Solving the strong *CP* problem via a $\bar{\theta}$-characterized mirror symmetry*

Pei-Hong Gu (顾佩洪)†

School of Physics, Jiulonghu Campus, Southeast University, Nanjing 211189, China

Abstract: In the standard model QCD Lagrangian, a term of *CP* violating gluon density is theoretically expected to have a physical coefficient $\bar{\theta}$, which is typically on the order of unity. However, the upper bound on the electric dipole moment of the neutron enforces the value of $\bar{\theta}$ to be extremely small. The significant discrepancy between theoretical expectations and experimental results in this context is widely recognized as the strong *CP* problem. To solve this puzzle in an appealing context of two Higgs doublets, we propose a $\bar{\theta}$-characterized mirror symmetry between two Higgs singlets with respective discrete symmetries. In our scenario, the parameter $\bar{\theta}$ can completely disappear from the full Lagrangian after the standard model fermions take a proper phase rotation as well as the Higgs doublets and singlets. Moreover, all of new physics for solving the strong *CP* problem can be allowed near the TeV scale.

Keywords: strong *CP* problem, $\bar{\theta}$-characterized mirror symmetry, two Higgs doublets

DOI: 10.1088/1674-1137/ad102b

I. INTRODUCTION

In the standard model QCD Lagrangian, a term of *CP* violating gluon density is theoretically expected to have a physical coefficient $\bar{\theta}$, which is typically on the order of unity. However, the upper bound on the electric dipole moment of the neutron enforces the value of $\bar{\theta}$ to be extremely small. The huge gap between the theoretical expectation and experimental result leads to the so-called strong *CP* problem [1−5]. In 1977, Peccei and Quinn determined that the *CP* violating $\bar{\theta}$-term can be effectively neutralized if the QCD Lagrangian contains a global symmetry [6]. Currently, this global symmetry is well known as the Peccei-Quinn (PQ) symmetry $U(1)_{\rm PQ}$. After the PQ global symmetry undergoes spontaneous breaking, a massless Goldstone boson typically emerges. However, in this case, the Goldstone boson gains mass due to the color anomaly [7−9], transforming into a pseudo Goldstone boson, commonly referred to as the axion [10, 11].

The simplest approach to the PQ symmetry appears to consider a two-Higgs-doublet model [6]. Unfortunately, this original PQ model was quickly ruled out in experiments. However, the axion has not been observed experimentally and is still an invisible particle. This implies that the interactions between the axion and SM particles should be at an extremely weak level [1−5]. For a successful realization of the PQ symmetry with an invisible axion, Kim-Shifman-Vainstein-Zakharov (KSVZ) [12, 13] and Dine-Fischler-Srednicki-Zhitnitsky (DFSZ) [14, 15] proposed their elegant methods to effeciently decrease the couplings of the axion to the SM particles. Currently, the absence of positive outcomes in axion search experiments imposes stringent constraints on the PQ symmetry within KSVZ-type and DFSZ-type models [12−15], indicating that it must be spontaneously broken at an energy scale significantly higher than the weak scale [1−5].

In the KSVZ-type and DFSZ-type models, the new particles except the invisible axion should be too heavy to verify in experiments unless the related couplings are artificially small. This implies that all experimental attempts to test the PQ symmetry can only depend on the axion-meson mixing and hence the axion searches [1−5]. Theoretically, when there is a substantial hierarchy between the PQ and electroweak symmetry breaking scales, the inevitable Higgs portal interaction requires an exceptionally small coupling. Otherwise, there must be significant cancellation between its contribution and the rarely quadratic term of the SM Higgs scalar [16]. In some sense, the invisible axion models pay a price of additional fine tuning to solve the strong *CP* problem.

