε_μ denotes the polarization vector of J/ψ , and the transition momentum satisfies $q_\mu \equiv (P_{B_c} - P_P)_\mu$. To cancel

the poles at $q^2 = 0$, invariant weak form factors $F_0(q^2)$, $F_1(q^2)$, $A_0(q^2)$, and $A_3(q^2)$ satisfy the following conditions:

$$\begin{aligned} F_1(0) &= F_0(0), \quad A_3(0) = A_0(0), \\ A_3(q^2) &= \frac{m_{B_c} + m_V}{2m_V} A_1(q^2) - \frac{m_{B_c} - m_V}{2m_V} A_2(q^2). \end{aligned} \quad (12)$$

In the $SU(4)$ symmetry, the interaction Lagrangians related to the processes are expressed as [48]

$$\mathcal{L}_{\mathcal{B}\mathcal{B}P} = \frac{g_{\mathcal{B}\mathcal{B}P}}{m_P} \langle \bar{\mathcal{B}} \gamma_5 \gamma_\mu \partial^\mu P \mathcal{B} \rangle, \quad (13)$$

$$\mathcal{L}_{\mathcal{B}\mathcal{B}V} = g_{\mathcal{B}\mathcal{B}V} \langle \bar{\mathcal{B}} \gamma_\mu V^\mu \mathcal{B} \rangle + \frac{f_{\mathcal{B}\mathcal{B}V}}{2m_V} \langle \bar{\mathcal{B}} \sigma_{\mu\nu} \partial_\mu V^\nu \mathcal{B} \rangle. \quad (14)$$

Here, $g_{\mathcal{B}\mathcal{B}P}$ and $g_{\mathcal{B}\mathcal{B}V}$ are the couplings of the pseudoscalar meson or vector meson and two baryons $\mathcal{B}\mathcal{B}$. Given that there are no experimental data to extract values for the needed couplings, we deduce them from the generic $SU(4)$ symmetry, *i.e.*, we use the empirical values $g_{NN\pi} = 13.5(5)$, $g_{\rho NN} = 3.25(90)$, $f_{\rho NN} = \kappa_\rho g_{\rho NN}$, $\kappa_\rho = 6.1(1)$

[49, 50] to obtain the couplings between doubly charmed and charmed baryons. For more details, please refer to Appendix A. Within $SU(4)$ symmetry, pseudoscalar and vector mesons are reclassified as 15-plet, whose expressions are [51]

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta_c}{\sqrt{12}} & \pi^+ & K^+ & \bar{D}^0 \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta_c}{\sqrt{12}} & K^0 & D^- \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta + \frac{\eta_c}{\sqrt{12}} & D_s^- \\ D^0 & D^+ & D_s^+ & -\frac{3\eta_c}{\sqrt{12}} \end{pmatrix}. \quad (15)$$

and

$$V_\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{6}} + \frac{J/\psi}{\sqrt{12}} & \rho^+ & K^{*+} & \bar{D}^{*0} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{6}} + \frac{J/\psi}{\sqrt{12}} & K^{*0} & D^{*-} \\ K^{*-} & \bar{K}^{*0} & -\sqrt{\frac{2}{3}}\omega + \frac{J/\psi}{\sqrt{12}} & D_s^{*-} \\ D^{*0} & D^{*+} & D_s^{*+} & -\frac{3J/\psi}{\sqrt{12}} \end{pmatrix}. \quad (16)$$

