Weak magnetic effect in quark-gluon plasma and local spin polarization*

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Abstract: We propose the weak magnetic effect, which emerges in quark-gluon plasma close to local thermal equilibrium as the dissipative correction to the quark phase space distribution function, as a novel contribution to the observed Lambda hyperon local spin polarization. With a finite field strength, which is consistent with previous estimate of the magnetic field in heavy-ion collisions, one is able to explain the experimentally observed Lambda local spin polarization through all centrality classes. Moreover, the weak magnetic effect plays an unambiguous role in the ordering between the second-order and the third-order modulations of the Lambda local spin polarization in experiments.

Keywords: heavy-ion collisions, magnetic field, Lambda local spin polarization

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I. INTRODUCTION

The thermodynamic characteristics can be effectively inferred through experimental measurements once a system reaches local thermal equilibrium. As can be seen from the equilibrium density operator, $\lg\hat\rho \sim \alpha\hat N - \beta_\mu\hat p^\mu + \omega_{\mu\nu}\hat M^{\mu\nu}/2$ (cf. Ref. [1]), the constants of motion $(\hat N, \hat p^\mu, \hat M^{\mu\nu})$, which correspond usually to the generators of a gauge symmetry group and the Poincaré group, are conjugate to the thermodynamic variables: chemical potential over temperature ($\alpha = \mu/T$), fluid four-velocity over temperature ($\beta_\mu = u_\mu/T$), and the thermal vorticity tensor ($\omega_{\mu\nu} = \omega^{\rm th}_{\mu\nu} \equiv -\frac{1}{2}(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu)$). Given that these constants of motion are experimental observables, through the measurements of charge, momentum and angular momentum of particles in the thermal system, one is allowed to extract the information of chemincal potential, temperature, and so on.

Such measurements have been carried out extensively, in particular in high-energy heavy-ion experiments. For instance, the observed hadron spectra – momentum distribution – reveals the information of temperature, baryon chemical potential and flow velocity in the quark-gluon plasma (QGP) [2, 3]. Some more differential measurements, including the hadron collective flow and flow correlations, indicate that the space-time distribution of temperature in the QGP fireball is originated hydrodynamically from an almond-shape geometry [4−6].

Recently, the study has been advanced to the analysis of hadron spin polarization, with the purpose of getting access to the rotational nature of QGP [7−12]. Indeed, with the assumption that QGP achieves local thermal equilibrium, the spin-1/2 particles are expected polarized owing to thermal vorticity [13−17]. Following the Cooper-Frye formula [13, 18],

$$P^\mu(\boldsymbol{p}) = -\frac{1}{8m}\epsilon^{\mu\alpha\beta\sigma}p_\sigma\frac{\int d\Sigma\cdot p n_F(1-n_F)\omega_{\alpha\beta}}{\int d\Sigma\cdot p n_F}, \tag{1}$$

with n_F the Fermi-Dirac distribution, $\epsilon^{\mu\alpha\beta\sigma}$ the totally antisymmetric Levi-Civita symbol, and $\omega_{\mu\nu} = \omega^{\rm th}_{\mu\nu}$, the global spin polarization of Lambda hyperons emitted from a hyper-surface Σ can be successfully characterized [9−11, 19−22].

However, thermal vorticity fails to describe the differential spectrum of the Lambda spin polarization, which is often referred to as the local spin polarization (see Ref. [15] for a recent review). Especially, projected along the beam axis (z-axis), the thermal voticity yields an azimuthal distribution of the Lambda spin polarization that has an opposite sign comparing to experimental observations [23−28]. The discrepency has intrigured extensive explorations of the spin generation from QGP beyond thermal vorticity and the local thermal equilibrium condition [29−38].