In this paper, we propose a new mechanism to solve

Received 18 October 2023; Accepted 24 November 2023; Published online 25 November 2023

* Supported in part by the National Natural Science Foundation of China (12175038) and in part by the Fundamental Research Funds for the Central Universities

† E-mail: phgu@seu.edu.cn

the strong *CP* problem in the appealing context of two Higgs doublets [17]. Specifically, we introduce a $\bar{\theta}$-characterized mirror symmetry only between two Higgs singlets with respective discrete symmetries. In this scenario, the parameter $\bar{\theta}$ can completely disappear from the full Lagrangian after the standard model fermions and the Higgs scalars take a proper phase rotation. Moreover, all of new physics for solving the strong *CP* problem can be allowed near the TeV scale.

II. STRONG CP PROBLEM

Before delving into the specifics of our proposed mechanism, let us first provide a succinct overview of the strong *CP* problem. The QCD Lagrangian in the SM can be characterized as follows:

$$\mathcal{L}_{\rm QCD} = \sum_q \bar{q}\left(\mathrm{i}\not{D} - m_q e^{\mathrm{i}\theta_q}\right) q - \frac{1}{4} G^{a\mu\nu} G^a_{\mu\nu} - \theta \frac{\alpha_s}{8\pi} G^{a\mu\nu} \tilde{G}^a_{\mu\nu}, \quad (1)$$

where θ_q denotes the phase from the Yukawa couplings of quark fields, θ denotes the QCD vacuum angle, $G^a_{\mu\nu}$ denotes the gluon field strength tensor, and $\tilde{G}^a_{\mu\nu}$ denotes its dual. Following the application of a chiral phase transformation to the quark fields, it can be described as follows:

$$q \to e^{-\mathrm{i}\gamma_5 \theta_q/2} q, \quad (2)$$

Furthermore, their mass terms can remove phases θ_q from the QCD Lagrangian, i.e.,

$$\mathcal{L}_{\rm QCD} = \sum_q \bar{q}\left(\mathrm{i}\not{D} - m_q\right) q - \frac{1}{4} G^{a\mu\nu} G^a_{\mu\nu} - \bar{\theta} \frac{\alpha_s}{8\pi} G^{a\mu\nu} \tilde{G}^a_{\mu\nu} \quad \text{with } \bar{\theta} \equiv \theta - \mathrm{ArgDet}\left(M_d M_u\right). \quad (3)$$

Here, M_d and M_u denote the respective mass matrices of the SM down-type and up-type quarks, respectively. To satisfy the upper limits on the electric dipole moment of the neutron, the value of $\bar{\theta}$ should be extremely small rather than the theoretically expected order of unity, i.e.,

$$|\bar{\theta}| < 10^{-10}. \quad (4)$$

This fine tuning of ten orders of magnitude is commonly termed as the strong *CP* problem.

III. $\bar{\theta}$-CHARACTERIZED MIRROR SYMMETRY

We now demonstrate our mechanism in a realistic model. All the scalars and fermions in the model are summarized in Table 1. The two Higgs singlets $\xi_{1,2}$ are distinguished by $Z_3^{(1)} \times Z_3^{(2)}$ discrete symmetries as well as the two Higgs doublets $\phi_{1,2}$. Besides three generations of the SM quarks q_L, d_R, and u_R and SM leptons l_L and e_R, we introduce three right-handed neutrinos N_R to realize a seesaw [18−22] mechanism for the generation of tiny neutrino masses and also a leptogenesis [23] mechanism for the explanation of cosmological baryon asymmetry. Here, the family indices of the fermions are omitted for simplicity. Moreover, there is a $\bar{\theta}$-characterized mirror symmetry between the two Higgs singlets $\xi_{1,2}$, i.e.

$$\xi_1 \xleftrightarrow{\bar{\theta}-\text{characterized mirror symmetry}} e^{-\mathrm{i}\bar{\theta}/3} \xi_2. \quad (5)$$

This may be a minimal version of the $\bar{\theta}$-characterized mirror symmetry [24].