Baryons represented as $\mathcal{B}$ belong to a 20-plet,

$$\begin{aligned} \mathcal{B}^{121} &= p, \quad \mathcal{B}^{122} = n, \quad \mathcal{B}^{132} = \frac{1}{\sqrt{2}}\Sigma^0 - \frac{1}{\sqrt{6}}\Lambda, \quad \mathcal{B}^{213} = \sqrt{\frac{2}{3}}\Lambda, \quad \mathcal{B}^{231} = \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda, \quad \mathcal{B}^{232} = \Sigma^-, \quad \mathcal{B}^{233} = \Xi^-, \\ \mathcal{B}^{311} &= \Sigma^+, \quad \mathcal{B}^{313} = \Xi^0, \quad \mathcal{B}^{141} = -\Sigma_c^{++}, \quad \mathcal{B}^{142} = \frac{1}{\sqrt{2}}\Sigma_c^+ + \frac{1}{\sqrt{6}}\Lambda_c, \quad \mathcal{B}^{143} = \frac{1}{\sqrt{2}}\Xi_c'^+ - \frac{1}{\sqrt{6}}\Xi_c^+, \quad \mathcal{B}^{241} = \frac{1}{\sqrt{2}}\Sigma_c^+ - \frac{1}{\sqrt{6}}\Lambda_c, \\ \mathcal{B}^{242} &= \Sigma_c^0, \quad \mathcal{B}^{243} = \frac{1}{\sqrt{2}}\Xi_c'^0 + \frac{1}{\sqrt{6}}\Xi_c^0, \quad \mathcal{B}^{341} = \frac{1}{\sqrt{2}}\Xi_c'^+ + \frac{1}{\sqrt{6}}\Xi_c^+, \quad \mathcal{B}^{342} = \frac{1}{\sqrt{2}}\Xi_c'^0 - \frac{1}{\sqrt{6}}\Xi_c^0, \quad \mathcal{B}^{343} = \Omega_c^0, \\ \mathcal{B}^{124} &= \sqrt{\frac{2}{3}}\Lambda_c, \quad \mathcal{B}^{234} = \sqrt{\frac{2}{3}}\Xi_c^0, \quad \mathcal{B}^{314} = \sqrt{\frac{2}{3}}\Xi_c^+, \quad \mathcal{B}^{144} = \Xi_{cc}^{++}, \quad \mathcal{B}^{244} = -\Xi_{cc}^+, \quad \mathcal{B}^{344} = \Omega_{cc}^+. \end{aligned} \quad (17)$$

These states are expressed by tensors $\mathcal{B}^{ijk}$, where the first two indices are anti-symmetric.

B. Amplitudes and decay rates

With the effective Lagrangians at hand, we can obtain the decay amplitudes shown in Fig. 2. For instance, the amplitude of $\bar{B}_c \rightarrow \Xi_{cc}^{++}\Xi_c^-$ is

