

Using $\Lambda_c^+ \rightarrow pK^-\pi^+$ as a spin polarimeter*

Dai-Hui Wei(魏代会)¹ Yong-Xu Yang(杨永翔)¹ Rong-Gang Ping(平荣刚)^{2,3†}

¹School of Physical Science and Technology, Guangxi Normal University, Guilin 541004, China

²Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China

³University of Chinese Academy of Science, Beijing 100049, China

Abstract: Polarization transfer measurement plays an important role in the search for new physics processes in charmed baryon decays. The measurement of the $\Lambda_c^+ \rightarrow pK^-\pi^+$ decay is suggested as a spin polarimeter. A general description of the decay is developed using Euler angles, and the polarization parameters are derived. Its relationship with parity violation is found using the phenomenological amplitude model. A Monte-Carlo simulation is performed, and the results show that charmed baryon polarization is well determined using a set of Monte-Carlo events with selected asymmetry parameters. The experimental measurement of these asymmetry parameters is suggested.

Keywords: Λ_c , baryon, polarization

DOI: 10.1088/1674-1137/ac5e93

I. INTRODUCTION

Measurements of the initial c -quark polarization in charmed-baryon decays play an important role in the search for new physics processes, for example, at the Large Hadron Collider (LHC) experiment. Analysis of spin transfer provides access to the nonperturbative QCD parameters relevant to the dynamics of the hadronization process and thus provides crucial information about the structure of new physics [1, 2]. Moreover, measurement of charmed baryon polarization can be used to search for the anomalous magnetic moment [3], magnetic dipole moments [4], and electromagnetic dipole moments [5–7] at the LHC experiment.

The decay $\Lambda_c^+ \rightarrow pK^-\pi^+$ can be selected as a spin polarimeter to analyze charmed baryon polarization [1]. Although two-body decays, such as $\Lambda_c^+ \rightarrow \Lambda\pi^+$, $\Sigma^+\pi^0$, $\Sigma^0\pi^+$, and pK_S^0 , having approximately 1%~2% branching fractions [8, 9], and their decay asymmetry parameters have recently been well measured [10], their individual branching fractions are only one sixth of that for $\Lambda_c^+ \rightarrow pK^-\pi^+$. The charged final states can be directly identified in the LHC experiment, and an analysis on the decay $\Lambda_c^+ \rightarrow pK^-\pi^+$ is ongoing [11].

Knowledge on spin transfer in the $\Lambda_c^+ \rightarrow pK^-\pi^+$ decay is essential for calibrating it as a polarimeter in exper-

iments. In the quark model, the lightest charmed baryon, Λ_c^+ , is composed of udc quarks and is classified into a mixed-symmetry 20 multiplet; hence, its spin is assigned as 1/2 [8]. Confirmation of the spin-1/2 assignment dates back to the 90s in the NA32 experiment [12]. Owing to limited $\Lambda_c^+ \rightarrow pK^-\pi^+$ events, the spin was not conclusively determined; however, the results were compatible with the spin-1/2 assignment. Recently, using the events from $e^+e^- \rightarrow \Lambda_c^+\bar{\Lambda}_c^-$ with $\Lambda_c^+ \rightarrow pK_S^0$, $\Lambda\pi^+$, $\Sigma^0\pi^+$, and $\Sigma^+\pi^0$, the spin-1/2 assignment was confirmed with a significance of approximately 6σ [13].

In this study, we propose to calibrate the Λ_c^+ polarimeter using the decays $e^+e^- \rightarrow \Lambda_c^+\bar{\Lambda}_c^-$ and $\Lambda_c^+ \rightarrow pK^-\pi^+$. These decays were once used to measure the absolute branching fraction of $\Lambda_c^+ \rightarrow pK^-\pi^+$ during the BESIII experiments [14]. Other than its large branching fraction, an advantage is that its polarization information is unambiguously known from the transfer of the unpolarized beams of the e^+e^- collider. The possible existence of Λ_c^+ transverse polarization (TP) can be determined using data events and is helpful for calibrating the Λ_c^+ polarimeter.