Based on the charge assignments in Table 1, the expressions for the allowed Yukawa and mass terms involving the fermions are as follows:

$$\mathcal{L}_{Y+M} = -y_d \bar{q}_L \phi_1 d_R - y_u \bar{q}_L \tilde{\phi}_2 u_R - y_e \bar{l}_L \phi_1 e_R - y_N \bar{l}_L \tilde{\phi}_2 N_R - \frac{1}{2} M_N \bar{N}_R N_R^c + \mathrm{H.c.} \quad \text{with } \tilde{\phi}_{1,2} = \mathrm{i}\tau_2 \phi^*_{1,2}, \quad (6)$$

Table 1. All scalars and fermions in the model. The two Higgs singlets $\xi_{1,2}$ are denoted by $Z_3^{(1)} \times Z_3^{(2)}$ discrete symmetries and the two Higgs doublets $\phi_{1,2}$. In addition to three generations of the SM quarks q_L, d_R , and u_R and SM leptons l_L and e_R, we introduce three right-handed neutrinos N_R to realize a seesaw mechanism for the generation of tiny neutrino masses and also a leptogenesis mechanism for the explanation of cosmological baryon asymmetry. Here, the family indices of the fermions are omitted for simplicity.

Scalars&Fermions	ξ_1	ξ_2	ϕ_1	ϕ_2	q_L	d_R	u_R	l_L	e_R	N_R
$SU(3)_c$	1	1	1	1	3	3	3	1	1	1
$SU(2)_L$	1	1	2	2	2	1	1	2	1	1
$U(1)_Y$	0	0	$+\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{6}$	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{2}$	-1	0
$Z_3^{(1)}$	$e^{\mathrm{i}\frac{2\pi}{3}}$	1	$e^{\mathrm{i}\frac{2\pi}{3}}$	1	1	$e^{\mathrm{i}\frac{4\pi}{3}}$	1	1	$e^{\mathrm{i}\frac{4\pi}{3}}$	1
$Z_3^{(2)}$	1	$e^{\mathrm{i}\frac{2\pi}{3}}$	$e^{\mathrm{i}\frac{2\pi}{3}}$	1	1	$e^{\mathrm{i}\frac{4\pi}{3}}$	1	1	$e^{\mathrm{i}\frac{4\pi}{3}}$	1

The full scalar potential at a renormalizable level is as follows:

$$\begin{aligned}V =&\mu_1^2\phi_1^\dagger\phi_1+\mu_2^2\phi_2^\dagger\phi_2+\lambda_1\left(\phi_1^\dagger\phi_1\right)^2+\lambda_2\left(\phi_2^\dagger\phi_2\right)^2\\&+\lambda_3\phi_1^\dagger\phi_1\phi_2^\dagger\phi_2+\lambda_4\phi_1^\dagger\phi_2\phi_2^\dagger\phi_1+\mu_\xi^2\left(\xi_1^*\xi_1+\xi_2^*\xi_2\right)\\&+\kappa_1\left[\left(\xi_1^*\xi_1\right)^2+\left(\xi_2^*\xi_2\right)^2\right]+\kappa_2\xi_1^*\xi_1\xi_2^*\xi_2\\&+\rho_\xi\left[\left(\xi_1^3+\mathrm{e}^{-\mathrm{i}\bar{\theta}}\xi_2^3\right)+\mathrm{H.c.}\right]+\epsilon_1\phi_1^\dagger\phi_1\left(\xi_1^*\xi_1+\xi_2^*\xi_2\right)\\&+\epsilon_2\phi_2^\dagger\phi_2\left(\xi_1^*\xi_1+\xi_2^*\xi_2\right)+\epsilon_3\left(\xi_1\xi_2\phi_1^\dagger\phi_2+\mathrm{H.c.}\right).\end{aligned}\tag{7}$$

The Yukawa couplings and the Majorana masses involving the right-handed neutrinos are responsible for the reaization of seesaw and leptogenesis. We do not examine the details of seesaw and leptogenesis which are beyond the goal of the present work. It should be noted that the $\bar{\theta}$-characterized mirror symmetry (5) is exactly complied in the classical Lagrangian where the kinetic terms are not given for simplicity.