$$\begin{aligned} M_a &= \int \frac{d^4k_1}{(2\pi)^4} g_{\Xi_{cc}\Xi_c D} g_{\Xi_c\Xi_c\eta_c} \frac{\bar{u}_{\Xi_{cc}}(p_2)\gamma_\nu(\not{k}_1 - m_{\Xi_c})\gamma_\mu v_{\Xi_c}(p_1)k_3^\mu k_2^\nu}{(k_1^2 - m_{\Xi_c}^2)(k_2^2 - m_D^2)(k_3^2 - m_{\eta_c}^2)} \mathcal{F}^2(k_1^2)(if_D) \left((p \cdot k_2 + k_3 \cdot k_2 - m_{B_c}^2 + m_\eta^2)F_1(k_2^2) + (m_{B_c}^2 - m_\eta^2)F_0(k_2^2) \right), \\ M_b &= i \int \frac{d^4k_1}{(2\pi)^4} g_{\Xi_{cc}\Xi_c D} g_{\Xi_c\Xi_c\psi} \frac{\bar{u}_{\Xi_{cc}}(p_2)(\not{k}_1 - m_{\Xi_c})\gamma_5(\gamma_\mu - i\kappa_\psi\sigma_{\alpha\mu}k_3^\alpha/(2m_\psi))v_{\Xi_c}(p_1)}{(k_1^2 - m_{\Xi_c}^2)(k_2^2 - m_D^2)(k_3^2 - m_\psi^2)} \mathcal{F}^2(k_1^2)(if_D)ik_{2\nu} \\ &\quad (-g^{\mu\nu} + \frac{k_3^\mu k_3^\nu}{m_\psi^2}) \left((m_{B_c} + m_\psi)A_1(k_2^2) - \frac{(p + k_3) \cdot k_2}{m_{B_c} + m_\psi}A_2(k_2^2) - 2m_\psi A_3(k_2^2) + 2m_\psi A_0(k_2^2) \right), \\ M_c &= i \int \frac{d^4k_1}{(2\pi)^4} g_{\Xi_{cc}\Xi_c D^*} g_{\Xi_c\Xi_c\eta_c} \frac{\bar{u}_{\Xi_{cc}}(p_2)(\gamma_\mu - i\kappa_{D^*}\sigma_{\alpha\mu}k_2^\alpha/(2m_{D^*}))(\not{k}_1 + m_{cqq})\gamma_5(\not{k}_1 + \not{p}_1)v_{\Xi_c}(p_1)}{-(k_1^2 - m_{\Xi_c}^2)(k_2^2 - m_{D^*}^2)(k_3^2 - m_{\eta_c}^2)} \\ &\quad \mathcal{F}^2(k_1^2)(-g^{\mu\nu} + \frac{k_2^\mu k_2^\nu}{m_{D^*}^2})(m_{D^*}f_{D^*}) \left((p + k_3 - \frac{(m_{B_c}^2 - m_\eta^2)}{k_2^2}k_2)^\nu F_1(k_2^2) + \frac{(m_{B_c}^2 - m_\eta^2)}{k_2^2}k_2^\nu F_0(k_2^2) \right), \\ M_d &= \int \frac{d^4k_1}{(2\pi)^4} \frac{\bar{u}_{\Xi_{cc}}(p_2)(\gamma_\mu - i\kappa_{D^*}\sigma_{\alpha'\mu}k_2^{\alpha'}/(2m_{D^*}))(\not{k}_1 + m_{\Xi_c})(\gamma_\nu - i\kappa_\psi\sigma_{\beta'\nu}k_3^{\beta'}/(2m_\psi))v_{\Xi_c}(p_1)}{-(k_1^2 - m_{\Xi_c}^2)(k_2^2 - m_{D^*}^2)(k_3^2 - m_\psi^2)} \\ &\quad (g_{\Xi_{cc}\Xi_c D^*} g_{\Xi_c\Xi_c\psi}) \mathcal{F}^2(k_1^2)(m_{D^*}f_{D^*}) \left(-g^{\mu\alpha} + \frac{k_2^\mu k_2^\alpha}{m_{D^*}^2} \right) \left(-g^{\nu\beta} + \frac{k_3^\nu k_3^\beta}{m_\psi^2} \right) \left(\frac{2}{m_{B_c} + m_\psi} \varepsilon_{\alpha\beta\rho\sigma} p^\rho k_2^\sigma V(k_2^2) \right. \\ &\quad \left. + i g_{\alpha\beta}(m_{B_c} + m_\psi)A_1(k_2^2) - i \frac{(p + k_3)_\alpha k_{2\beta}}{m_{B_c} + m_\psi}A_2(k_2^2) - i \frac{k_{2\alpha} k_{2\beta}}{k_2^2}2m_\psi A_3(k_2^2) + i \frac{k_{2\beta}}{k_2^2}2m_\psi k_{2\alpha} A_0(k_2^2) \right). \end{aligned} \quad (18)$$

where $k_3 = p_1 + k_1$ is the momentum of $J/\psi(\eta_c)$, and $k_2 = p_2 - k_1$ is the momentum of $D(D^*)$, respectively. Hadrons have finite sizes; therefore, the monopole form factor is adopted at each vertex,

$$\mathcal{F}(k_1^2) = \left(\frac{\Lambda^2 - m_i^2}{\Lambda^2 - k_1^2} \right)^2, \quad (19)$$

where Λ is the so-called cutoff mass, which governs the range of suppression. We use the expression $\Lambda = m_E + \alpha$ [39], where m_E is the mass of the exchanged particle and $\alpha = \eta\Lambda_{\text{QCD}}$. Usually the parameter η is expected to be of order unity and depends not only on the exchanged particle but also on the external particles involved in the

strong-interaction vertex. In principle, it cannot be determined from first-principles calculations. However, we can set the parameter using the measured decay rates. In this B_c meson system [52], we take a naive value for α in the range of 100–300 MeV. The decay rate for the non-leptonic transition $\bar{B}_c \rightarrow P_1 P_2$ is expressed in terms of the decay amplitude $\mathcal{M}(\bar{B}_c \rightarrow P_1 P_2)$

$$\Gamma(\bar{B}_c \rightarrow \mathcal{B}_{cc} + \bar{\mathcal{B}}_{cqq}) = \frac{|\mathbf{P}_1|}{8\pi m_{B_c}^2} |\mathcal{M}|^2, \quad (20)$$

where $\mathbf{P}_1$ is the magnitude of the three momenta of the final state meson, expressed as $|\mathbf{P}_1| = \frac{1}{2m_{B_c}}$

$$\sqrt{\lambda(m_{B_c}^2, m_{B_{cc}}^2, m_{\Xi_c}^2)}; \lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ac.$$