Copious intermediate states, for example, $\bar{K}^*(892)^0$, $\Delta(1232)$, and the excited Λ^* and Σ^* states, are observed in the Dalitz plot of the weak decay $\Lambda_c^+ \rightarrow pK^-\pi^+$ [15, 16]. An amplitude model can be applied to the decay to study the decay dynamics [17]. However, a measurement of

Received 13 September 2021; Accepted 17 March 2022; Published online 10 May 2022

* Partly supported by the National Natural Science Foundation of China NSFC (11875262, 11875115), Joint Large-Scale Scientific Facility Funds of the NSFC and Chinese Academy of Sciences (U2032110)

† E-mail: pingrg@ihep.ac.cn

©2022 Chinese Physical Society and the Institute of High Energy Physics of the Chinese Academy of Sciences and the Institute of Modern Physics of the Chinese Academy of Sciences and IOP Publishing Ltd

charmed-baryon polarization requires an analysis of the angular distributions of decayed particles in experiments. The possibility of using $\Lambda_c^+ \rightarrow pK^-\pi^+$ as a spin polarimeter has been demonstrated in [17] with an amplitude model determined via full phase space amplitude analysis. In this study, a model-independent method is developed for the three-body decay $\Lambda_c^+ \rightarrow pK^-\pi^+$ to calibrate it with data events from the spin polarimeter used in other experiments.

II. CONSTRUCTION OF HELICITY AMPLITUDE

FOR $\Lambda_c^+ \rightarrow pK^-\pi^+$

We consider the decay $\Lambda_c^+(p_0, \lambda_0) \rightarrow p(p_1, \lambda)K^-(p_2)\pi^+(p_3)$ using an isobar model. It proceeds via a two-step sequential decay, where p_i ($i = 0, 1, 2, 3$) denote the momenta of individual particles, λ_0 denotes the z -projection of the Λ_c^+ spin, and λ_1 denotes the proton helicity. Currently, the measurement shows that the resonances $\Delta(1232)^{++}$, $\bar{K}^*(890)^0$, and $\Lambda(1520)$ dominate the decays. Their branching fractions add up to 5.2%, as shown in Table 1, whereas the total branching fraction for $\Lambda_c^+ \rightarrow pK^-\pi^+$ is measured to be $(6.23 \pm 0.23)\%$.

The Dalitz-plot decomposition of the $\Lambda_c^+ \rightarrow pK^-\pi^+$ decay has been discussed with the helicity amplitude in Ref. [18]. In this study, we construct the amplitudes for these two-step consequential decays using the covariant tensor formalism, which is then related to the calculation of the helicity amplitude. The first-step decay occurs by emitting a W boson from a charmed quark and changing the charmed quark into a strange quark. This decay induces a weak interaction, and the decay does not conserve parity. The vertex forms of the tensor formalism, which include parity-violation terms, are given in Table 1 in terms of the linear combination with complex numbers f_i ($i = 1, 2, \dots, 10$). The second-step decays of intermediate states conserve spin and parity, and their vertex forms are also given in Table 1. To calculate the amplitudes, we present the amplitude for the sequential decay in the heli-

Table 1. Branching fractions and the vertex forms of the decay for resonances in $\Lambda_c^+ \rightarrow pK^-\pi^+$.

| $\Lambda_c^+ \rightarrow$ | Branching fraction | Vertex form |
|---|---------------------|--|
| $\Delta(1232)^{++}K^-$ | $(1.07 \pm 0.25)\%$ | $f_1 \gamma_5 p_0^\mu + f_2 p_0^\mu$ |
| $\Lambda(1520)\pi^+$ | $(2.2 \pm 0.5)\%$ | $f_3 p_0^\mu + f_4 \gamma_5 p_0^\mu$ |
| $p\bar{K}^*(892)^0$ | $(1.94 \pm 0.27)\%$ | $f_5 p_0^\mu + f_6 \gamma_\mu$ $+ f_7(p_2^\nu + p_3^\nu)\sigma_{\mu\nu}$ $+ \gamma_5(f_8 p_0^\mu + f_9 \gamma_\mu)$ $+ \gamma_5 f_{10}(p_2^\nu + p_3^\nu)\sigma_{\mu\nu}$ |
| $\Delta^{++} \rightarrow p\pi^+$ | 33.1% | $f_8 p_{1\mu}$ |
| $\Lambda(1520) \rightarrow pK^-$ | 22.5% | $f_9 \gamma_5 p_{1\mu}$ |
| $\bar{K}^*(892)^0 \rightarrow K^-\pi^+$ | 33.3% | $f_{11} p_{2\mu}$ |

city system.