We clarify that the fields in Table 1 with the $\bar{\theta}$-characterized mirror symmetry in Eq. (5) to aid in solving the strong CP problem. After the two Higgs singlets, the two Higgs doublets and the three generations of fermions take the phase rotations as below,

$$\left(\begin{array}{l}\xi_1\to\xi_1\\ \xi_2\to\mathrm{e}^{+\mathrm{i}\bar{\theta}/3}\xi_2\end{array}\right),\quad\left(\begin{array}{l}\phi_1\to\mathrm{e}^{+\mathrm{i}\bar{\theta}/3}\phi_1\\ \phi_2\to\phi_2\end{array}\right),$$

$$\left(\begin{array}{l}q_L\to q_L\\ d_R\to\mathrm{e}^{-\mathrm{i}\bar{\theta}/3}d_R\\ u_R\to u_R\\ l_L\to l_L\\ e_R\to\mathrm{e}^{-\mathrm{i}\bar{\theta}/3}e_R\\ N_R\to N_R\end{array}\right),\tag{8}$$

the QCD Lagrangian (3) and the scalar potential (7) can simultaneously remove the parameter $\bar{\theta}$ [1)] as follows:

$$\mathcal{L}_{\mathrm{QCD}}\ \Rightarrow\ \sum_q\bar{q}\left(\mathrm{i}\,\not{D}-m_q\right)q-\frac{1}{4}G^{a\mu\nu}G^a_{\mu\nu},\tag{9}$$

$$\begin{aligned}V\Rightarrow&\mu_1^2\phi_1^\dagger\phi_1+\mu_2^2\phi_2^\dagger\phi_2+\lambda_1\left(\phi_1^\dagger\phi_1\right)^2+\lambda_2\left(\phi_2^\dagger\phi_2\right)^2\\&+\lambda_3\phi_1^\dagger\phi_1\phi_2^\dagger\phi_2+\lambda_4\phi_1^\dagger\phi_2\phi_2^\dagger\phi_1+\mu_\xi^2\left(\xi_1^*\xi_1+\xi_2^*\xi_2\right)\\&+\kappa_1\left[\left(\xi_1^*\xi_1\right)^2+\left(\xi_2^*\xi_2\right)^2\right]+\kappa_2\xi_1^*\xi_1\xi_2^*\xi_2\\&+\rho_\xi\left[\left(\xi_1^3+\xi_2^3\right)+\mathrm{H.c.}\right]+\epsilon_1\phi_1^\dagger\phi_1\left(\xi_1^*\xi_1+\xi_2^*\xi_2\right)\\&+\epsilon_2\phi_2^\dagger\phi_2\left(\xi_1^*\xi_1+\xi_2^*\xi_2\right)+\epsilon_3\left(\xi_1\xi_2\phi_1^\dagger\phi_2+\mathrm{H.c.}\right),\end{aligned}\tag{10}$$

The Yukawa and mass terms (6) can remain invariant with the unshown kinetic terms.

IV. PHYSICAL SCALARS

When the Higgs scalars ξ_1, ξ_2, ϕ_1 , and ϕ_2 develop their nonzero vacuum expectation values v_{ξ_1}, v_{ξ_2}, v_{ϕ_1} , and v_{ϕ_2}, respectively, they are expressed as follows:

$$\xi_1=\left(v_{\xi_1}+h_{\xi_1}+\mathrm{i}P_{\xi_1}\right)/\sqrt{2},\tag{11}$$

$$\xi_2=\left(v_{\xi_2}+h_{\xi_2}+\mathrm{i}P_{\xi_2}\right)/\sqrt{2},\tag{12}$$

$$\phi_1=\left[\begin{array}{c}\phi_1^+\\ \left(v_{\phi_1}+h_{\phi_1}+\mathrm{i}P_{\phi_1}\right)/\sqrt{2}\end{array}\right],\tag{13}$$

$$\phi_2=\left[\begin{array}{c}\phi_2^+\\ \left(v_{\phi_2}+h_{\phi_2}+\mathrm{i}P_{\phi_2}\right)/\sqrt{2}\end{array}\right].\tag{14}$$