In this Letter, with respect to the conversion of quark spin from QGP fluid, we introduce the dissipative effect induced by a weak external magnetic field. It has been

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noticed that, for the field strength $|eB| \ll m_\pi^2$, with m_π the pion mass, while the bulk evolution of QGP is barely affected, quarks can be driven slightly out of local thermal equilibrium [39, 40]. The out-of-equilibrium effect leads to correction in the quark distribution function, which for quark species with electric charge Q, is [40]

$$\delta f_{\rm EM}(x,\boldsymbol{p}) = n_F(1-n_F)\frac{\sigma_{\rm el}}{T\chi_{\rm ch}} Q p_\mu F^{\mu\nu}\beta_\nu\,, \tag{2}$$

where $\sigma_{\rm el}$ is the electrical conductivity, $\chi_{\rm ch}$ is an effective charge susceptibility, and $F^{\mu\nu}$ the electromagnetic field strength tensor. Eq. (2) is a scalar function solution with quark spin averaged. Correspondingly, when the weak magnetic field is taken into account, conversion of quark spin is modified, and the Cooper-Frye formula in Eq. (1) would become

$$P^\mu(\boldsymbol{p}) = -\frac{1}{8m}\epsilon^{\mu\alpha\beta\sigma}p_\sigma \frac{\int d\Sigma\cdot p\left[n_F(1-n_F)+(1-2n_F)\delta f_{\rm EM}\right]\omega_{\alpha\beta}}{\int d\Sigma\cdot p\,(n_F+\delta f_{\rm EM})}. \tag{3}$$

In realistic heavy-ion collisions, since the magnetic field is created and orientated out of the reaction plane, the $p_\mu F^{\mu\nu}u_\nu$ factor in Eq. (2) results in a dipole structure in the azimuthal angle distribution. Therefore, the extra $\delta f_{\rm EM}$ does not contribute to the globle spin polarization, in terms of local spin polarization, however, it is potentially significant. Eq. (3) is the major result of this Letter.

Throughout the Letter, we use the natural units. Our convention of matrix is $g^{\mu\nu}=(+,-,-,-)$, and for the Levi-Civita symbol we take $\epsilon^{0123}=1$.

II. DERIVATION OF EQ. (3)

In the presence of an electromagnetic field, in QGP medium the evolution of quarks with spin degrees of freedom satisfies the Boltzmann-Vlasov equation [41–43],

$$p^\mu\partial_\mu\mathscr{F} + QF^{\mu\nu}p_\mu\frac{\partial\mathscr{F}}{\partial p^\nu} = -\mathscr{C}[\mathscr{F}] = -(p\cdot u)\frac{\mathscr{F}-\mathscr{F}_{\rm eq}}{\tau_R}. \tag{4}$$

Note that the quark distribution function is a Hermian matrix in the spin subspace, $\mathscr{F}=\mathscr{F}^\dagger$. For QGP close to local thermal equilibrium, we are allowed to linearize the collision kernal via the relaxation time approximation. In principle, relaxation time τ_R is a function of $p\cdot u$, depending on microscopic dynamics. In this Letter, we take the so-called quadratic ansatz in Ref. [44]: $\tau_R = \bar\tau p\cdot u/T$, with $\bar\tau$ a free parameter to be determined by the transport coefficients[1]. Because quarks in QGP are dominated by strong and electromagnetic forces, the collision kernel, as well as the parameter $\bar\tau$, are expected even with respect to the charge conjugation symmetry. Accordingly, subject to a charge conjugation transformation, Eq. (4) applies to anti-quarks. In the following, we shall focus on the derivation of quarks, while results associated with anti-quarks can be inferred upon charge conjugate symmetry.

By meanings of the Chapman-Enskog method, with the knowledge of the equilibrium quark distribution function [13],

$$\mathscr{F}_{\rm eq} = \frac{1}{2m}\overline{U}(p)X(x,p)U(p)\,, \tag{5}$$

Eq. (4) can be solved order by order [40, 41]. In Eq. (5), $U(p)$ is the amalgamated spinor solution for free fermions with both spin up and spin down, $U(p)=(u_+(p),u_-(p))$, and $\overline{U}(p)=U^\dagger\gamma^0$. The matrix $X(x,p)$ is identified as $X(x,p)\equiv\left(e^{\beta\cdot p}\exp\left[-\frac{1}{2}\omega_{\mu\nu}\Sigma^{\mu\nu}\right]+1\right)^{-1}$, with $\Sigma^{\mu\nu}=\frac{i}{4}[\gamma^\mu,\gamma^\nu]$ the generator of relativistic rotation realized through the gamma matrix. At the leading order of $|eB|/T^2$, the solution can be found as, $\mathscr{F}=\mathscr{F}_{\rm eq}+\delta\mathscr{F}_{\rm EM}$, where the dissipative correction is