III. NUMERICAL ANALYSIS

The calculation of branching ratios requires the following decay constants of several pseudoscalar and vector mesons:

$$\begin{aligned} f_D &= 0.208(15) \text{ GeV}, \quad f_{D_s} = 0.273(12) \text{ GeV}, \\ f_{D^*} &= 0.245(20) \text{ GeV}, \quad f_{D_s^*} = 0.273(19) \text{ GeV}. \end{aligned} \quad (21)$$

Furthermore, the masses of mesons and baryons are extracted from PDG [53]. Numerous strong couplings are deduced from $SU(4)$ symmetry (see Appendix A).

In the physical region, each of the transition form factors can be accurately interpolated using the following function [54]:

$$f(q^2) = \alpha_1 + \alpha_2 q^2 + \frac{\alpha_3 q^4}{m_{B_c}^2 - q^2},$$

where the parameters $\alpha_{1,2,3}$ are taken from Table 2.

Using these expressions and results, we can determine the production of doubly charmed baryons from $\bar{B}_c$ mesons, obtaining the number of widths and branching ratios presented in Table 3. Note that the branching ratio can reach values on the order of 10^{-5} . In our analysis, we set the parameter $\alpha = \Lambda_{\text{QCD}}$ in Λ with a value of 250 MeV [55]. For completeness, the branching ratios as a function of α are presented in Fig. 3. Moreover, we verify the

dominant sub-process of productions $\bar{B}_c \rightarrow M_1 M_2 \xrightarrow{EP} \bar{B}_c \mathcal{B}_{cc}$ ($M_1 M_2 = J/\psi D, \eta_c D, J/\psi D^*$ or ηD^*). Based on the results, we draw several conclusions as follows.

- Our calculations show that the production $\bar{B}_c \rightarrow \Xi_{cc}^{++} \Xi_c'^-$ occupies the largest branching ratio among the processes considered in this study; this must be confirmed experimentally. Once verified, it could be an ideal candidate to produce doubly charmed baryons in the future. The dominant amplitude contribution comes from the sub-processes $\bar{B}_c \rightarrow J/\psi D \xrightarrow{\Xi_c} \Xi_{cc}^{++} \Xi_c'^-$ (the branching ratio of this subprocess is 2.1×10^{-5}) and $\bar{B}_c \rightarrow \eta_c D^* \xrightarrow{\Xi_c} \Xi_{cc}^{++} \Xi_c'^-$ (the branching ratio of this subprocess is 1.1×10^{-5}). Therefore, the major contribution of branching ratios comes from these two sub-processes and their cross ones.

- It seems that the leading contributions of strong couplings are subprocesses with $\mathcal{B}\mathcal{B}V$ or $\mathcal{B}\mathcal{B}P$ vertices consistent with the processes, *i.e.*, for $\bar{B}_c \rightarrow \Xi_{cc}^{++} \Xi_c'^-$, the isolated contribution from a pair of $\mathcal{B}\mathcal{B}P$ or $\mathcal{B}\mathcal{B}V$ is trivial, merely on the order of 10^{-8} .

- Unlike the first four Cabibbo allowed processes, the last four processes are Cabibbo suppressed. However, the branching ratios are still considerable. Especially, the undiscovered Ω_{cc} can reach values on the order of 10^{-6} .

- Table 3 shows the rates of decay widths: $\Gamma(\bar{B}_c \rightarrow \Xi_{cc}^{++} \Xi_c'^-)/\Gamma(\bar{B}_c \rightarrow \Xi_{cc}^{++} \bar{\Lambda}_c^-) \approx 16.4$, $\Gamma(\bar{B}_c \rightarrow \Xi_{cc}^{++} \Xi_c'^-)/$

Table 2. Interpolation parameters of form factors in Eq. (10); α_1 is dimensionless, and $\alpha_{2,3}$ have dimensions of GeV^{-1} .