Three would-be-Goldstone bosons:

$$G_W^\pm=\left(v_{\phi_1}\phi_1^\pm+v_{\phi_2}\phi_2^\pm\right)/\sqrt{v_{\phi_1}^2+v_{\phi_2}^2},\tag{15}$$

$$G_Z=\left(v_{\phi_1}P_{\phi_1}+v_{\phi_2}P_{\phi_2}\right)/\sqrt{v_{\phi_1}^2+v_{\phi_2}^2},\tag{16}$$

eaten by the longitudinal components of the SM gauge bosons $W^\pm$ and Z. Therefore, besides a pair of massive charged scalars,

$$\begin{aligned}&H^\pm=\left(v_{\phi_1}\phi_2^\pm-v_{\phi_2}\phi_1^\pm\right)/\sqrt{v_{\phi_1}^2+v_{\phi_2}^2}\ \text{ with}\\&m_{H^\pm}^2=-\left[\lambda_4+\epsilon_3v_\xi^2/\left(2v_1v_2\right)\right]\left(v_1^2+v_2^2\right),\end{aligned}\tag{17}$$

we eventually obtain seven massive neutral scalars including four scalars and three pseudo scalars, i.e.,

$$\begin{aligned}&h_{\phi_1},\ h_{\phi_2},\ h_\xi=\left(h_{\xi_1}+h_{\xi_2}\right)/\sqrt{2},\\&S_\xi=\left(h_{\xi_1}-h_{\xi_2}\right)/\sqrt{2};\end{aligned}\tag{18}$$

$$\begin{aligned}&a_\phi=\left(v_{\phi_1}P_{\phi_2}-v_{\phi_2}P_{\phi_1}\right)/\sqrt{v_{\phi_1}^2+v_{\phi_2}^2},\\&a_\xi=\left(P_{\xi_1}+P_{\xi_2}\right)/\sqrt{2},\ P_\xi=\left(P_{\xi_1}-P_{\xi_2}\right)/\sqrt{2}.\end{aligned}\tag{19}$$

1) The parameter $\bar{\theta}$ can still appear at loop level, however, such loop corrections are much smaller than the experimental constraints [25].

With the minimum of the scalar potential, we obtain the mass-squared matrix of three scalars h_{ϕ_1}, h_{ϕ_2} and h_ξ, i.e.

$$\mathcal{L} \supset -\frac{1}{2}\left[h_{\phi_1}\ h_{\phi_2}\ h_\xi\right]\begin{bmatrix} 2\lambda_1 v_{\phi_1}^2 - \frac{1}{2}\epsilon_3 v_\xi^2 \frac{v_{\phi_2}}{v_{\phi_1}} & (\lambda_3+\lambda_4)v_{\phi_1}v_{\phi_2} + \frac{1}{2}\epsilon_3 v_\xi^2 & \sqrt{2}\left(\epsilon_1 + \frac{1}{2}\epsilon_3\right)v_{\phi_1}v_\xi \\ (\lambda_3+\lambda_4)v_{\phi_1}v_{\phi_2} + \frac{1}{2}\epsilon_3 v_\xi^2 & 2\lambda_2 v_{\phi_2}^2 - \frac{1}{2}\epsilon_3 v_\xi^2 \frac{v_{\phi_1}}{v_{\phi_2}} & \sqrt{2}\left(\epsilon_2 + \frac{1}{2}\epsilon_3\right)v_{\phi_2}v_\xi \\ \sqrt{2}\left(\epsilon_1 + \frac{1}{2}\epsilon_3\right)v_{\phi_1}v_\xi & \sqrt{2}\left(\epsilon_2 + \frac{1}{2}\epsilon_3\right)v_{\phi_2}v_\xi & 2\kappa_1 v_\xi^2 + \frac{3}{\sqrt{2}}\rho_\xi v_\xi - \frac{1}{2}\epsilon_3 v_{\phi_1}v_{\phi_2} \end{bmatrix}\begin{bmatrix} h_{\phi_1} \\ h_{\phi_2} \\ h_\xi \end{bmatrix}. \tag{20}$$