$$\delta\mathscr{F}_{\rm EM} = -\frac{\bar\tau}{T}QF^{\mu\nu}p_\mu\frac{\partial}{\partial p^\nu}\mathscr{F}_{\rm eq} = \frac{1}{2m}\overline{U}(p)Y(x,p)U(x,p)\,, \tag{6}$$

with

$$\begin{aligned}Y(x,p) &\equiv -\frac{\bar\tau}{T}QF^{\mu\nu}p_\mu\frac{\partial X}{\partial p^\nu}\\ &= \frac{\bar\tau}{T}QF^{\mu\nu}p_\mu\beta_\nu e^{\beta\cdot p}X^2\exp\left(-\frac{1}{2}\omega_{\alpha\beta}\Sigma^{\alpha\beta}\right).\end{aligned} \tag{7}$$

In deriving Eq. (6), we have used the Dirac equation $(\not p-m)U(p)=0$, which implies that $F^{\mu\nu}p_\mu\frac{\partial}{\partial p^\nu}U(p)=0$ and $F^{\mu\nu}p_\mu\frac{\partial}{\partial p^\nu}\overline{U}(p)=0$. Hermiticity of $\delta\mathscr{F}_{\rm eq}$ can be verified, with respect to the condition $Y^\dagger=\gamma^0Y\gamma^0$.

In accordance with the dissipative correction in the conserved current of hydrodynamics, the dissipative correction in the phase-space distribution function satisfies the Landau's matching condition. Especially, for a QGP medium with local charge neutrality condition, the leading order dissipative correction to the current is driven by the external electromagnetic field, as,

$$\Delta j^\mu = \sigma_{\rm el}E^\mu = \sum Q_f\int\frac{d^3\boldsymbol{p}}{p^0}p^\mu{\rm tr}_2[\delta\mathscr{F}_{\rm EM}-\delta\overline{\mathscr{F}}_{\rm EM}]\,, \tag{8}$$

where the trace, ${\rm tr}_2$, is taken in the spin subspace and the

1) We have numerically tested the effect of linear ansatz, for which the relaxation time is independent of momentum, $\tau_R=\bar\tau$. With respect to local spin polarization of the lambda hyperon, the results are not changed qualitatively.

summation takes into account of flavors and colors. To evaluate the trace, using the cyclicity in a trace and $U(p)\overline{U}(p)=\sum_{\rm spin}u(p)\bar{u}(p)=\not{p}+m$, and the property that trace of three gamma matrices vanishes, one finds for quarks with electric charge Q,

$$\begin{aligned}\mathrm{tr}_2\delta\mathscr{F}_{\rm EM} &= \frac{1}{2}\mathrm{tr}_2 Y = \frac{\bar\tau}{2T}QF^{\mu\nu}p_\mu\beta_\nu e^{\beta\cdot p}\mathrm{tr}_2\\ &\left[X^2\exp\left(-\frac{1}{2}\omega_{\alpha\beta}\Sigma^{\alpha\beta}\right)\right]\\ &=\frac{\bar\tau}{2T}QF^{\mu\nu}p_\mu\beta_\nu e^{\beta\cdot p}\sum_{m,n=1}(-1)^{m+n+2}e^{-(m+n)\beta\cdot p}\mathrm{tr}_2\\ &\left[\exp\left(\frac{m+n-1}{2}\omega_{\alpha\beta}\Sigma^{\alpha\beta}\right)\right]\\ &=-\frac{\bar\tau}{2T}QF^{\mu\nu}p_\mu\beta_\nu\left(4n'_F+\frac{\omega_{\alpha\beta}\omega^{\alpha\beta}}{2}n'''_F\right). \end{aligned}\tag{9}$$