	$\bar{B}_c \rightarrow \eta_c$			$\bar{B}_c \rightarrow J/\psi$			
	$F_1(0)$	$F_0(0)$	$V_0(0)$	$A_0(0)$	$A_1(0)$	$A_2(0)$	$A_3(0)$
α_1	0.632(13)	0.632(13)	0.836(41)	0.574(33)	0.551(25)	0.561(28)	0.574(33)
α_2	0.032(24)	0.025(11)	0.045(80)	0.030(43)	0.018(38)	0.021(35)	0.030(43)
α_3	0.061(13)	0.034(7)	0.028(110)	0.055(25)	0.036(61)	0.037(77)	0.055(25)

Table 3. Branching ratios of doubly charmed baryons from $\bar{B}_c$ ($\bar{B}_c \rightarrow M_1 M_2 \xrightarrow{EP} \bar{B}_c \mathcal{B}_{cc}$) with $\Lambda = m_{B_c} + 0.25 \text{ GeV}$. The exchanged particles (EP) can be $\Lambda_c, \Xi_c, \Sigma_c$. The dominant contribution from $M_1 M_2$ are also presented.

Process	EP	Decay width ($\times 10^{-19}$)	Branching ratio ($\times 10^{-6}$)	Dominant $M_1 M_2$
$\bar{B}_c \rightarrow \Xi_{cc}^{++} \Xi_c'^-$	Ξ_c	$4.88^{+1.07}_{-0.92}$	$3.78^{+0.83}_{-0.71}$	$\eta_c D^*$
$\bar{B}_c \rightarrow \Xi_{cc}^{++} \Xi_c^0$	Ξ_c	$5.51^{+1.27}_{-1.06}$	$4.27^{+0.98}_{-0.82}$	$\eta_c D^*$
$\bar{B}_c \rightarrow \Xi_{cc}^{++} \Xi_c'^0$	Ξ_c'	$83.1^{+20.1}_{-17.3}$	$64.4^{+15.6}_{-13.4}$	$J/\psi D, \eta_c D^*$
$\bar{B}_c \rightarrow \Xi_{cc}^{++} \Xi_c^0$	Ξ_c'	$117.9^{+29.2}_{-24.5}$	$91.4^{+22.6}_{-19.0}$	$J/\psi D, \eta_c D^*$
$\bar{B}_c \rightarrow \Xi_{cc}^{++} \bar{\Lambda}_c^-$	Λ_c	$0.30^{+0.08}_{-0.07}$	$0.23^{+0.06}_{-0.05}$	$\eta_c D^*$
$\bar{B}_c \rightarrow \Xi_{cc}^{++} \bar{\Sigma}_c^0$	Σ_c	$11.9^{+3.29}_{-2.72}$	$9.25^{+2.55}_{-2.11}$	$J/\psi D, \eta_c D^*$
$\bar{B}_c \rightarrow \Xi_{cc}^{++} \bar{\Sigma}_c^-$	Σ_c	$5.37^{+1.50}_{-1.23}$	$4.16^{+1.16}_{-0.95}$	$J/\psi D, \eta_c D^*$
$\bar{B}_c \rightarrow \Omega_{cc}^{++} \Xi_c^0$	Ξ_c	$0.13^{+0.04}_{-0.03}$	$0.10^{+0.03}_{-0.02}$	$\eta_c D^*$

