Chinese Physics C Vol. 49, No. 9 (2025) 095104

Quasinormal modes of accelerating spacetime*

Tao Zhou (周陶)† Peng-Cheng Li (李鹏程)‡

School of Physics and Optoelectronics, South China University of Technology, Guangzhou 510641, China

Abstract: We calculate the exact values of the quasinormal frequencies for massless perturbations with spin $s \leq 2$ propagating in purely accelerating spacetime. Using two different methods, we transfer the perturbation equations into hypergeometric differential equations and obtain identical quasinormal frequencies. These purely imaginary spectra are shown to be independent of the spin of the perturbation and match those of the acceleration modes identified in accelerating black holes in the Minkowski limit. This implies that the acceleration modes originate from the purely accelerating spacetime and that the presence of black holes would deform the spectra. Additionally, we compute the quasinormal frequencies of scalar, electromagnetic, and gravitational perturbations in D-dimensional de Sitter spacetime and compare them with previous findings to validate our method.

Keywords: quasinormal modes, black hole perturbations, C-metric

DOI: 10.1088/1674-1137/add10d **CSTR:** CSTR:32044.14.ChinesePhysicsC.49095104

I. INTRODUCTION

The first detection of gravitational waves (GWs) from the merger of two black holes (BHs) [1], along with the release of the first image of the supermassive BH in the galaxy M87 [2], has provided strong evidence that BHs are real celestial objects rather than just theoretical concepts. When a BH is perturbed, the relaxation can be described by a superposition of exponentially damped sinusoids termed quasinormal modes (QNMs) [3–5]. Thus, during the ringdown stage of the coalescence of two astrophysical BHs, the GWs can be expressed as a superposed QNM of the remnant BH. According to the no-hair theorem, the frequencies and decay rates of these QNMs are uniquely determined by the final BH's physical parameters [6]. The measurement of the QNMs from GW observations facilitates the test of general relativity and provides insights into the nature of remnants formed in compact binary mergers. This program is known as BH spectroscopy [7]. Moreover, the QNMs can also be used to determine the linear stability of a perturbed BH. For example, an analysis of the QNMs of massless scalar fields in the exterior of Reissner-Nordström-de Sitter (RNdS) BHs can be used to determine whether the strong cosmic censorship conjecture is violated [8].

Recently, the QNMs of the C-metric have attracted increasing attention in various physical contexts [9–17]. The C-metric describes an axially symmetric and stationary spacetime containing two causally separated BHs that accelerate away from each other in opposite spatial directions under the action of conical singularities along the axis [18–20]. This model may be useful for understanding the behavior of moving and accelerating BHs, such as those resulting from a BH superkick or a cosmic string connecting two BHs [21, 22]. These studies on the QNMs of accelerating BHs rely on the C-metric's Petrov type D classification, which allows the separation of perturbation equations for various test fields [23–26]. Notably, QNMs of charged and rotating accelerating BHs can be classified into three families: photon sphere, near-extreme, and acceleration modes. Photon sphere modes correspond to peaks in the potential barrier, while near extreme modes relate to the BHs' near-horizon geometry. Acceleration modes depend solely on the acceleration horizon and are absent in non-accelerating spacetimes. This new mode types were first identified for scalar perturbations of charged accelerating BHs [10]. Later, a similar phenomenon was also verified for scalar [14] and gravitational perturbations [17] of rotating accelerating BHs.

In this study, we investigate the origin of the acceleration modes. Given that these modes have a weak dependence on the BH's charge or rotational parameter and depend solely on the surface gravity in the Minkowski limit rather than the BH's surface gravity, we speculate that the acceleration modes may originate from purely accelerating spacetime without a BH. Notably, pure de Sitter (dS)

Received 7 March 2025; Accepted 27 April 2025; Published online 28 April 2025

* Part supported by NSFC (12205104), the Fundamental Research Funds for the Central Universities (2023ZYGXZR079), the Guangzhou Science and Technology Project (2023A04J0651) and the startup funding of South China University of Technology

† E-mail: 202220130252@mail.scut.edu.cn

‡ E-mail: pchli2021@scut.edu.cn (Corresponding author)

spacetime also exhibit dS modes [27–29], which become deformed when BHs appear [8, 30–32]. Given the significant similarity between accelerating spacetime, which features an accelerating horizon, and dS spacetime, which has a cosmological horizon, it is reasonable to associate acceleration modes with their counterpart in pure accelerating spacetime.

In this study, we calculate the QNMs of massless perturbations with spin $s\leq 2$ in purely accelerating spacetime. By taking the Minkowski limit of the master equations describing perturbations of accelerating BHs − specifically by setting the mass, electric charge, or spin to zero − we can derive the equations that govern perturbations in the empty accelerating spacetime. Two methods are employed to solve the perturbation equations. The first method follows the approach in [27], where a direct coordinate transformation shows that the perturbation equations are expressed as a hypergeometric type. QNMs can then be easily obtained by imposing appropriate boundary conditions. This method works straightforward for scalar perturbations [10]. The second approach analyzes the properties of the Minkowski limit of the Teukolsky-like equations governing various massless field perturbations of spinning accelerating BHs [24]. This approach involves identifying all singular points and demonstrating that the equations are of the Fuchsian type, with all singular points being regular. If exactly three regular singular points are present, the equations can be transformed into the standard form of the hypergeometric differential equations (HDEs) [33, 34]. By imposing appropriate boundary conditions, we can derive the QNMs. To validate this approach, we apply it to calculate dS modes and compare our results with those in [27]. We confirm that our results for dS modes of scalar, electromagnetic, and gravitational perturbations are consistent with those of previous findings. An advantage of this method is that it does not require complex coordinate transformations to convert the equations into HDEs.

The remainder of this paper is organized as follows: In Sec. II, we explicitely show the derivation of the QNMs of massless scalar perturbations in purely accelerating spacetime by solving the Minkowski limit of the Klein-Gordon (KG) equation in the background of the C-metric, employing the two methods described above. In Sec. III, we calculate the QNMs of massless perturbations with spin $s\leq 2$ by solving the Minkowski limit of the Teukolsky-like equations in the spinning C-metric spacetime. In Sec. IV, we discuss the application of the new method to calculate dS modes of scalar, electromagnetic, and gravitational perturbations in D dimensional dS spacetime and compare the results with those in [27]. Finally, in Sec. V, we summarize the study, and in Appendix A, we introduce the basics of Fuchsian equations relevant to this study. By convention, we employ geometric units $c=G=1$ and the metric signature $(-,+,+,+)$.

II. SCALAR FIELD

Initially, we examine the QNMs of scalar perturbations in purely accelerating spacetime. We consider a massless neutral scalar filed minimally coupled to gravity living in the spacetime of a C-metric. The evolution of the scalar perturbations is described by the KG equation, which has been shown to be separable through conformal transformation, as demonstrated in [10]. The C-metric describing a single BH can be expressed in terms of spherical-type coordinates as given in [20]:

$$\mathrm{d}s^2=\frac{1}{(1-\mathcal{A}r\cos\theta)^2}\left(-f(r)\mathrm{d}t^2+\frac{\mathrm{d}r^2}{f(r)}+\frac{r^2\mathrm{d}\theta^2}{P(\theta)}+P(\theta)r^2\sin^2\theta\mathrm{d}\varphi^2\right),\tag{1}$$

where

$$f(r)=\left(1-\frac{2M}{r}\right)\left(1-\mathcal{A}^2r^2\right),\tag{2}$$

$$P(\theta)=1-2\mathcal{A}M\cos\theta.\tag{3}$$

Here $\mathcal{A}$ is the parameter describing acceleration, while M is the mass of the BH. As $\mathcal{A}\to 0$, this metric asymptotes to Schwarzschild metric. As explained in [35], taking the Minkowski limit $M=0$, the metric (1) in the region $r<1/\mathcal{A}$ can be transformed into the uniformly accelerated metric (also known as *Rindler* metric) through an appropriate coordinate transformation. Notably, $\mathcal{A}$ is interpreted as the acceleration of a test particle located at the origin $r=0$.