In the above expression, prime over n_F indicates derivative with respect to $p\cdot u/T$, hence for instance, $n'_F=-n_F(1-n_F)$. A couple of comments are in order. First, we have used the approximated identity in the derivation [13]: $\mathrm{tr}_2\left[\exp\left(\frac{n}{2}\omega_{\alpha\beta}\Sigma^{\alpha\beta}\right)\right]\approx 4+\frac{n^2}{2}\omega^{\alpha\beta}\omega_{\alpha\beta}$. Secondly, substituting the trace back to the matching condition Eq. (8) yields an expression that relates $\bar\tau$ to the electrical conductivity $\sigma_{\rm el}$. In the limit $\omega\to 0$, one arrives at the standard relation $\sigma_{\rm el}=\bar\tau\chi_{\rm el}$ [40]. Accordingly, up to a factor of two that amounts to a summation over spin, one identifies the first term in Eq. (9) as the scalar function $\delta f_{\rm EM}$. In sum, as expected, in a non-rotating system the spin averaged quark distribution function reduces to $\frac{1}{2}\mathrm{tr}_2\mathscr{F}=n_F+\delta f_{\rm EM}$.

The Cooper-Frye formula in Eq. (1) is a variant of the the Pauli-Lubanski pseudo-vector,

$$\Pi^{\rm PL}_\mu=-\frac{1}{2m}\epsilon_{\mu\rho\sigma\tau}p^\tau S^{\rho\sigma},\tag{10}$$

where $S^{\rho\sigma}$ corresponds to the angular momentum operator. Spin polarization of particles emitted from QGP, in particular, can be described when $\Pi^{\rm PL}_\mu$ is evaluated on a hypersurfce that meats the freeze out condition in heavy-ion collisions, and as the averaged expectation of particles in phase space. As has been shown previously [13], because $S^{\mu\nu}$ receives contributions only from spin, which with respect to the quark distribution function in QGP out of local thermal equilibrium, can be solved from the spin density tensor [41],

$$\begin{aligned}s^{\lambda,\rho\sigma}(x)&=\frac{1}{2}\int\frac{d^3\boldsymbol{p}}{2p^0}\mathrm{tr}_2\left(\mathscr{F}\overline{U}(p)\{\gamma^\lambda,\Sigma^{\rho\sigma}\}U(p)\right)\\ &=\int\frac{d^3\boldsymbol{p}}{2p^0}\left[p^\lambda\Theta^{\rho\sigma}+p^\rho\Theta^{\sigma\lambda}+p^\sigma\Theta^{\lambda\rho}\right],\end{aligned}\tag{11}$$

where the rank-2 tensor function is,

$$\Theta^{\mu\nu}(x)\equiv\mathrm{tr}_2\left[(X+Y)\Sigma^{\mu\nu}\right]=\left[n_F(1-n_F)+(1-2n_F)\delta f_{\rm EM}\right]\omega^{\mu\nu}.\tag{12}$$

Accordingly, one finds average with respect to quark spectrum,

$$P_\mu=\langle\Pi^{\rm PL}_\mu\rangle=-\frac{1}{2m}\epsilon_{\mu\rho\sigma\tau}p^\tau\frac{1}{\mathrm{tr}_2\mathscr{F}}\frac{ds^{0,\rho\sigma}}{d^3\boldsymbol{p}}=-\frac{1}{4m}\epsilon_{\mu\rho\sigma\tau}p^\tau\frac{\Theta^{\rho\sigma}}{\mathrm{tr}_2\mathscr{F}}.\tag{13}$$

Following the Cooper-Frye prescription [45] and integrate over a hypersurface, Eqs. (12) and (13) lead to Eq. (3). Note that the derivation does not reply on the the explicit form of $\omega^{\mu\nu}$, hence Eq. (3) applies as well when other contributions are taken into account beyond the thermal vorticity tensor, including the shear-induced polarization.

III. LOCAL SPIN POLARIZATION IN HEAVY-ION COLLISIONS

In realistic heavy-ion collisions, in the collision region there must be an electromagentic field created pointing out of reaction plane, namely, $\mathbf{B}=B_y\hat{y}$. Although the field strength is expected strong initially, it drops drastically [46−53]. It is very likely that the magnetic field becomes weak when the QGP medium is formed and starts to expand hydrodynamically [52, 53]. Although the magnetic effect in QGP is too weak to influence the bulk medium evolution, it has been found significant to some of the particular signatures, including the anisotropy in the thermal photon spectrum [39, 40] and the thermal dilepton polarization [54].