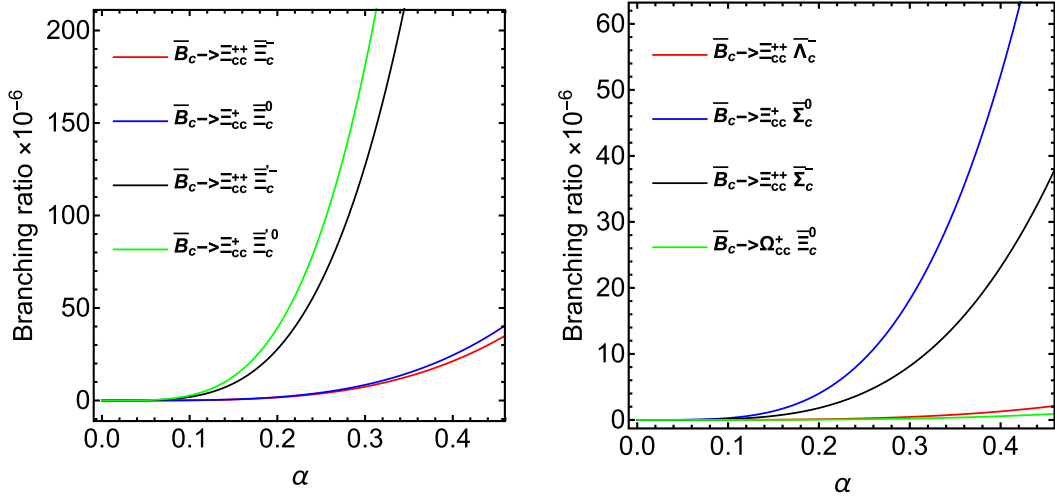


Fig. 3. (color online) Branching ratios as a function of α in Λ ; α varies in the range of 100–300 MeV. The left image contains four Cabibbo allowed processes, whereas the right one shows Cabibbo suppressed processes with transition $\bar{b} \rightarrow \bar{c}d$.

$\Gamma(\bar{B}_c \rightarrow \Xi_{cc}^{++} \bar{\Sigma}_c^-) \approx 15.5$; these values are consistent with the straightforward symmetry analysis expressed by Eq. (2), resulting in $|V_{cs}|^2/|V_{cd}|^2 \approx 19.9$. This confirms that the $SU(3)$ symmetry is basically maintained in these processes.

- According to a systematic error analysis that encompasses the uncertainties of form factors, decay constants, and coupling constants, the deviation of the central value can reach 20%. Further analysis reveals that the main uncertainties come from the coupling constant $\mathcal{B}\mathcal{B}V$, *i.e.*, $g_{\rho NN} = 3.25(90)$, $g_{\Xi_{cc}^+ \Sigma_c D^*} = -4.60(113)$; once these are excluded, the error will be less than 10%.

- The decay channels $\bar{B}_c \rightarrow \bar{B}_c \mathcal{B}_{cc}$ have been calculated for the first time in this study. Previous studies reported the branching ratios for processes such as $\bar{B}^0 \rightarrow \Lambda_c^+ \bar{p}$ in the framework of perturbative QCD (pQCD) [56]; the effective operator involved, $\bar{b} \rightarrow \bar{c}ud$, is similar to ours, $\bar{b} \rightarrow \bar{c}c\bar{s}$. However, it is well established that complete theoretical predictions within factorization schemes require non-perturbative inputs. In the present case, the wave functions of doubly charmed baryons remain unavailable, as there are currently no theoretical or experimental studies on their LCDAs. Consequently, we adopt the effective Lagrangian approach, which circumvents the need for explicit knowledge of hadronic LCDAs.

IV. CONCLUSIONS

The observation of Ξ_{cc}^{++} by the LHCb Collaboration has opened a new field of research on the nature of baryons containing two heavy quarks. Doubly charmed baryons can form an $SU(3)$ triplet, $\Xi_{cc}^{++}, \Xi_{cc}^+$, and Ω_{cc}^+ . Therefore, after the experimental observation of Ξ_{cc}^{++} , it is of

prime importance to search for additional production modes of this particle and the other two. It is worth noting that the charmonium pentaquarks were discovered from B -meson decays [57, 58]. This inspired us to search for doubly charmed baryons through meson decays.

In this paper, we have presented a theoretical study on the production of doubly charmed baryons from $\bar{B}_c$ meson decays. The decay widths and branching ratios have been obtained using the effective Lagrangian method; the results are summarized in Table 3. We expect these results to be helpful for exploring new decay modes in the future.

APPENDIX A. COUPLINGS

The Lagrangian of $\mathcal{B}\mathcal{B}P$ in $SU(3)$ symmetry can be expressed as

$$\mathcal{L}_{\mathcal{B}\mathcal{B}P}^{SU3} = a_1 \bar{\mathcal{B}}_j^i \gamma_5 \gamma_\mu \partial^\mu P_i^j \mathcal{B}_k^i + a_2 \bar{\mathcal{B}}_j^i \gamma_5 \gamma_\mu \partial^\mu P_i^k \mathcal{B}_k^j, \quad (A1)$$

where $a_1 = D + F$, $a_2 = D - F$, and $\frac{D}{D+F} = \alpha_D = 0.64$ [59]. Expanding the Lagrangian, we take the couplings of $NN\pi$ and $K\Lambda N$, and the rate is

$$\frac{g_{NN\pi}}{g_{K\Lambda N}} = \frac{\sqrt{3}}{2\alpha_D - 3}. \quad (A2)$$