Ref. [10] found that under the conformal transformation $\tilde{g}_{\mu\nu}\to\Omega^2g_{\mu\nu}$, $\tilde{\Phi}\to\Omega^2\Phi$, where $\Omega=1-\mathcal{A}r\cos\theta$, the KG equation $\nabla^\mu\nabla_\mu\Phi=0$ becomes separable by choosing the ansatz

$$\tilde{\Phi}=\sum_{lm}\mathrm{e}^{-\mathrm{i}\omega_{lm}t}\mathrm{e}^{\mathrm{i}m\varphi}\frac{\phi_{lm}(r)}{r}\chi_{lm}(\theta),\tag{4}$$

where ω_{lm} is the quasinormal frequency, and m is the magnetic (azimuthal) quantum number. Subsequently, we omit the subscript lm for simplicity. As seen from metric (1), conical singularities appear along the axis at both $\theta=0$ and $\theta=\pi$. To eliminate the conical singularity at $\theta=\pi$, the period of the azimuthal coordinate φ can be appropriately specified as $2\pi C$, where $C=1/P(\pi)$. Consequently, the magnetic quantum number m would not be an integer; instead, m must be of the form $m=m_0P(\pi)$ with m_0 being an integer [24], given that one has $\mathrm{e}^{\mathrm{i}m(\varphi+2\pi C)}=\mathrm{e}^{\mathrm{i}m\varphi}$.

This leads to the following set of separated equations:

$$\frac{d^2\phi(r)}{dr_*^2} + \left(\omega^2 - V_r\right)\phi(r) = 0, \tag{5}$$

$$\frac{d^2\chi(\theta)}{dz^2} - \left(m^2 - V_\theta\right)\chi(\theta) = 0, \tag{6}$$

where

$$dr_* = \frac{dr}{f(r)}, \quad dz = \frac{d\theta}{P(\theta)\sin\theta}, \tag{7}$$

and

$$V_r = f(r)\left(\frac{\lambda}{r^2} - \frac{f(r)}{3r^2} + \frac{f'(r)}{3r} - \frac{f''(r)}{6}\right), \tag{8}$$

$$V_\theta = P(\theta)\left(\lambda\sin^2\theta - \frac{P(\theta)\sin^2\theta}{3} + \frac{\sin\theta\cos\theta P'(\theta)}{2} + \frac{\sin^2\theta P''(\theta)}{6}\right). \tag{9}$$

Here r_* is the tortoise coordinate, and λ is a separation constant. Taking the Minkowski limit $M\to 0$, the separated wave Eqs. (5) and (6) governing the scalar perturbations in purely accelerating spacetime are reduced to

$$\left(1-\mathcal{A}^2r^2\right)\frac{d}{dr}\left[\left(1-\mathcal{A}^2r^2\right)\frac{d\phi(r)}{dr}\right] + \left[\omega^2 - \left(1-\mathcal{A}^2r^2\right)\left(\frac{\lambda}{r^2} - \frac{1}{3r^2}\right)\right]\phi(r) = 0, \tag{10}$$

and

$$\sin\theta\frac{d}{d\theta}\left(\sin\theta\frac{d\chi(\theta)}{d\theta}\right) + \left[(\lambda - \frac{1}{3})\sin^2\theta - m^2\right]\chi(\theta) = 0. \tag{11}$$

Notably, no difference is observed between m and m_0 in the Minkowski limit. The angular Eq. (11) has the same form as the equation of the Laplacian spherical harmonics Y_{lm} in spherical coordinates. Therefore, we can write the angular equation as

$$\sin\theta\frac{d}{d\theta}\left(\sin\theta\frac{d\chi(\theta)}{d\theta}\right) + \left[\ell(\ell+1)\sin^2\theta - m^2\right]\chi(\theta) = 0, \tag{12}$$

from which the separation constant λ is related to the eigenvalue of the spherical Laplacian operator by

$$\lambda = \ell(\ell+1) + \frac{1}{3}. \tag{13}$$

A. Direct conversion to HDEs

We follow the steps in [27] to show that the radial Eq. (10) can be transformed into the HDE by a change of variable. Initially, by introducing two dimensionless quantities

$$x = r\mathcal{A}, \quad \text{and} \quad \omega = \mathcal{A}\tilde{\omega}, \tag{14}$$

Eq. (10) can be written as

$$\left(x^2-1\right)\frac{d^2\phi}{dx^2} + 2x\frac{d\phi}{dx} + \left(\frac{\tilde{\omega}^2}{x^2-1} + \frac{\ell(\ell+1)}{x^2}\right)\phi = 0. \tag{15}$$

Then, by changing the variable $y = x^2$, the equation is transformed into

$$4y(y-1)\frac{d^2\phi}{dy^2} + (6y-2)\frac{d\phi}{dy} + \left(\frac{\ell(\ell+1)}{y} + \frac{\tilde{\omega}^2}{y-1}\right)\phi = 0. \tag{16}$$

Furthermore, we make the ansatz

$$\phi = y^A(1-y)^B\tilde{\phi}, \tag{17}$$

where the parameters A and B are chosen as

$$A = \begin{cases} \dfrac{\ell+1}{2}, \\ -\dfrac{\ell}{2}, \end{cases} \qquad B = \pm\frac{i\tilde{\omega}}{2}, \tag{18}$$

to find that the function $\tilde{\phi}$ must be the solution of the HDE

$$y(1-y)\frac{d^2\tilde{\phi}}{dy^2} + \left[\gamma - (\alpha+\beta+1)y\right]\frac{d\tilde{\phi}}{dy} - \alpha\beta\tilde{\phi} = 0, \tag{19}$$

with the parameters α, β, and γ equal to

$$\alpha = A + B, \quad \beta = A + B + \frac{1}{2}, \quad \gamma = 2A + \frac{1}{2}. \tag{20}$$

One can verify that Eq. (19) indeed has three regular singular points $y = 0$, 1, and ∞, which correspond to the standard form of the HDE. By contrast, Eq. (15) has three regular singular points $x = 0$, 1, and -1, while ∞ is an ordinary point. The structure of the regular singular points changes with the variable transformation from x to y. The subsequent transformation (17) is employed to obtain the standard form of the HDE. There are several possible combinations for the values of A and B, and therefore of α, β, and γ. Thus, the solutions of Eq. (A12) can be expressed in several equivalent forms. In the following, we study in detail the case $A=\dfrac{\ell+1}{2}$ and $B=\dfrac{\mathrm{i}\tilde{\omega}}{2}$.

Similar to dS QNMs, the QNMs are solutions to the equations of motion that satisfy the following physical boundary conditions: (i) the field is regular at the origin, and (ii) the field is purely outgoing near the acceleration horizon. Depending on the parameter values, the HDEs have different solutions [27, 36]. The regular behavior at $r=0$ selects the solution of Eq. (15) to be

$$\phi = y^{\frac{\ell+1}{2}}(1-y)^{\frac{\mathrm{i}\tilde{\omega}}{2}}\,{}_2F_1(\alpha,\beta;\gamma;y), \tag{21}$$

where ${}_2F_1(\alpha,\beta;\gamma;y)$ denotes the hypergeometric function. To examine the behavior of the solution at $y\to 1$ (the acceleration horizon), we utilize the linear transformation formulas for the hypergeometric functions [36]. If the quantity $\gamma-\alpha-\beta=-\mathrm{i}\tilde{\omega}$ is not an integer, then the relation between two hypergeometric functions with variables y and $1-y$ is

$$\begin{aligned}\phi =& \frac{\Gamma(\gamma)\Gamma(\gamma-\alpha-\beta)}{\Gamma(\gamma-\alpha)\Gamma(\gamma-\beta)}y^{\frac{\ell+1}{2}}(1-y)^{\frac{\mathrm{i}\tilde{\omega}}{2}}\\ &\times {}_2F_1(\alpha,\beta;\alpha+\beta-\gamma+1;1-y)\\ &+\frac{\Gamma(\gamma)\Gamma(\alpha+\beta-\gamma)}{\Gamma(\alpha)\Gamma(\beta)}y^{\frac{\ell+1}{2}}(1-y)^{-\frac{\mathrm{i}\tilde{\omega}}{2}}\\ &\times {}_2F_1(\gamma-\alpha,\gamma-\beta;\gamma-\alpha-\beta+1;1-y),\end{aligned} \tag{22}$$

where $\Gamma(y)$ denotes the Gamma function. We can verify that the first term on the right hand side of Eq. (22) represents an ingoing wave, while the second term represents an outgoing wave. To satisfy the boundary conditions of QNMs, the first term must be discarded. This is achieved by imposing[1)]

$$\gamma-\alpha=-n, \quad \text{or} \quad \gamma-\beta=-n, \quad n=0,1,2,\cdots, \tag{23}$$

such that $\Gamma(\gamma-\alpha)$ or $\Gamma(\gamma-\beta)$ becomes infinity. However, from Eq. (20), if $\gamma-\alpha-\beta=-\mathrm{i}\tilde{\omega}$ is not an integer, then neither $\gamma-\alpha=(\ell+2-\mathrm{i}\tilde{\omega})/2$ nor $\gamma-\beta=(\ell+1-\mathrm{i}\tilde{\omega})/2$ can be an integer. Given that $(\ell+2)/2$ and $(\ell+1)/2$ are integers or half-integers, and $-\mathrm{i}\tilde{\omega}/2$ is neither an integer nor a half-integer, their sum cannot be an integer. Hence the condition (23) cannot be satisfied if $\gamma-\alpha-b$ is not an integer.