In the presence of a weak magnetic field, as indicated by Eq. (3), although the physics that quarks get polarized due to the rotation in QGP is not modified, the quark spin configuration in phase space can be dramatically different. To show this, we apply Eq. (3) to the hydrodynamical modeling of QGP evolution, and calculate the spin polarization of s-quarks emitted from the cheminal freeze-out hypersurface. Because the spin of a Lambda hyperon is dominantly carried by the s-quark, as u-, d- and s-quark coalesce [55], the calculation is also expected a good approximation to the polarization of Lambda hyperon. Of course, as a consequence of the approximate chiral symmetry breaking during the coalescence, we leave the s-quark mass m_s as a free parameter.

With respect to the isobar collision systems with $\sqrt{s_{NN}}=200$ GeV, we carry out hydrodynamical simulations using the 3+1D MUSIC program [56]. Initial conditions are chosen and calibrated, according to the centrality class determined by the charged particle multiplicity and a longitudinal profile that reproduces the rapidity dependent direct flow [57]. In order to capture the thermal vorticity associated with the elliptic and triangular initial geometry, we deform the optical Glauber model distribution with a finite ellipticity and triangularity, following the prescription introduced in Ref. [58]. In addition to the standard set of parameters that has been used in the hydrodynamical modeling, such as the shear and the bulk viscosities, we choose the mean value of electrical conductivity $\sigma_{\rm el}/T = 1$, with respect to the pQCD expectation $\sigma_{\rm el}/T \in [0.2, 2]$ [59, 60][1)]. To avoid the complexity from the unknown space-time evolution, for the magnetic field we take $\mathbf{B} = B\Gamma(\eta_s)\hat{y}$, with B a constant parameter and the space-time rapidity dependence $\Gamma(\eta_s)$ determined from the retarded potential solution [51]. In this way, $|eB|$ is the time-average field strength in the center of the QGP.

Lambda local spin polarization along the beam axis can be quantified when it is decomposed into sine Fourier modulations. Under the local thermal equilibrium condition, the local thermal vorticities in QGP are expected from the medium response to the elliptic and the triangular geometries, respectively. Shown in Fig. 1, we solve the Lambda local spin polarization for RuRu collisions with impact parameter $b = 7$ fm and $m_s = 0.8$ GeV. Indeed, in Fig. 1 (a) where $w_{\mu\nu}$ is identified as $w^{\rm th}_{\mu\nu}$, without magnetic field one finds negative second-order and third-order modulations of the local spin polarization. It is interesting to notice that the magnitude for the second order is larger, in consistency with the hierarchy between elliptic flow and triangular flow.

In order to resolve the sign problem of the Lambda local spin polarization, a number of contributions have been proposed beyond thermal vorticity [32−36]. Of particular interest, the shear tensor $\xi_{\mu\nu} = \frac{1}{2}(\partial_\mu\beta_\nu + \partial_\nu\beta_\mu)$, which appears in systems in local thermal equilibrium, gives rise to the so-called shear-induced polarization $\omega^{\rm SIP}_{\mu\nu}$. Corresponding to the two types of formulation, $\omega^{\rm SIP}_{\mu\nu}$(LY) by Liu and Yin [33], and $\omega^{\rm SIP}_{\mu\nu}$(BBP) by Becattini, Bucciantini and Palermo [34], with the replacement $\omega_{\mu\nu} \to \omega^{\rm th}_{\mu\nu} + \omega^{\rm SIP}_{\mu\nu}$ in Eq. (3), we re-calculate the Lambda local spin polarization. Results are shown in Fig. 1 (b) and (c).

Let us simply summarize our observations from Fig. 1. First, in all cases, with the magnetic field introduced, both modulations in the local spin polarization get increased monotonically. Eventually, with a weak magnetic field, they become positive regardless of the contribution from the shear-induced polarization. We emphasize that the sign change of the local polarization reflects the more significant influence of $\delta f_{\rm EM}$ in the weighted integral with respect to local vorticity at freeze-out hypersurface, Eq. (2), namely, $\delta f_{\rm EM}$ can overwhelm n_F in terms of the conversion of local spin. However, it does not imply $|\delta f_{\rm EM}| > |n_F|$. Secondly, the second-order modulation appears more sensitive to the magnetic field than the third-order one, while the sensitivity is reduced by the shear-induced polarization. Lastly, and quite importantly, although the shear-induced polarization can make the local Lambda spin polarization positive [61], the ordering that the second-order modulation is greater than the third-order one, is an unambiguous signature associated with the magnetic field. All of these qualitative features are not affected by the s-quark mass, though for smaller m_s, we do find that the spin conversion from the QGP is earsier, which effectively reduces the required field strength.