The helicity amplitudes for $\Lambda_c^+(p_0, \lambda_0) \rightarrow \Delta(1232)^{++}K^- \rightarrow p(p_1, \lambda_1)K(p_2)\pi^+(p_3)$ reads as

$$F_{\lambda_0, \lambda_1}^\Delta(m_{p\pi^+}) = \text{Tr} \left[u_{\lambda_0}(p_0) \bar{u}_{\lambda_1}(p_1) p_0^\mu (f_1 \gamma_5 + f_2) \times S_{\mu\nu}^{3/2}(p_1 + p_3, m_\Delta, \Gamma_\Delta) f_8 p_{1\nu}^\nu \right], \quad (1)$$

where $u_\lambda(p)$ is the Dirac spinor, normalized with $\bar{u}u = 2m$, and $\lambda_0(\lambda_1)$ is the helicity value for the $\Lambda_c^+(p)$ particle, that is,

$$u_\lambda(p) = \frac{\not{p} + m}{\sqrt{p^0 + m}} u_\lambda(0), \text{ with} \\ u_{+1/2}(0) = \left(\cos \frac{\theta}{2}, \sin \frac{\theta}{2} e^{i\phi}, 0, 0 \right)^T, \\ u_{-1/2}(0) = \left(-\sin \frac{\theta}{2} e^{-i\phi}, \cos \frac{\theta}{2}, 0, 0 \right)^T, \quad (2)$$

where θ, ϕ are the polar and azimuthal angles of the momentum in the Dirac spinor, respectively. $S_{\mu\nu}^{3/2}$ is the propagator for $\Delta(1232)$, which is given in Appendix B. In this decay, the two terms with the coupling constants f_1 and f_2 correspond to the violated and conserved parity contributions, respectively.

Similarly, the helicity amplitude for $\Lambda_c^+ \rightarrow \Lambda(1520)\pi^+ \rightarrow pK^-\pi^+$ can be expressed as

$$F_{\lambda_0, \lambda_1}^\Lambda(m_{pK^-}) = \text{Tr} \left[u_{\lambda_0}(p_0) \bar{u}_{\lambda_1}(p_1) p_0^\mu (f_3 + f_4 \gamma_5) \times S_{\mu\nu}^{3/2}(p_1 + p_2, m_\Lambda, \Gamma_\Lambda) f_9 p_{1\nu}^\nu \right], \quad (3)$$

where $S_{\mu\nu}^{3/2}$ is the propagator for $\Lambda(1520)$. In this decay, the two terms with the coupling constants f_3 and f_4 correspond to the violated and conserved parity contributions, respectively.

The helicity amplitude for $\Lambda_c^+ \rightarrow p\bar{K}^*(892)^0 \rightarrow pK^-\pi^+$ can be expressed as

$$F_{\lambda_0, \lambda_1}^{K^*} (m_{K^-\pi^+}) = \text{Tr} \left[u_{\lambda_0}(p_0) \bar{u}_{\lambda_1}(p_1) \times [f_5 p_0^\mu + f_6 \gamma_\mu + f_7(p_2^\nu + p_3^\nu)\sigma_{\mu\nu} + \gamma_5(f_8 p_0^\mu + f_9 \gamma_\mu + f_{10}(p_2^\beta + p_3^\beta)\sigma_{\mu\beta})] \times S_{\mu\alpha}^1(p_2 + p_3, m_{K^*}, \Gamma_{K^*}) f_{11} p_{2\alpha}^\alpha \right], \quad (4)$$

where $S_{\mu\alpha}^1$ is the propagator for $\bar{K}^*(892)^0$, which is given in Appendix B. In this decay, the terms combined with the coupling constants f_8, f_9 , and f_{10} correspond to the parity-violation contributions, while the terms combined with f_5, f_6 , and f_7 correspond to the parity-conserved contributions.

III. POLARIZATION IN $\Lambda_c^+ \rightarrow pK^-\pi^+$ DECAYS

The spin of Λ_c^+ is conclusively determined to be $1/2$, and its spin density matrix (SDM) can be expressed with a 2×2 matrix. This is usually written in terms of Pauli σ matrices as

$$\rho(\Lambda_c^+) = \frac{1}{2}(1 + \vec{\mathcal{P}} \cdot \vec{\sigma}) = \frac{1}{2} \begin{bmatrix} 1 + \mathcal{P}_z & \mathcal{P}_x - i\mathcal{P}_y \\ \mathcal{P}_x + i\mathcal{P}_y & 1 - \mathcal{P}_z \end{bmatrix}, \quad (5)$$

where the mean polarization vector $\vec{\mathcal{P}} = (\mathcal{P}_x, \mathcal{P}_y, \mathcal{P}_z)$, which determines the Λ_c^+ polarization.