Furthermore, the Lagrangian of $\mathcal{B}\mathcal{B}P$ in $SU(4)$ symmetry is

$$\begin{aligned} \mathcal{L}_{\mathcal{B}\mathcal{B}P}^{SU4} &= g_0 (b_1 \bar{\mathcal{B}}_{[jk]}^i \gamma_5 \gamma_\mu \partial^\mu P_i^{[kl]} \mathcal{B}_l^{[kl]} + b_2 \bar{\mathcal{B}}_{[kl]}^i \gamma_5 \gamma_\mu \partial^\mu P_i^{[jk]} \mathcal{B}_j^{[jk]}) \\ &= g_0 (g_1 \bar{\mathcal{B}}^{[ij]k} \gamma_5 \gamma_\mu \partial^\mu P_i^{[l]jk} \mathcal{B}_{[l]jk} + g_2 \bar{\mathcal{B}}^{[ij]k} \gamma_5 \gamma_\mu \partial^\mu P_i^{[l]jk} \mathcal{B}_{[l]jk}). \end{aligned} \quad (A3)$$

Here, we show two equivalent tensor representations of baryons, $\mathcal{B}_{[\alpha\beta]}^i \varepsilon^{jk\alpha\beta}$ and $\mathcal{B}^{(ij)k}$, where $[\]$ and $\{ \}$ represent symmetric and anti-symmetric indices, respectively; $\varepsilon^{jk\alpha\beta}$ is the total antisymmetric tensor. Expanding the Lagrangian, we obtain

$$\frac{g_{NN\pi}}{g_{K\Lambda N}} = \frac{b_2}{-\frac{b_1+4b_2}{2\sqrt{3}}} = \frac{-2\sqrt{3}}{\frac{b_1}{b_2}+4}. \quad (\text{A4})$$

Comparing the results in Eqs. (A2) and (A4), we have

$$\frac{b_1}{b_2} = 2 - 4\alpha_D. \quad (\text{A5})$$

This is similar for the case of $\mathcal{B}\mathcal{B}V$ couplings. Generally, the Lagrangian of $\mathcal{B}\mathcal{B}V$ in $SU(3)$ symmetry can be expressed as

$$\mathcal{L}_{\mathcal{B}\mathcal{B}V}^{SU3} = c_1 \bar{\mathcal{B}}_j^i \gamma_\mu (V^\mu)_k^j \mathcal{B}_i^k + c_1 \bar{\mathcal{B}}_k^j \gamma_\mu (V^\mu)_i^j \mathcal{B}_j^k + c_2 \bar{\mathcal{B}}_k^j \gamma_\mu (V^\mu)_j^i \mathcal{B}_i^k. \quad (\text{A6})$$

Expanding the Lagrangian and comparing the coupling ρNN with $K^* \Lambda N$, we obtain

$$g_{\rho NN} = -\sqrt{3} g_{K^* \Lambda N}. \quad (\text{A7})$$

The Lagrangian of $\mathcal{B}\mathcal{B}V$ in $SU(4)$ symmetry is given below,

$$\begin{aligned} \mathcal{L}_{\mathcal{B}\mathcal{B}V}^{SU4} &= g'_0 (g'_1 \bar{\mathcal{B}}_{[jk]}^i \gamma_\mu (V^\mu)_l^j \mathcal{B}_i^{[kl]} + g'_2 \bar{\mathcal{B}}_{[kl]}^i \gamma_\mu (V^\mu)_j^j \mathcal{B}_i^{[kl]} \\ &\quad + g'_3 \bar{\mathcal{B}}_{[kl]}^i \gamma_\mu (V^\mu)_j^j \mathcal{B}_i^{[kl]}) \\ &= g'_0 (h_1 \bar{\mathcal{B}}^{(ij)k} \gamma_\mu (V^\mu)_i^j \mathcal{B}_{[l]jk} + h_2 \bar{\mathcal{B}}^{[ij]k} \gamma_\mu (V^\mu)_i^j \mathcal{B}_{[lk]j}). \end{aligned}$$