The experimental observables, especially the centrality dependent Lambda local spin polarization can be well captured in the presence of the weak magnetic field. Although our current calculation is approximate, we achieve excellent characterizations with $m_s = 0.8$ GeV and $w_{\mu\nu} = \omega^{\rm th}_{\mu\nu} + w^{\rm SIP}_{\mu\nu}$(BBP). Shown in Fig. 2 (a), the STAR measured $n = 2$ modulation of the Lambda local spin polarization in the isobar systems can be reproduced [62], with the the centrality dependent $|eB|$ extracted and given in Fig. 2 (b) (blue symbols). Error bars in Fig. 2 (b) stem from the experimental uncertainties, as well as $\sigma_{\rm el}/T \in [0.2, 2]$. For comparison, in Fig. 2 (b), the estimated field strength from the direct photon v_2 in AuAu collisions are shown as well [40]. It is interesting to notice that the extracted field strength grows as one proceeds towards peripheral collisions. With the same field strength, the third-order modulation of the Lambda local spin polarization can be described as well, with the results shown as the green shaded band, in Fig. 2 (a).

In the centrality class 20%-60%, the STAR collaboration also measured the transverse momentum dependent local spin polarization for the Lambda hyperon [62]. Although the ordering in Fig. 2 (a) is not obvious, as shown in Fig. 3, from the p_T dependent Lambda local spin polarization, in the low p_T region one clearly observes a larger $n = 2$ modulation than $n = 3$. As we emphasized previously, this ordering indicates the presence of a weak magnetic field. Moreover, our theoretical calculation gives consistent description of the p_T dependence. The deviations at large p_T, where hydrodynamics becomes invalid and perturbative contributions to spin local polarization can be important, should not be a surprise.

IV. SUMMARY AND DISCUSSION

For the conversion of quark spin from QGP, and con-

1) While from lattice calculations, one expects a smaller electrical conductivity (one order of magnitude smaller), the combined factor $\frac{\sigma_{\rm el}}{T}\frac{eB}{m_\pi^2}$, remains small in our calculations, which does not change the formulation. On the other hand, it should be aware that the exact value of the electrical conductivity in realistic QGP systems is undetermined as it depends on strong coupling constant, QGP local thermal equalibration, etc.

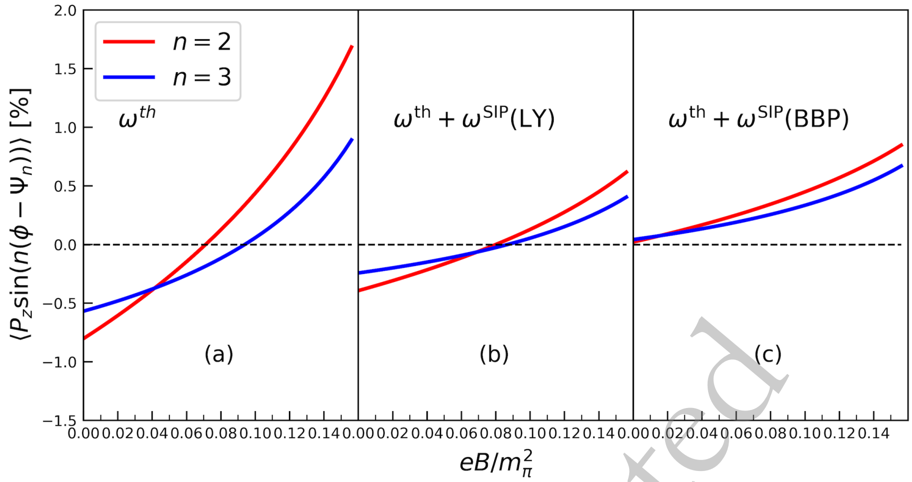

Fig. 1. (color online) Dependence of the second-order modulation (red) and the third-order modulation (blue) of Lambda local spin polarization on the magnetic field.