Hereafter, we consider the following:

$$v_{\xi_1} = v_{\xi_2} \equiv v_\xi, \tag{21}$$

which can be easily deduced from the minimization of the scalar potential. By diagonalizing the mass-squared matrix (20), we obtain three mass eigenstates $H_{1,2,3}$ with Yukawa couplings. For simplicity, we do not perform this diagonalization in the present work. For the forth scalar S_ξ without Yukawa couplings, it corresponds to a mass eigenstate with the following mass square,

$$m_{S_\xi}^2 = 2\kappa_1 v_\xi^2 + \frac{3}{\sqrt{2}}\rho_\xi v_\xi - \frac{1}{2}\epsilon_3 v_{\phi_1}v_{\phi_2}. \tag{22}$$

We then consider the pseudo scalars a_ϕ, a_ξ and P_ξ. Their mass-squared matrix is given by

$$\mathcal{L} \supset -\frac{1}{2}\left[a_\phi\ a_\xi\ P_\xi\right]\begin{bmatrix} -\frac{1}{2}\epsilon_3 v_\xi^2\left(\frac{v_{\phi_1}}{v_{\phi_2}} + \frac{v_{\phi_2}}{v_{\phi_1}}\right) & -\frac{1}{4}\epsilon_3 v_\xi\sqrt{v_{\phi_1}^2 + v_{\phi_2}^2} & 0 \\ -\frac{1}{4}\epsilon_3 v_\xi\sqrt{v_{\phi_1}^2 + v_{\phi_2}^2} & -9\sqrt{2}\rho_\xi v_\xi - \epsilon_3 v_{\phi_1}v_{\phi_2} & 0 \\ 0 & 0 & -9\sqrt{2}\rho_\xi v_\xi - \epsilon_3 v_{\phi_1}v_{\phi_2} \end{bmatrix}\begin{bmatrix} a_\phi \\ a_\xi \\ P_\xi \end{bmatrix}. \tag{23}$$

Clearly, P_ξ is already a mass eigenstate and its mass square is just

$$m_{P_\xi}^2 = -9\sqrt{2}\rho_\xi v_\xi - \epsilon_3 v_{\phi_1}v_{\phi_2}. \tag{24}$$

For a_ϕ and a_ξ, they mix with each other, and their mass eigenstates are as follows:

$$a_1 = a_\phi\cos\alpha - a_\xi\sin\alpha \ \text{ with}$$
$$m_{a_1}^2 = \frac{m_{a_\phi}^2 + m_{a_\xi}^2 + \sqrt{\left(m_{a_\phi}^2 - m_{a_\xi}^2\right)^2 + 4\Delta^4}}{2}, \tag{25}$$

$$a_2 = a_\phi\sin\alpha + a_\xi\cos\alpha \ \text{ with}$$
$$m_{a_2}^2 = \frac{m_{a_\phi}^2 + m_{a_\xi}^2 - \sqrt{\left(m_{a_\phi}^2 - m_{a_\xi}^2\right)^2 + 4\Delta^4}}{2}, \tag{26}$$

Here, $m_{a_\phi}^2$, $m_{a_\xi}^2$, and Δ^2 are defined by

$$m_{a_\phi}^2 = -\frac{1}{2}\epsilon_3 v_\xi^2\left(\frac{v_{\phi_1}}{v_{\phi_2}} + \frac{v_{\phi_2}}{v_{\phi_1}}\right),$$
$$m_{a_\xi}^2 = -9\sqrt{2}\rho_\xi v_\xi - \epsilon_3 v_{\phi_1}v_{\phi_2},$$

$$\Delta^2 = -\frac{1}{4}\epsilon_3 v_\xi\sqrt{v_{\phi_1}^2 + v_{\phi_2}^2}, \tag{27}$$

while α is the mixing angle and is determined by

$$\tan 2\alpha = \frac{2\Delta^2}{m_{A_\phi}^2 - m_{A_\xi}^2}. \tag{28}$$

The pseudo scalars $a_{1,2}$ couple to the axial currents of the SM quarks, and thus, they act as heavy axions [24].