We again show two equivalent formulas. Similarly, one can obtain

$$\frac{g_{\rho NN}}{g_{K^* \Lambda N}} = \frac{g'_2}{-\frac{g'_1+4g'_2}{2\sqrt{3}}} = \frac{-2\sqrt{3}}{\frac{g'_1}{g'_2}+4}, \quad (\text{A9})$$

and finally, we have

$$\frac{g'_1}{g'_2} = -2. \quad (\text{A10})$$

According to the couplings $g_{NN\pi} = 13.5(5)$, $g_{\rho NN} = 3.25(90)$, $\kappa_\rho = 6.1(1)$ as well as the relations given in Eqs. (A5) and (A10), we can determine the required couplings:

$$\begin{aligned} g_{\Xi_c \Xi_c \eta_c} &= \frac{1}{3\sqrt{6}}(b_1 - 5b_2) = -10.2(3), & g_{\Xi_c \Xi_c' \eta_c} &= 0, \\ g_{\Lambda_c \Lambda_c \eta_c} &= \frac{1}{3\sqrt{6}}(b_1 - 5b_2) = -10.2(3), & g_{\Lambda_c \Sigma_c \eta_c} &= 0, \\ g_{\Sigma_c \Sigma_c \eta_c} &= \frac{1}{\sqrt{6}}(b_2 - b_1) = 8.6(3), \\ g_{\Xi_c' \Xi_c' \eta_c} &= \frac{1}{\sqrt{6}}(b_2 - b_1) = 8.6(3), \\ g_{\Xi_c^{++} \Xi_c D_s} &= -\frac{1}{\sqrt{3}}(b_1 + b_2) = -3.4(2), \\ g_{\Xi_c^{++} \Xi_c' D_s} &= b_2 = 13.5(5), \\ g_{\Xi_c^{++} \Xi_c D_s} &= -\frac{1}{\sqrt{3}}(b_1 + b_2) = -3.4(2), \\ g_{\Xi_c^{++} \Xi_c' D_s} &= -b_2 = -13.5(5), \\ g_{\Xi_c^{++} \Lambda_c D} &= \frac{1}{\sqrt{3}}(b_1 + b_2) = 3.4(2), \\ g_{\Xi_c^{++} \Sigma_c D} &= b_2 = 13.5(5), \\ g_{\Xi_c^{++} \Sigma_c D} &= -\sqrt{2}b_2 = -19.1(7), \\ g_{\Omega_c^{++} \Xi_c D} &= -\frac{1}{\sqrt{3}}(b_1 + b_2) = -3.4(2), \\ g_{\Xi_c \Xi_c J/\psi} &= \frac{1}{3\sqrt{6}}(g_1 - 5g_2) = -3.10(75), & g_{\Xi_c \Xi_c' J/\psi} &= 0, \\ g_{\Lambda_c \Lambda_c J/\psi} &= \frac{1}{3\sqrt{6}}(g_1 - 5g_2) = -3.10(75), & g_{\Lambda_c \Sigma_c J/\psi} &= 0, \\ g_{\Sigma_c \Sigma_c J/\psi} &= \frac{g_2 - g_1}{\sqrt{6}} = 3.98(90), \\ g_{\Xi_c' \Xi_c J/\psi} &= \frac{g_2 - g_1}{\sqrt{6}} = 3.98(90), \\ g_{\Xi_c^{++} \Xi_c D_s^*} &= -\frac{1}{\sqrt{3}}(g_1 + g_2) = 1.88(45), \\ g_{\Xi_c^{++} \Xi_c' D_s^*} &= g_2 = 3.25(90), \\ g_{\Xi_c^{++} \Xi_c D_s^*} &= -\frac{1}{\sqrt{3}}(g_1 + g_2) = 1.88(45), \\ g_{\Xi_c^{++} \Xi_c' D_s^*} &= -g_2 = -3.25(90), \\ g_{\Xi_c^{++} \Lambda_c D^*} &= \frac{1}{\sqrt{3}}(g_1 + g_2) = -1.88(45), \\ g_{\Xi_c^{++} \Sigma_c D^*} &= g_2 = 3.25(90), \\ g_{\Xi_c^{++} \Sigma_c D^*} &= -\sqrt{2}g_2 = -4.60(113), \\ g_{\Omega_c^{++} \Xi_c D^*} &= -\frac{1}{\sqrt{3}}(g_1 + g_2) = 1.88(45). \end{aligned} \quad (\text{A11})$$

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