Instead, we must assume $\gamma-\alpha-\beta$ is an integer. If $\gamma-\alpha-\beta=-n_1$, where $n_1=1,2,3,\ldots$ [2)], we can express the radial function (21) as

$$\begin{aligned}\phi =& y^{\frac{\ell+1}{2}}(1-y)^{\frac{\mathrm{i}\tilde{\omega}}{2}}\left\{\frac{\Gamma(\alpha+\beta-n_1)\Gamma(n_1)}{\Gamma(\alpha)\Gamma(\beta)}(1-y)^{-n_1}\right.\\ &\times\sum_{s=0}^{n_1-1}\frac{(\alpha-n_1)_s(\beta-n_1)_s}{s!(1-n_1)_s}(1-y)^s\\ &-\frac{(-1)^{n_1}\Gamma(\alpha+\beta-n_1)}{\Gamma(\alpha-n_1)\Gamma(\beta-n_1)}\\ &\times\sum_{s=0}^{\infty}\frac{(\alpha)_s(\beta)_s}{s!(n+s)!}(1-y)^s[\ln(1-y)-\psi(s+1)\\ &\left.-\psi(s+n+1)+\psi(\alpha+s)+\psi(\beta+s)]\right\},\end{aligned} \tag{24}$$

where $\psi(y)=\mathrm{d}\Gamma(y)/\mathrm{d}y$. Notably, the first term in the curly brackets represents an outgoing wave, while the second term represents an ingoing wave. Thus, to satisfy the boundary conditions of QNMs, we must impose the condition

$$\alpha-n_1=-n, \text{ or } \beta-n_1=-n, \quad n=0,1,2,\cdots, \tag{25}$$

to retain the outgoing wave. By combining this condition with $\gamma-\alpha-\beta=-n_1$, we can find the accelerating QN frequencies are equal to

$$\mathrm{i}\tilde{\omega}=\ell+1+2n, \quad \mathrm{i}\tilde{\omega}=\ell+2+2n, \tag{26}$$

which can also be expressed as

$$\mathrm{i}\tilde{\omega}=\ell+\tilde{n}, \quad \tilde{n}=1,2,3,\cdots. \tag{27}$$

The result shows that the previous assumption is self-consistent. We can verify that our results match those of the acceleration modes for a scalar field propagating in the spacetime of the charged [10] and spinning C-metrics [14] when taking the Minkowski limit. Therefore, we conclude that the acceleration modes found for accelerating BHs originate from the empty accelerating spacetime and become deformed when BHs are present in the spacetime.

B. Fuchsian equations

In this subsection, we derive the QNMs of the scalar field in the empty accelerating spacetime by using the established connection between HDEs and Fuchsian equations. A linear differential equation where every singular point, including the point at infinity, is a regular singular-

1) Note that repeated symbols in each subsection of this paper may represent different quantities.

2) We do not need to consider $\gamma-\alpha-\beta$ is equal to zero or a positive value, since the former means the scalar perturbations are always static and the latter makes the scalar perturbations unstable.

ity is called a Fuchsian equation or an equation of Fuchsian type. Particularly, a Fuchsian equation with three regular singular points reduces to the HDE. Appendix A provides a brief review of the basics of Fuchsian equations relevant to this study, referring to Refs. [33] and [34]. Understanding Fuchsian equations enables us to transform an equation with three arbitrary regular singular points into the standard form of the HDE in a systematic approach.

Initially, the radial Eq. (15) can be expressed in the form of Eq. (A1) as follows:

$$p(x)=\frac{2x}{x^2-1},\tag{28}$$

$$q(x)=\frac{\tilde{\omega}^2x^2+\ell(\ell+1)(x^2-1)}{(x^2-1)^2x^2}.\tag{29}$$

We can readily identify the three regular singular points, which are denoted by $a_1=0$, $a_2=1$, and $a_3=-1$. Using Eqs. (A2) and (A3), by expanding $p(x)$ and $q(x)$ around these singular points, we can determine the following coefficients:

$$A_1=0,\quad B_1=-\ell(\ell+1),\quad C_1=0,\tag{30}$$

$$A_2=1,\quad B_2=\frac{\tilde{\omega}^2}{4},\quad C_2=\frac{-\tilde{\omega}^2+2\ell(\ell+1)}{4},\tag{31}$$

$$A_3=1,\quad B_3=\frac{\tilde{\omega}^2}{4},\quad C_3=\frac{\tilde{\omega}^2-2\ell(\ell+1)}{4}.\tag{32}$$

Given that $x=\infty$ is an ordinary point, the parameters A_r, B_r, and C_r, with $r=1,2,3$, should satisfy the constraint (A8). We can verify from these expressions that the solution satisfies the required boundary conditions.

Subsequently, from the indicial Eq. (A5), we can determine the characteristic exponents of each regular singular point, given by

$$\begin{aligned}&\alpha_1=\ell+1,\quad \alpha_2=-\ell,\\&\beta_1=\frac{\mathrm{i}\tilde{\omega}}{2},\quad \beta_2=-\frac{\mathrm{i}\tilde{\omega}}{2},\\&\gamma_1=\frac{\mathrm{i}\tilde{\omega}}{2},\quad \gamma_2=-\frac{\mathrm{i}\tilde{\omega}}{2}.\end{aligned}\tag{33}$$

One can verify that these results satisfy the constraint given by Eq. (A10). This is achieved through the transformation described in Eq. (A11), that is

$$y=\frac{2x}{x+1},\quad \phi=\left(\frac{x}{x+1}\right)^{\ell+1}\left(\frac{x-1}{x+1}\right)^{\frac{\mathrm{i}\tilde{\omega}}{2}}g(y),\tag{34}$$

where the radial Eq. (15) can be transoformed into the standard form of HDE, Eq. (A12), with the parameters given by Eq. (A13)

$$\alpha=\ell+1+\mathrm{i}\tilde{\omega},\quad \beta=\ell+1,\quad \gamma=2\ell+2.\tag{35}$$

As discussed in the previous subsection, one of the solutions of Eq. (15) that is regular at $r=0$ is given by

$$\phi=\left(\frac{y}{2}\right)^{\ell+1}(y-1)^{\frac{\mathrm{i}\tilde{\omega}}{2}}{}_2F_1(\alpha,\beta;\gamma;y).\tag{36}$$

As previously described, we need to change the variable of hypergeometric function from y to $1-y$ to analyze the behavior of the solution at the acceleration horizon. If the quantity $\gamma-\alpha-\beta=-\mathrm{i}\tilde{\omega}$ is not an integer, then we have

$$\begin{aligned}\phi=&\frac{\Gamma(\gamma)\Gamma(\gamma-\alpha-\beta)}{\Gamma(\gamma-a)\Gamma(\gamma-\beta)}\left(\frac{y}{2}\right)^{\ell+1}(y-1)^{\frac{\mathrm{i}\tilde{\omega}}{2}}\\&\times{}_2F_1(\alpha,\beta;\alpha+\beta-\gamma+1;1-y)\\&+\frac{\Gamma(\gamma)\Gamma(\alpha+\beta-c)}{\Gamma(\alpha)\Gamma(\beta)}\left(\frac{y}{2}\right)^{\ell+1}(1-y)^{-\frac{\mathrm{i}\tilde{\omega}}{2}}(-1)^{\frac{\mathrm{i}\tilde{\omega}}{2}}\\&\times{}_2F_1(\gamma-\alpha,\gamma-\beta;\gamma-\alpha-\beta+1;1-y).\end{aligned}\tag{37}$$

The second term on the right hand side of Eq. (37) represents an outgoing wave, which satisfies the boundary condition at the acceleration horizon. To achieve this, we impose

$$\gamma-\alpha=-n,\quad\text{or}\quad\gamma-\beta=-n,\quad n=0,1,2,\cdots.\tag{38}$$