For the $\Lambda_c^+(\lambda_0) \rightarrow p(\lambda_1)K^-\pi^+$ decay, the three final-state particles are located on the same decay plane in the rest frame of the Λ_c^+ system. We take the x axis to be along the proton flying direction, the z axis is the normal to the decay plane, and the y axis is vertical to the xz plane; hence, they form a right-hand system. The orientation of this system is described using three Euler angles (α, β, γ) , with which the three successive rotations carry the Λ_c^+ production helicity system to this system. The SDM of the proton can be calculated using

$$\rho(p) = N \cdot \rho(\Lambda_c^+) \cdot N^\dagger, \quad (6)$$

where N is a Λ_c^+ decay matrix, which can be expressed as

$$N_{\lambda_1, \lambda_0} = \sum_{\mu=\pm 1/2} D_{\lambda_0, \mu}^{1/2*}(\alpha, \beta, \gamma) F_{\mu, \lambda_1}(m_{pK}, m_{p\pi}, m_{K\pi}), \quad (7)$$

where the decay matrix F_{μ, λ_1} is dependent on the resonance structures in the $pK^-\pi^+$ final states, which can be expressed by the coherent sum of intermediate states, as given in the previous section; that is,

$$|F_{\mu, \lambda_1}(m_{pK}, m_{p\pi}, m_{K\pi})|^2 = |F_{\mu, \lambda_1}^\Delta(m_{p\pi^*}) + F_{\mu, \lambda_1}^\Lambda(m_{pK^-}) + F_{\mu, \lambda_1}^{K^*}(m_{K^-\pi^*})|^2. \quad (8)$$

For a study on intermediate states in Λ_c^+ decays, F_{μ, λ_1} must be related to the decay amplitude of different resonances observed in the $pK^-\pi^+$ final states [18]. In this study, we do not model the decay matrix F_{μ, λ_1} . Instead we treat it as a model-independent parameter and then relate them to the polarization parameters; the values can be determined with the data events. This treatment has the advantage of model-independent measurement on the Λ_c^+ polarization parameters.

Because the helicity amplitudes are isolated from the angular distributions represented by Euler angles, we can integrate out the masses in the above model-dependent amplitude over the three body space $d\Phi_3$. It then relates

to the square of the model-independent amplitude,

$$|F_{\mu, \lambda_1}|^2 = \int d\Phi_3 |F_{\mu, \lambda_1}(m_{pK}, m_{p\pi}, m_{K\pi})|^2, \quad (9)$$

where the invariant masses, m_{pK} , $m_{p\pi}$, and $m_{K\pi}$, are treated as Dalitz plot variables, and only two are independent in the integral. Because the mass variables are integrated out, we will suppress this mass dependence of the helicity amplitudes in what follows.

In practice, integration over the phase space can be estimated as an average using the Monte-Carlo method. If the number of unweighted phase-space Monte-Carlo (MC) events is sufficiently large, the helicity amplitude can be estimated as

$$|F_{\mu, \lambda_1}|^2 = \frac{\widetilde{\Phi}_3}{N} \sum_{i=1}^N |F_{\mu, \lambda_1}(m_{pK}, m_{p\pi}, m_{K\pi})|^2, \quad (10)$$

where $m_{p\pi^*}^{(i)}$, $m_{pK^-}^{(i)}$, and $m_{K^-\pi^*}^{(i)}$ are invariant masses calculated with the i -th phase-space event, and $\widetilde{\Phi}_3 = \int d\Phi_3$ is the normalization factor when generating the unweighted MC events.

The angular distribution of protons can be calculated by taking the trace of its SDM matrix.

$$\mathcal{W}(\alpha, \beta, \gamma) = \mathcal{A}_0 + \vec{\mathcal{P}} \cdot \vec{\mathcal{A}}, \quad (11)$$

where \mathcal{A}_0 corresponds to an unpolarized cross section, and $\vec{\mathcal{A}}$ represents the analyzing power. These are determined with the Λ_c^+ decay matrix as¹⁾

$$\mathcal{A}_0 = \text{Tr}[N \cdot N^\dagger], \quad (12)$$

$$\vec{\mathcal{A}} = \text{Tr}[N \cdot \vec{\sigma} \cdot N^\dagger]. \quad (13)$$