V. CONCLUSION

In this paper, we propose a novel $\bar{\theta}$-characterized mirror symmetry to naturally solve the strong *CP* problem. In our scenario, the scalars include two Higgs singlets and two Higgs doublets, while the fermions include three generations of the SM fermions and the right-handed neutrinos. The $\bar{\theta}$-characterized mirror symmetry is only involved in the two Higgs singlets with respective discrete symmetries. The parameter $\bar{\theta}$ can completely disappear from the full Lagrangian after the fermions and the Higgs scalars take a proper phase rotation. Our mechanism ensures that the new physics for solving the strong *CP* problem is near the TeV scale.

References

[1] H.Y. Cheng, Phys. Rep. **158**, 1 (1988)
[2] J.E. Kim and G. Carosi, Rev. Mod. Phys. **82**, 557 (2010)
[3] L. Di Luzio, M. Giannotti, E. Nardi *et al*., Phys. Rep. **870**, 1 (2020)
[4] P. Sikivie, Rev. Mod. Phys. **93**, 015004 (2021)
[5] R.L. Workman *et al*. (Particle Data Group), Prog. Theor. Exp. Phys. **2022**, 083C01 (2022)
[6] R.D. Peccei and H.R. Quinn, Phys. Rev. Lett. **38**, 1440 (1977)
[7] S.L. Adler, Phys. Rev. **177**, 2426 (1969)
[8] J.S. Bell and R. Jackiw, Nuovo Cim. **60A**, 47 (1969)
[9] S.L. Adler and W.A. Bardeen, Phys. Rev. **182**, 1517 (1969)
[10] S. Weinberg, Phys. Rev. Lett. **40**, 223 (1978)
[11] F. Wilczek, Phys. Rev. Lett. **40**, 279 (1978)
[12] J.E. Kim, Phys. Rev. Lett. **43**, 103 (1979)
[13] M.A. Shifman, A. Vainshtein, and V.I. Zakharov, Nucl. Phys. B **166**, 493 (1980)
[14] M. Dine, W. Fischler, and M. Srednicki, Phys. Lett. B **104**, 199 (1981)
[15] A. Zhitnitsky, Sov. J. Nucl. Phys. **31**, 260 (1980)
[16] J.D. Clarke and R.R. Volkas, Phys. Rev. D **93**, 035001 (2016)
[17] G.C. Branco, P.M. Ferreira, L. Lavoura, M.N. Rebelo, M. Sher, and J. Silva, Phys. Rept. **516**, 1 (2012)
[18] P. Minkowski, Phys. Lett. B **67**, 421 (1977)
[19] T. Yanagida, *Proceedings of the Workshop on Unified Theory and the Baryon Number of the Universe*, ed. O. Sawada and A. Sugamoto (Tsukuba 1979).
[20] M. Gell-Mann, P. Ramond, and R. Slansky, *Supergravity*, ed. F. van Nieuwenhuizen and D. Freedman (North Holland 1979).
[21] R.N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44**, 912 (1980)
[22] J. Schechter and J.W.F. Valle, Phys. Rev. D **22**, 2227 (1980)
[23] M. Fukugita and T. Yanagida, Phys. Lett. B **174**, 45 (1986)
[24] P.H. Gu, arXiv: 2307.04236[hep-ph]
[25] J. Ellis and M.K. Gaillard, Nucl. Phys. B **150**, 141 (1979)