One can verify that this condition cannot be satisfied, indicating that we should instead assume that $\gamma-\alpha-\beta$ is an integer. If $\gamma-\alpha-\beta=-n_1$, where $n_1=1,2,3,...$, the solution can be expressed as

$$\begin{aligned}\phi=&\left(\frac{y}{2}\right)^{\ell+1}(y-1)^{\frac{\mathrm{i}\tilde{\omega}}{2}}\left\{\frac{\Gamma(\alpha+\beta-n_1)\Gamma(n_1)}{\Gamma(\alpha)\Gamma(\beta)}(1-y)^{-n_1}\right.\\&\times\sum_{s=0}^{n_1-1}\frac{(\alpha-n_1)_s(\beta-n_1)_s}{s!(1-n_1)_s}(1-y)^s-\frac{(-1)^{n_1}\Gamma(\alpha+\beta-n_1)}{\Gamma(\alpha-n_1)\Gamma(\beta-n_1)}\\&\times\sum_{s=0}^{\infty}\frac{(\alpha)_s(\beta)_s}{s!(n+s)!}(1-y)^s[\ln(1-y)-\psi(s+1)\\&\left.-\psi(s+n+1)+\psi(\alpha+s)+\psi(\beta+s)]\right\}.\end{aligned}\tag{39}$$

Notably, the first term in the curly brackets represents an outgoing wave, while the second term represents an ingoing wave. Thus, to satisfy the boundary conditions of QNMs, we must impose

$$\alpha - n_1 = -n, \text{ or } \beta - n_1 = -n, \quad n = 0, 1, 2, \cdots. \tag{40}$$

Combining this condition with $\gamma - \alpha - \beta = -n_1$, the accelerating QN frequencies are equal to

$$\mathrm{i}\tilde{\omega} = \ell + n + 1, \tag{41}$$

which is equivalent to Eq. (27). Thus, we derive consistent results from two distinct methods, although the radial Eqs. (21) and (36) differ in these cases.

III. MASSLESS PERTURBATIONS OF ANY SPIN

In this section, we calculate the QNMs of massless perturbations of any spin in the accelerating spacetime. The master equation describing massless perturbations of the spinning C-metric due to fields of any spin has been derived in [24] by following the approach of Teukolsky [37] within the context of the Newman-Penrose formalism [38]. Notably, similar to the Teukolsky equation for the Kerr BH, the master equation can be separated into its radial and angular parts because the spinning C-metric is of Petrov type D. By taking the Minkowski limit, we can obtain the equations capturing the dynamics of scalar, Dirac, electromagnetic, and gravitational perturbations in the empty accelerating spacetime.

The metric of spinning C-metric expressed in Boyer-Lindquist-type coordinates (t, r, θ, ϕ) is given by [39]

$$\begin{aligned}\mathrm{d}s^2 = \frac{1}{\Omega^2}\Big\{ &-\frac{1}{\Sigma}\left(Q - a^2 P\sin^2\theta\right)\mathrm{d}t^2 \\ &+ \frac{2a\sin^2\theta}{\Sigma}\left[Q - P\left(r^2+a^2\right)\right]\mathrm{d}t\mathrm{d}\phi \\ &+ \frac{\sin^2\theta}{\Sigma}\left[P\left(r^2+a^2\right)^2 - a^2 Q\sin^2\theta\right]\mathrm{d}\phi^2 \\ &+ \frac{\Sigma}{Q}\mathrm{d}r^2 + \frac{\Sigma}{P}\mathrm{d}\theta^2\Big\},\end{aligned} \tag{42}$$

where a is the rotation parameter, and the functions Ω, Σ, P, and Q are defined by

$$\begin{aligned}&\Omega = 1 - \mathcal{A}r\cos\theta, \quad \Sigma = r^2 + a^2\cos^2\theta, \\ &P = 1 - 2\mathcal{A}M\cos\theta + a^2\mathcal{A}^2\cos^2\theta, \\ &Q = \Delta\left(1 - \mathcal{A}^2 r^2\right), \quad \Delta = r^2 - 2Mr + a^2.\end{aligned} \tag{43}$$

The master equation describing the dynamics of a massless field with spin weight s in this spacetime is given by

$$\begin{aligned}&\left[(\nabla^\mu - s\Gamma^\mu)\left(\nabla_\mu - s\Gamma_\mu\right) + 4s^2\Psi_2\right]\psi = 0, \\ &s = 0, \pm 1/2, \pm 1, \pm 3/2, \pm 2,\end{aligned} \tag{44}$$

where $\Psi_2 = -(1 + \mathrm{i}a\mathcal{A})M\Omega^3/(r - \mathrm{i}a\cos\theta)^3$ is the nonvanishing Weyl scalar in the spinning C-metric background, and a "connection vector" is introduced by

$$\begin{aligned}\Gamma^t =& \frac{\Omega^2}{\Sigma}\Big\{\frac{1}{Q^2}[M(\mathcal{A}^2 r^4 + a^2) + r(1 + a^2\mathcal{A}^2)(\Delta - Mr)] \\ &+ \mathrm{i}\frac{a}{P}[(1 + a^2\mathcal{A}^2)\cos\theta - \mathcal{A}M(1 + \cos^2\theta)]\Big\}, \\ \Gamma^r =& -\frac{\Omega}{\Sigma}\left(\frac{1}{2}\Omega\partial_r Q + 2\mathcal{A}\cos\theta Q\right), \\ \Gamma^\theta =& \frac{2\mathcal{A}\Omega P r\sin\theta}{\Sigma}, \\ \Gamma^\phi =& -\frac{\Omega^2}{\Sigma}\left[\frac{a\partial_r Q}{2Q} + \mathrm{i}\frac{\cos\theta(2P-1)}{P\sin^2\theta}\right. \\ &\left. + \mathrm{i}\frac{\mathcal{A}M(\cos^2\theta - \mathcal{A}^2 a^2\cos\theta + 1)}{P\sin^2\theta}\right].\end{aligned} \tag{45}$$

Ref. [24] demonstrated that the master Eq. (44) admits separable solutions of the form

$$\psi(t, r, \theta, \phi) = \sum_{lm}\Omega^{(1+2s)}\mathrm{e}^{-\mathrm{i}\omega_{lm}t}\mathrm{e}^{\mathrm{i}m\phi}R_{lm}(r)S_{lm}(\theta), \tag{46}$$

where ω_{lm} is the wave frequency, and m is the azimuthal number. Additionally, we omit the subscript lm for simplicity in the following discussions. The radial equation is then expressed as

$$Q^{-s}\frac{\mathrm{d}}{\mathrm{d}r}\left(Q^{s+1}\frac{\mathrm{d}R(r)}{\mathrm{d}r}\right) + V_{(\mathrm{rad})}R(r) = 0, \tag{47}$$

with

$$\begin{aligned}V_{(\mathrm{rad})} =& -2r\mathcal{A}^2(r - M)(1+s)(1+2s) \\ &+ \frac{((r^2+a^2)\omega - am)^2}{Q} - 2\mathrm{i}s\left[-\frac{am\partial_r Q}{2Q}\right. \\ &\left. + \frac{\omega M(r^2 - a^2)}{\Delta} - \frac{\omega r\sigma_0}{1 - \mathcal{A}^2 r^2}\right] + 2K,\end{aligned} \tag{48}$$

where K is the separation constant, and $\sigma_0 = (1 + a^2\mathcal{A}^2)$. By introducing

$$H(r) = (r^2 + a^2)^{1/2}Q^{s/2}, \tag{49}$$

and the "tortoise" coordinate r_*, where

$$\frac{\mathrm{d}r}{\mathrm{d}r_*} = \frac{Q}{r^2 + a^2}, \tag{50}$$

the radial equation can be transformed into the one-dimensional Schrödinger-like equation

$$\frac{d^2}{dr_*^2}H(r)+\tilde{V}H(r)=0, \tag{51}$$

with the potential

$$\begin{aligned}\tilde{V}=&\left[\frac{\left(r^2+a^2\right)\omega-am}{r^2+a^2}-\mathrm{i}G\right]^2-\frac{dG}{dr_*}\\&-\frac{2Q}{(r^2+a^2)^2}\left[r\mathcal{A}^2(r-M)(1+s)(1+2s)\right.\\&\left.-K-2\mathrm{i}\omega rs-\frac{\mathrm{i}r(\left(r^2+a^2\right)\omega-am)}{(r^2+a^2)}\right],\end{aligned} \tag{52}$$

where

$$G=\frac{s\left[(r-M)\left(1-r^2\mathcal{A}^2\right)-r\mathcal{A}^2\Delta\right]}{(r^2+a^2)}+\frac{rQ}{(r^2+a^2)^2}. \tag{53}$$

Moreover, the angular equation is given by

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{dY(\theta)}{d\theta}\right)+V^R_{(\mathrm{ang})}(\theta)Y(\theta)=0, \tag{54}$$

with

$$\begin{aligned}V_{(\mathrm{ang})}(\theta)=&\frac{1-2K+s(2-\sigma_0)}{P}\\&+\frac{1}{P^2}\left\{-\frac{(w\cos\theta-\sigma_0 s)^2}{\sin^2\theta}\right.\\&-(z+w-4s\mathcal{A}M)^2+(z\cos\theta-s\sigma_0)^2\\&-(\mathcal{A}M\cos\theta-1)^2+1-\sigma_0+\mathcal{A}^2M^2\\&\left.+4s(\sigma_0-1)\cos\theta(2s\mathcal{A}M-w)\right\},\end{aligned} \tag{55}$$

where $Y(\theta)=\sqrt{P}S(\theta)$, $z=a\omega+sM\mathcal{A}$, and $w=-m+2sM\mathcal{A}$.