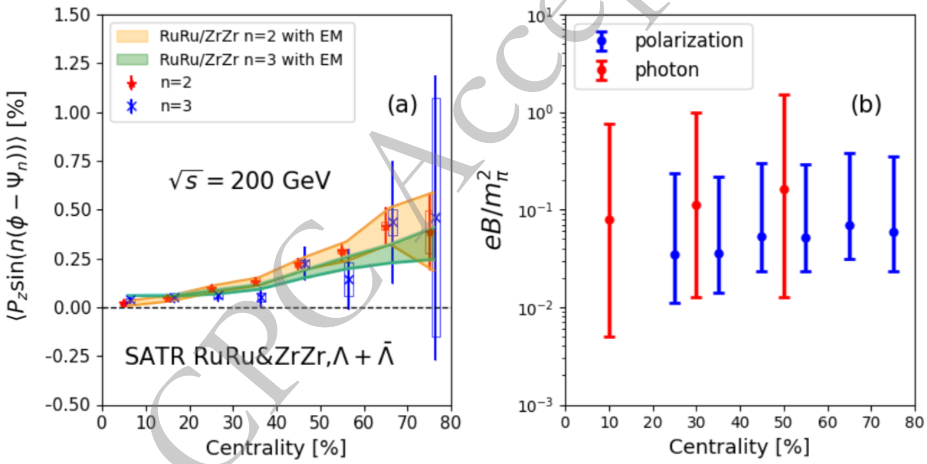

Fig. 2. (color online) (a) Centrality dependent Lambda local spin polarization from the isobar collisions: symbols are experimental results from the STAR collaboration [62] while shaded band correspond to theoretical results with magnetic field for $\omega = \omega^{\rm th} + \omega^{\rm SIP}$(BBP), respectively. (b) Centrality dependence of the magnetic field extracted according to the Lambda local spin polarization (blue symbols) and the direct photon elliptic flow (red symbols) [40].

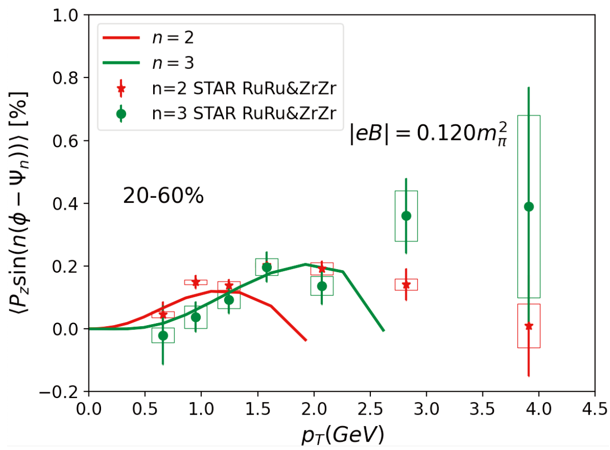

Fig. 3. (color online) Transverse momentum dependent Lambda local spin polarization for the $n=2$ (red) and $n=3$ (green) modes of the isobar collisions in the centrality class 20%-60%.

sequently the spin polarization of Lambda hyperon, we find a novel effect originated from the weak magnetic field. Unlike the coupling via a finite magnetic moment of the hyperon μ_Λ, namely, with $\omega_{\mu\nu} \to \omega_{\mu\nu} + QF_{\mu\nu}\mu_\Lambda/T$ [14], this novel effect reflects a small dissipative correction in the quark phase space distribution. The weak magnetic effect can be crucial to the observed Lambda local spin polarization, not only because it correctly captures the sign observed in experiments, but also it is responsible to the ordering between the second- and the third-order modulations of the local spin polarization. Using hydrodynamical simulations, the experimentally measured Lambda local spin polarization can be well understood. The required fields in different centralities, as shown in Fig. 2 (b), are found consistent with the estimation made based on the direct photon elliptic flow. These are indeed weak fields, as they satisfy $|eB| \ll m_\pi^2$. In fact, using the mean value from Fig. 2 (b), one finds the time integrated value $\mathscr{B} = \int dteB \lesssim 50$ MeV, which is too weak to induce the splitting in the global polarization between Λ and $\bar{\Lambda}$ [63].

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