Taking the Minkowski limit, $M=a=0$, the angular equation reduces to

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{dY(\theta)}{d\theta}\right)+\left[s-2K-\frac{(s\cos\theta+m)^2}{\sin^2\theta}\right]Y(\theta)=0. \tag{56}$$

Comparing this equation to that of the spin-weighted spherical harmonics ${}_sY_{\ell m}$ [40]

$$\begin{aligned}&\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}{}_sY_{\ell m}\right)\\&+\left[\ell(\ell+1)-s^2-\frac{(m+s\cos\theta)^2}{\sin^2\theta}\right]{}_sY_{\ell m}=0,\end{aligned} \tag{57}$$

we can immediately find that

$$2K=s+s^2-\ell(\ell+1). \tag{58}$$

Thus, the separation constant of the angular equation in the Minkowski limit is known, and the QN frequencies can be obtained solely by considering the radial equation. In this limit, the radial equation (51) simplifies to

$$\begin{aligned}&\left(1-\mathcal{A}^2r^2\right)^2\frac{d^2H(r)}{dr^2}-\left(1-\mathcal{A}^2r^2\right)2r\mathcal{A}^2\frac{dH(r)}{dr}\\&+\left(\omega^2+\frac{2\mathrm{i}s\omega}{r}-\frac{\ell(\ell+1)}{r^2}+\ell(\ell+1)\mathcal{A}^2-s^2\mathcal{A}^2\right)H(r)=0,\end{aligned} \tag{59}$$

which in terms of the dimensionless quantities (15) can be expressed in the form of Eq. (A1)

$$\begin{aligned}&\frac{d^2H}{dx^2}+\frac{2x}{x^2-1}\frac{dH}{dx}+\left(-\frac{\ell(\ell+1)}{(x^2-1)^2x^2}\right.\\&\left.+\frac{2\mathrm{i}s\tilde{\omega}}{x(x^2-1)^2}+\frac{\tilde{\omega}^2-s^2+\ell(\ell+1)}{(x^2-1)^2}\right)H=0.\end{aligned} \tag{60}$$

A. Fuchsian equation

In the following analysis, we show that Eq. (60) is a Fuchsian equation with three regular singular points 0, 1, and −1, which can be readily transformed into the standard form of the HDE. From Eq. (A1), we know

$$p(x)=\frac{2x}{x^2-1}, \tag{61}$$

$$\begin{aligned}q(x)=&-\frac{\ell(\ell+1)}{(x^2-1)^2x^2}+\frac{2\mathrm{i}s\tilde{\omega}}{x(x^2-1)^2}\\&+\frac{\tilde{\omega}^2-s^2+\ell(\ell+1)}{(x^2-1)^2}.\end{aligned} \tag{62}$$

We can verify that the three points 0, 1, and −1 are the regular singular points of the equation, and the infinity is an ordinary point. Furthermore, we can expand $p(x)$ and $q(x)$ around each regular singular point as Eq. (A2) and Eq. (A3), respectively, with the expansion coefficients given by

$$A_1=0,\quad B_1=-\ell(\ell+1),\quad C_1=2\mathrm{i}s\tilde{\omega}, \tag{63}$$

$$\begin{aligned}&A_2=1,\\&B_2=-\frac{1}{4}(s-\mathrm{i}\tilde{\omega})^2,\\&C_2=\frac{1}{4}\left(s^2-\tilde{\omega}^2-4\mathrm{i}s\tilde{\omega}+2\ell(\ell+1)\right),\end{aligned} \tag{64}$$

and

$$\begin{aligned}
A_3 &= 1,\\
B_3 &= -\frac{1}{4}(s+\mathrm{i}\tilde{\omega})^2,\\
C_3 &= \frac{1}{4}\left(-s^2+\tilde{\omega}^2-4\mathrm{i}s\tilde{\omega}-2\ell(\ell+1)\right).
\end{aligned} \tag{65}$$

The three regular singular points are labeled as $a_1 = 0$, $a_2 = 1$, and $a_3 = -1$. Subsequently, from the indicial Eq. (A5), we can determine the characteristic exponents of each regular singular point, which are given by

$$\begin{aligned}
&\alpha_1 = \ell+1, \quad \alpha_2 = -\ell,\\
&\beta_1 = \frac{1}{2}(-s+\mathrm{i}\tilde{\omega}), \quad \beta_2 = \frac{1}{2}(s-\mathrm{i}\tilde{\omega}),\\
&\gamma_1 = \frac{1}{2}(s+\mathrm{i}\tilde{\omega}), \quad \gamma_2 = \frac{1}{2}(-s-\mathrm{i}\tilde{\omega}).
\end{aligned} \tag{66}$$

Applying the transformation given in Eq. (A11), which is expressed as

$$\begin{aligned}
y &= \frac{2x}{x+1},\\
H &= \left(\frac{x}{x+1}\right)^{\ell+1}\left(\frac{x-1}{x+1}\right)^{\frac{1}{2}(-s+\mathrm{i}\tilde{\omega})} g(y),
\end{aligned} \tag{67}$$

the radial Eq. (60) can be transformed into the standard form of HDE Eq. (A12), with the parameters given by Eq. (A13):

$$\begin{aligned}
\alpha &= \ell+1+\mathrm{i}\tilde{\omega},\\
\beta &= \ell+1-s,\\
\gamma &= 2\ell+2.
\end{aligned} \tag{68}$$

The subsequent analysis is parallel to the scalar field case; therefore, we present only the main steps. Imposing regularity at the origin of the accelerating spacetime selects the hypergeometric function as the appropriate solution to the HDE. To examine the behavior of this solution at the acceleration horizon, namely, $y = 1$, we need to change the variable in the hypergeometric function from y to $1-y$. Depending on the values of the parameter $\gamma-\alpha-\beta$, the resulting expression can have different forms, corresponding to either Eq. (37) or (39). To satisfy the boundary condition that the field is purely outgoing near the acceleration horizon, condition (25) must also be imposed, leading to

$$\mathrm{i}\tilde{\omega} = n+\ell+1. \tag{69}$$

We obtain the same QN frequency as that in the previous section. Notably, the spectra are purely imaginary and independent of the spin of the perturbations. Moreover, the result for $s = -2$ matches that of the acceleration modes of gravitational perturbations in the spinning C-metric [17] after taking the Minkowski limit.

B. Direct conversion

Similar to the analysis of the scalar field in the previous section, the radial equation (60) can be converted into the standard form of the HDE through an appropriate transformation; however, this process is more complicated than the case of the scalar field. A direct approach is to make the change of variable $y = x^2$, which leads to

$$4y(y-1)\frac{\mathrm{d}^2H}{\mathrm{d}y^2}+(6y-2)\frac{\mathrm{d}H}{\mathrm{d}y}+\left(-\frac{\ell(\ell+1)}{y}+\frac{2\mathrm{i}s\tilde{\omega}}{\sqrt{y}\cdot(y-1)}+\frac{\tilde{\omega}^2-s^2+\ell(\ell+1)}{y-1}\right)H = 0. \tag{70}$$

In the case of the scalar fields $s = 0$, via the change of variable $y = x^2$, the regular singular points of the equation change from $(0,1,-1)$ to $(0,1,\infty)$. The equation can then be converted into the form of the HDE. However, for $s \neq 0$, due to the appearance of the $\sqrt{y}$ term in the equation, the point $y = 0$ is no longer a regular singular point, further complicating the situation. Nevertheless, the similar issue was encountered in dS spacetime and a coordinate transformation was proposed [41]:

$$z = \frac{1-x}{1+x}, \tag{71}$$

which may be useful in this study. Applying this change of variable, the radial equation (60) becomes

$$\begin{aligned}
&\frac{\mathrm{d}^2H(z)}{\mathrm{d}z^2}+\frac{1}{z}\frac{\mathrm{d}H(z)}{\mathrm{d}z}\\
&-\left(\frac{\left(s^2-\tilde{\omega}^2\right)(z-1)^2+2\mathrm{i}s\tilde{\omega}\left(z^2-1\right)+4z\ell(\ell+1)}{4z^2(z-1)^2}\right)\\
&\times H(z) = 0.
\end{aligned} \tag{72}$$

Notably, the regular singular points for this equations are $(0,1,-1)$, making it natural to transform the equation into the standard form of the HDE. However, with the new coordinate z, the positions of the origin of the accelerating spacetime and the acceleration horizon are interchanged, which complicates subsequent analysis. To address this issue, we introduce

$$y = 1-z, \tag{73}$$

such that the origin remains at $y=0$ and the acceleration is at $y=1$. Thus, Eq. (73) becomes

$$\frac{d^2H}{dy^2}+\frac{1}{y-1}\frac{dH}{dy} -\left(\frac{\left(s^2-\tilde{\omega}^2\right)y^2+2is\tilde{\omega}y(y-2)+4(1-y)\ell(\ell+1)}{4y^2(y-1)^2}\right) \times H=0. \tag{74}$$

Consider the ansatz for $H(y)$,

$$H(y)=y^A(1-y)^B\tilde{H}(y), \tag{75}$$

where

$$A=\begin{cases}\ell+1,\\ -\ell,\end{cases}\qquad B=\begin{cases}\frac{1}{2}(-s+i\tilde{\omega}),\\ \frac{1}{2}(s-i\tilde{\omega}),\end{cases} \tag{76}$$

we get that the function $\tilde{H}(y)$ must be a solution of the HDE

$$y(1-y)\frac{d^2\tilde{H}}{dy^2}+\left[\gamma-(\alpha+\beta+1)y\right]\frac{d\tilde{H}}{dy}-\alpha\beta\tilde{H}=0, \tag{77}$$

with the parameters given by

$$\begin{aligned}\alpha&=A+B+\frac{s+i\tilde{\omega}}{2},\\ \beta&=A+B-\frac{s+i\tilde{\omega}}{2},\\ \gamma&=2A.\end{aligned} \tag{78}$$

There are four equivalent choices for A and B, and the one with $A=\ell+1$ and $B=\frac{1}{2}(-s+i\tilde{\omega})$ yields exactly the same parameters as those of the HDE in Eq. (68). Given that the two HDEs are identical, their QNM spectra are also expected to be identical.

IV. DE SITTER MODES

In this section, we apply the transformation techniques of HDEs and Fuchsian equations with three regular singular points to calculate the dS modes, demonstrating the validity and convenience of this method. We consider scalar, electromagnetic, and gravitational perturbations in D dimensional dS spacetime. As shown below, our results are identical with those presented in [27].

A. Scalar field

For simplicity, we consider only a massless scalar field minimally coupled to gravity living in the pure dS spacetime. The QN frequencies are represented in the radial part of the master equation [42], which is given by

$$x(1-x)\frac{d^2R}{dx^2}-\frac{1}{2}[(D+1)x-(D-1)]\frac{dR}{dx} +\frac{1}{4}\left(\frac{\tilde{\omega}^2}{1-x}-\frac{\ell(\ell+D-3)}{x}\right)R=0. \tag{79}$$

In this and in the subsequent subsections, $x=r^2/L$ and $\tilde{\omega}=\omega L$, where L is the radius of the dS space and D is the spacetime dimension. We can observe that Eq. (79) has three regular singular points, which are labeled by $a_1=0$, $a_2=1$, and ∞. By rewriting the radian equation in the form of Eq. (A1), we can expand $p(x)$ and $q(x)$ around a_1 and a_2 as Eq. (A2) and Eq. (A3), with the expansion coefficients given by

$$\begin{aligned}A_1&=\frac{D-1}{2},\\ B_1&=-\frac{\ell(D+\ell-3)}{4},\\ C_1&=\frac{-(D-3)\ell+\tilde{\omega}^2-\ell^2}{4},\end{aligned} \tag{80}$$

and

$$\begin{aligned}A_2&=1,\\ B_2&=\frac{\tilde{\omega}^2}{4},\\ C_2&=\frac{(D-3)\ell-\tilde{\omega}^2+\ell^2}{4}.\end{aligned} \tag{81}$$

From Eqs. (A5) and (A6), which are identical, we determine the characteristic exponents at each regular singular point, which are given by

$$\begin{aligned}&\alpha_1=\frac{\ell}{2},\quad \alpha_2=\frac{1}{2}(3-D-\ell),\\ &\beta_1=\frac{i\tilde{\omega}}{2},\quad \beta_2=-\frac{i\tilde{\omega}}{2},\\ &\gamma_1=0,\quad \gamma_2=\frac{D-1}{2}.\end{aligned} \tag{82}$$

Moreover, through the transformation, Eq. (A11), the radian equation can be converted into the standard HDE with the parameters given by Eq. (A13). Comparison with the scalar field analysis in dS spacetime [27] shows that both the transformations connecting the radial equation to the HDE and the resulting HDEs are identical (see Eqs. (46)–(48) of [27]). This clearly implies that the QNM spectra in both cases are identical.

B. Electromagnetic field

For electromagnetic fields moving in the D dimensional dS spacetime, the separation of the equations of motion has been studied in [43]. Depending on the choices of gauge for the electromagnetic field, two sets of physical solutions to the field equations arise, referred to as physical modes I and II. Correspondingly, there are two sets of radial equations and QNM spectra [27].

The radial equation of physical mode I is given by

$$\frac{\mathrm{d}^2R^{(I)}}{\mathrm{d}x^2}+\left[\frac{D+1}{2x}-\frac{1}{1-x}\right]\frac{\mathrm{d}R^{(I)}}{\mathrm{d}x}+\frac{1}{4}\left[\frac{\tilde{\omega}^2}{x(1-x)^2}-\frac{(\ell-1)(\ell+D-2)}{x^2(1-x)}-\frac{3(D-2)}{x(1-x)}\right]\times R^{(I)}=0. \tag{83}$$

Notably, this equation also has three regular singular points $a_1=0$, $a_2=1$, and ∞. Consistent with the previous approach, we can express this equation in the form of Eq. (A1) and expand $p(x)$ and $q(x)$ around a_1 and a_2 as Eqs. (A2) and (A3), with the expansion coefficients given by

$$\begin{aligned}A_1&=\frac{D+1}{2},\\ B_1&=-\frac{1}{4}(\ell-1)(D+\ell-2),\\ C_1&=\frac{1}{4}\left(-D(\ell+2)+\tilde{\omega}^2-\ell^2+3\ell+4\right),\end{aligned} \tag{84}$$

and

$$\begin{aligned}A_2&=1,\\ B_2&=\frac{\tilde{\omega}^2}{4},\\ C_2&=\frac{1}{4}\left(D(\ell+2)-\tilde{\omega}^2+\ell^2-3\ell-4\right).\end{aligned} \tag{85}$$

From indentical Eqs. (A5) and (A6), the characteristic exponents at each regular singular point are given by

$$\begin{aligned}&\alpha_1=\frac{\ell-1}{2},\quad \alpha_2=\frac{1}{2}(-D-\ell+2),\\ &\beta_1=\frac{\mathrm{i}\tilde{\omega}}{2},\quad \beta_2=-\frac{\mathrm{i}\tilde{\omega}}{2},\\ &\gamma_1=\frac{D-2}{2},\quad \gamma_2=\frac{D-2}{2}.\end{aligned} \tag{86}$$

We compare these results with those in [27], demonstrating that both the transformations connecting the radial equation to the HDE and the resulting HDEs are identical (see Eqs. (7)–(10) of [27]). This implies that the resulting QNM spectra are also identical.

The radial equation of physical mode II is given by

$$\frac{\mathrm{d}^2R^{(II)}}{\mathrm{d}x^2}+\frac{D(1-x)+x-3}{2(1-x)x}\frac{\mathrm{d}R^{(II)}}{\mathrm{d}x}+\frac{1}{4x(1-x)}\left[\frac{\tilde{\omega}^2}{1-x}-\frac{(\ell+1)(D+\ell-4)}{x}\right]R^{(II)}=0. \tag{87}$$

Similar to physical mode I, this equation has three regular singular points $a_1=0$, $a_2=1$, and ∞. Consistent with the previous approach, we can express the equation in the form of Eq. (A1) and then expand $p(x)$ and $q(x)$ around a_1 and a_2 as Eqs. (A2) and (A3), with the expansion coefficients given by

$$\begin{aligned}A_1&=\frac{D-3}{2},\\ B_1&=-\frac{1}{4}(\ell+1)(D+\ell-4),\\ C_1&=\frac{1}{4}\left(-D(\ell+1)+\tilde{\omega}^2-\ell^2+3\ell+4\right),\end{aligned} \tag{88}$$

and

$$\begin{aligned}A_2&=1,\\ B_2&=\frac{\tilde{\omega}^2}{4},\\ C_2&=\frac{1}{4}\left(D(\ell+1)-\tilde{\omega}^2+\ell^2-3\ell-4\right).\end{aligned} \tag{89}$$

From indicial Eqs. (A5) and (A6), the characteristic exponents at each regular singular point are given by

$$\begin{aligned}&\alpha_1=\frac{\ell+1}{2},\quad \alpha_2=\frac{1}{2}(-D-\ell+4),\\ &\beta_1=\frac{\mathrm{i}\tilde{\omega}}{2},\quad \beta_2=-\frac{\mathrm{i}\tilde{\omega}}{2},\\ &\gamma_1=0,\quad \gamma_2=\frac{D-3}{2}.\end{aligned} \tag{90}$$

Similar to the physical mode I, both the transformations connecting the radial equation to the HDE and the resulting HDEs are exactly the same as those in [27] (see Eqs. (22)–(24) therein). This implies that the resulting QNM spectra would be identical.

C. Gravitational perturbations

In dimensions higher than four, the gravitational perturbations of D dimensional dS spacetime are classified into three types based on their tensorial behavior on the $(D-2)$ sphere: tensor, vector, and scalar types [44]. By contrast, in four dimensions, gravitational perturbations consist of only the tensor type. The radial equations of these perturbations in this background can be uniformly expressed as [45]

$$\frac{\mathrm{d}^2R_G}{\mathrm{d}x^2}+\frac{1-3x}{2x(1-x)}\frac{\mathrm{d}R_G}{\mathrm{d}x}+\frac{1}{4x(1-x)^2}\left[\tilde{\omega}^2+\tilde{\alpha}(1-x)-\frac{\tilde{\beta}(\tilde{\beta}+1)(1-x)}{x}\right]\times R_G=0, \tag{91}$$

where $\tilde{\alpha}$ and $\tilde{\beta}$ are introduced by

$$\tilde{\alpha}=\begin{cases}\dfrac{(D-2)D}{4}, & \text{tensor type,}\\[2mm] \dfrac{(D-4)(D-2)}{4}, & \text{vector type,}\\[2mm] \dfrac{(D-6)(D-4)}{6}, & \text{scalar type,}\end{cases} \tag{92}$$

and

$$\tilde{\beta}=\frac{2\ell+D-4}{2}. \tag{93}$$

This equation has three regular singular points $a_1=0$, $a_2=1$, and ∞. Consistent with previous approaches, we can express the equation in the form of Eq. (A1) and then expand $p(x)$ and $q(x)$ around a_1 and a_2 as Eqs. (A2) and (A3), with the expansion coefficients given by

$$\begin{aligned}A_1&=\frac{1}{2},\\ B_1&=-\frac{1}{4}\tilde{\beta}(\tilde{\beta}+1),\\ C_1&=\frac{1}{4}\left(\tilde{\alpha}-\tilde{\beta}^2-\tilde{\beta}+\omega^2\right),\end{aligned} \tag{94}$$

and

$$\begin{aligned}A_2&=1,\\ B_2&=\frac{\tilde{\omega}^2}{4},\\ C_2&=-\frac{1}{4}\left(\tilde{\alpha}-\tilde{\beta}^2-\tilde{\beta}+\omega^2\right).\end{aligned} \tag{95}$$

From indicial Eqs. (A5) and (A6), the characteristic exponents at the three regular singular points are given by

$$\begin{aligned}&\alpha_1=\frac{1+\tilde{\beta}}{2},\quad \alpha_2=-\frac{\tilde{\beta}}{2},\\ &\beta_1=\frac{\mathrm{i}\tilde{\omega}}{2},\quad \beta_2=-\frac{\mathrm{i}\tilde{\omega}}{2},\\ &\gamma_1=\frac{1}{4}\left(1+\sqrt{4\tilde{\alpha}+1}\right),\quad \gamma_2=\frac{1}{4}\left(1-\sqrt{4\tilde{\alpha}+1}\right).\end{aligned} \tag{96}$$

Additionally, both the transformations connecting the radial equation to the HDE and the resulting HDEs are exactly the same as those in [27] (see Eqs. (34)–(36) therein). Therefore, the QNM spectra in these two cases would also be identical.

V. SUMMARY

This study investigated the QNMs of perturbations with spin $s\leq 2$ in purely accelerating spacetime to examine the origin of the acceleration modes recently discovered for scalar and gravitational perturbations of accelerating BHs [10, 14, 17]. The perturbation equations were obtained by taking the Minkowski limit of the master equations describing various types of perturbations of accelerating BHs. We employed two methods to obtain the QNM spectra by solving the radial part of the perturbation equations. The first method involves a change of variable to convert the radial equation into the form of the standard HDEs. This approach is effective for scalar perturbations, such as those describing dS mode in pure dS spacetime [27]. However, for other types of perturbations, the operation becomes more complicated. The second method utilizes the simple connection between Fuchsian equations with three regular singular points–of which the radial perturbation equations are an example–and HDEs. Both methods were shown to yield identical QNM spectra.

Notably, the resulting QNM spectra of the purely accelerating spacetime are imaginary and independent of the spin of the perturbations and match those of the acceleration modes of accelerating BHs after taking the Minkowski limit. This correspondence implies that the acceleration modes originate from the purely accelerating spacetime and become deformed in the presence of BHs. A similar situation arises in the case of dS modes [8, 30–32], which were first calculated in pure dS spacetime and then identified within the scalar QNMs of Schwarzschild-dS BHs [30]. Additionally, we applied the second method to compute the QNMs of scalar, electromagnetic, and gravitational perturbations in D dimensional dS spacetime and obtained results identical to those reported in [27], ultimately verifying the validity of this method.

Given that acceleration modes are closely linked to the acceleration parameter and exhibit robustness to BH parameters, investigating their observability may reveal signatures of acceleration of BHs within GW signals, potentially supporting the detection of moving or accelerating BHs.

ACKNOWLEDGMENTS

We gratefully acknowledge Wei Xiong for his valuable assistance during the initial stages of this study.

APPENDIX A: SOME BASICS OF FUCHSIAN EQUATIONS

Given a second-order linear ordinary differential equation

$$f''(x)+p(x)f'(x)+q(x)f(x)=0, \tag{A1}$$

where a_r $(r=1,2,\cdots,n)$ and ∞ are regular singular points. Moreover, given that a_r is a regular singular point, it must also be a first-order pole of $p(x)$. Therefore, we have

$$p(x)=\sum_{r=1}^{n}\frac{A_r}{x-a_r}+\varphi(x), \tag{A2}$$

where A_r is the residue of $p(x)$ at $x=a_r$, and $\varphi(x)$ is analytical in complex plane. When $x\to\infty$, $\varphi(x)$ approaches zero, as $x=\infty$ is the regular singular point of the equation. This implies that $\varphi(x)$ can be set to zero.

Based on similar analysis, $q(x)$ can be expanded as

$$q(x)=\sum_{r=1}^{n}\left\{\frac{B_r}{(x-a_r)^2}+\frac{C_r}{x-a_r}\right\}, \tag{A3}$$

with

$$\sum_{r=1}^{n}C_r=0. \tag{A4}$$

When considering the series solution of Eq. (A1), the coefficient of the lowest power of x satisfies the *indicial equation*. The roots of this equation determine the *characteristic exponents*, which in turn specify the form of the solution. For each regular singular point a_r $(a_r\neq\infty)$ of Eq. (A1), the corresponding two characteristic exponents satisfy the following indicial equation:

$$\rho^2+(A_r-1)\rho+B_r=0 \quad (r=1,2,\cdots,n). \tag{A5}$$

Moreover, if the point at infinity $x=\infty$ is a regular singular point, its characteristic exponents obey the equation

$$\rho^2+\left(1-\sum_{r=1}^{n}A_r\right)\rho+\sum_{r=1}^{n}(B_r+a_rC_r)=0. \tag{A6}$$

From these two equations, we can obtain a key constraint in this case:

$$\sum\text{roots of all indicial equations}=n-1. \tag{A7}$$

For example, for the HDE, the sum of all characteristic exponents is one, as $n=2$.

However, if $x=\infty$ is an ordinary point rather than a singular point of Eq. (A1), the parameters A_r, B_r and C_r satisfy the following equations:

$$\begin{aligned}\sum_{r=1}^{n}A_r&=2,\\ \sum_{r=1}^{n}C_r&=0,\\ \sum_{r=1}^{n}(B_r+a_rC_r)&=0,\\ \sum_{r=1}^{n}\left(2a_rB_r+a_r^2C_r\right)&=0.\end{aligned} \tag{A8}$$

For Fuchsian equations with three regular singular points, denoted by a, b, and c ($\neq\infty$, thus ∞ is an ordinary point), the equations can expressed in the following form based on the preceding relations:

$$\begin{aligned}&\frac{d^2f}{dx^2}+\left\{\frac{1-\alpha_1-\alpha_2}{x-a}+\frac{1-\beta_1-\beta_2}{x-b}+\frac{1-\gamma_1-\gamma_2}{x-c}\right\}\frac{df}{dx}\\&+\left\{\frac{\alpha_1\alpha_2(a-b)(a-c)}{x-a}+\frac{\beta_1\beta_2(b-c)(b-a)}{x-b}\right.\\&\left.+\frac{\gamma_1\gamma_2(c-a)(c-b)}{x-c}\right\}\times\frac{f}{(x-a)(x-b)(x-c)}=0,\end{aligned} \tag{A9}$$

where (α_1, α_2), (β_1, β_2), and (γ_1, γ_2) are the corresponding characteristic exponents of the regular singular points a, b, and c. From Eqs. (A5) and (A8), we find

$$\alpha_1+\alpha_2+\beta_1+\beta_2+\gamma_1+\gamma_2=1. \tag{A10}$$

Through the following transformation:

$$\begin{aligned}y&=\frac{(b-c)(x-a)}{(b-a)(x-c)},\\ f&=\left(\frac{x-a}{x-c}\right)^{\alpha_1}\left(\frac{x-b}{x-c}\right)^{\beta_1}g,\end{aligned} \tag{A11}$$

Eq. (A9) can be converted into the standard form of HDE,

$$y(1-y)\frac{d^2g}{dy^2}+\left[\gamma-(\alpha+\beta+1)y\right]\frac{dg}{dy}-\alpha\beta g=0, \tag{A12}$$

whose parameters are related to those of the Fuchsian equations, given by

$$\alpha=\gamma_1+\alpha_1+\beta_1,\quad \beta=\gamma_2+\alpha_1+\beta_1,\quad \gamma=1+\alpha_1-\alpha_2. \tag{A13}$$

The detailed derivation of these equations using Riemann's P-functions can be found in [33].

References

[1] B. P. Abbott *et al*., Phys. Rev. Lett. **116**, 061102 (2016)
[2] K. Akiyama *et al*. (Event Horizon Telescope Collaboration), Astrophys. J. Lett. **875**, L1 (2019)
[3] K. D. Kokkotas and B. G. Schmidt, Living Rev. Rel. **2**, 2 (1999)
[4] R. A. Konoplya and A. Zhidenko, Rev. Mod. Phys. **83**, 793 (2011)
[5] E. Berti, V. Cardoso, and A. O. Starinets, Class. Quant. Grav. **26**, 163001 (2009)
[6] J. D. Bekenstein, in *2nd International Sakharov Conference on Physics* (1996) pp. 216–219, arXiv: gr-qc/9605059 [gr-qc]
[7] O. Dreyer, B. J. Kelly, B. Krishnan *et al*., Class. Quant. Grav. **21**, 787 (2004)
[8] V. Cardoso, J. a. L. Costa, K. Destounis *et al*., Phys. Rev. Lett. **120**, 031103 (2018)
[9] M. Nozawa and T. Kobayashi, Phys. Rev. D **78**, 064006 (2008)
[10] K. Destounis, R. D. B. Fontana, and F. C. Mena, Phys. Rev. D **102**, 044005 (2020)
[11] R. D. B. Fontana and F. C. Mena, JHEP **10**, 047 (2022), arXiv: 2203.13933[gr-qc]
[12] K. Destounis, G. Mascher, and K. D. Kokkotas, Phys. Rev. D **105**, 124058 (2022)
[13] B. Gwak, Eur. Phys. J. Plus **138**, 582 (2023)
[14] W. Xiong and P. C. Li, Phys. Rev. D **108**, 044064 (2023), arXiv: 2305.04040[gr-qc]
[15] Y. Lei, H. Shu, K. Zhang *et al*., JHEP **02**, 140 (2024)
[16] J. Barragán Amado and B. Gwak, JHEP **2024**, 189 (2024)
[17] T. Chen, R. G. Cai, and B. Hu, Phys. Rev. D **109**, 084049 (2024)
[18] W. Kinnersley and M. Walker, Phys. Rev. D **2**, 1359 (1970)
[19] J. F. Plebanski and M. Demianski, Annals Phys. **98**, 98 (1976)
[20] J. B. Griffiths, P. Krtous, and J. Podolsky, Class. Quant. Grav. **23**, 6745 (2006)
[21] S. W. Hawking and S. F. Ross, Phys. Rev. Lett. **75**, 3382 (1995)
[22] D. Merritt, M. Milosavljevic, M. Favata *et al*., Astrophys. J. Lett. **607**, L9 (2004)
[23] S. W. Hawking and S. F. Ross, Phys. Rev. D **56**, 6403 (1997)
[24] D. Bini, C. Cherubini, and A. Geralico, J. Math. Phys. **49**, 062502 (2008)
[25] D. Kofroň, Phys. Rev. D **92**, 124064 (2015)
[26] V. P. Frolov, P. Krtouš, and D. Kubizňák, Phys. Rev. D **97**, 101701 (2018)
[27] A. Lopez-Ortega, Gen. Rel. Grav. **38**, 1565 (2006a)
[28] A. Lopez-Ortega, Gen. Rel. Grav. **38**, 743 (2006b)
[29] A. Lopez-Ortega, Gen. Rel. Grav. **44**, 2387 (2012)
[30] A. Jansen, Eur. Phys. J. Plus **132**, 546 (2017)
[31] K. Destounis, Phys. Lett. B **795**, 211 (2019)
[32] R. A. Konoplya and A. Zhidenko, Phys. Rev. D **106**, 124004 (2022)
[33] Z. X. Wang and D. R. Guo, *Special Functions*, (World Scientific, 1989)
[34] J. Gray, *Linear Differential Equations and Group Theory from Riemann to Poincare*, (Birkhäuser Boston, 2008)
[35] J. B. Griffiths and J. Podolský,, *Exact Space-Times in Einstein's General Relativity*, (Cambridge: Cambridge University Press, 2009)
[36] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions With Formulas, Graphs, and Mathematical Tables*, (United States: Government Printing Office, 1965)
[37] S. A. Teukolsky, Astrophys. J. **185**, 635 (1973)
[38] E. Newman and R. Penrose, J. Math. Phys. **3**, 566 (1962)
[39] J. B. Griffiths and J. Podolsky, Class. Quant. Grav. **22**, 3467 (2005)
[40] E. T. Newman and R. Penrose, J. Math. Phys. **7**, 863 (1966)
[41] H. Suzuki and E. Takasugi, Mod. Phys. Lett. A **11**, 431 (1996)
[42] E. Abdalla, K. H. C. Castello-Branco, and A. Lima-Santos, Phys. Rev. D **66**, 104018 (2002)
[43] L. C. B. Crispino, A. Higuchi, and G. E. A. Matsas, Phys. Rev. D **63**, 124008 (2001)
[44] H. Kodama and A. Ishibashi, Prog. Theor. Phys. **111**, 29 (2004)
[45] J. Natario and R. Schiappa, Adv. Theor. Math. Phys. **8**, 1001 (2004)