Using the Λ_c^+ decay matrix, we get

$$\begin{aligned} \mathcal{A}_0 &= |F_{-1/2, -1/2}|^2 + |F_{-1/2, 1/2}|^2 + |F_{1/2, -1/2}|^2 + |F_{1/2, 1/2}|^2, \\ \mathcal{A}_x &= 2(\mathcal{I}_1 + \mathcal{I}_2)(\sin \alpha \cos \gamma - \cos \alpha \cos \beta \sin \gamma) \\ &\quad + 2(\mathcal{R}_1 + \mathcal{R}_2)(\cos \alpha \cos \beta \cos \gamma + \sin \alpha \sin \gamma) \\ &\quad + \mathcal{G} \cos \alpha \sin \beta, \\ \mathcal{A}_y &= -2(\mathcal{I}_1 + \mathcal{I}_2)(\sin \alpha \cos \beta \sin \gamma + \cos \alpha \cos \gamma) \\ &\quad + 2(\mathcal{R}_1 + \mathcal{R}_2)(\sin \alpha \cos \beta \cos \gamma - \cos \alpha \sin \gamma) \\ &\quad + \mathcal{G} \sin \alpha \sin \beta, \\ \mathcal{A}_z &= 2(\mathcal{I}_1 + \mathcal{I}_2) \sin \beta \sin \gamma + \mathcal{G} \cos \beta \\ &\quad - 2(\mathcal{R}_1 + \mathcal{R}_2) \sin \beta \cos \gamma. \end{aligned} \quad (14)$$

1) Here we drop out an inessential factor $1/2$ for simplicity.

Here, we define

$$\mathcal{G} = |F_{1/2,-1/2}|^2 + |F_{1/2,1/2}|^2 - |F_{-1/2,-1/2}|^2 - |F_{-1/2,1/2}|^2.$$

If parity is conserved in $\Lambda_c^+ \rightarrow pK^-\pi^+$, $\mathcal{G} = 2|F_{1/2,1/2}|^2 -$

$2|F_{-1/2,-1/2}|^2$. \mathcal{R}_1 and \mathcal{I}_1 are the real and imaginary parts of the product of the amplitudes, $F_{1/2,-1/2}^*F_{-1/2,-1/2}$, respectively, while \mathcal{R}_2 and \mathcal{I}_2 are those of the product $F_{1/2,1/2}^*F_{-1/2,1/2}$. These are determined using the square of the helicity amplitude, for instance,

$$\begin{aligned} 2\mathcal{R}_1 &= \int d\Phi_3 \left[|F_{1/2,-1/2}(\dots) + F_{-1/2,-1/2}(\dots)|^2 - |F_{1/2,-1/2}(\dots)|^2 - |F_{-1/2,-1/2}(\dots)|^2 \right], \\ 2\mathcal{I}_1 &= \int d\Phi_3 \left[|F_{1/2,-1/2}(\dots)|^2 - |F_{-1/2,-1/2}(\dots)|^2 - |F_{1/2,-1/2}(\dots) + i^* F_{-1/2,-1/2}(\dots)|^2 \right], \end{aligned} \quad (15)$$

where (...) implies the integration variables ($m_{pK}, m_{p\pi}, m_{K\pi}$), as given in Eq. (8). These are dependent on the coupling constants f_i , as shown in Table 1.

In the above equations, \mathcal{A}_0 corresponds to the unpolarized cross section of Λ_c^+ decays, while $\mathcal{R}_{1,2}$, $\mathcal{I}_{1,2}$, and \mathcal{G} correspond to contributions due to parity violation. We introduce three parameters characterizing the asymmetric distribution of Λ_c^+ decays into $pK^-\pi^+$ final states, namely,

$$\mathcal{G}_0 = \mathcal{G}/\mathcal{A}_0, \quad \mathcal{G}_1 = (\mathcal{I}_1 + \mathcal{I}_2)/\mathcal{A}_0, \quad \mathcal{G}_2 = (\mathcal{R}_1 + \mathcal{R}_2)/\mathcal{A}_0,$$

and reformulate the angular distribution as

$$\mathcal{W}(\alpha, \beta, \gamma) = \mathcal{A}_0 [1 + \mathcal{G}_0 \vec{\mathcal{P}} \cdot \vec{\mathcal{R}} + \mathcal{G}_1 \vec{\mathcal{P}} \cdot \vec{\mathcal{S}} + \mathcal{G}_2 \vec{\mathcal{P}} \cdot \vec{\mathcal{T}}], \quad (16)$$

where $\vec{\mathcal{P}}$ is the Λ_c^+ polarization vector,

$$\begin{aligned} \vec{\mathcal{P}} &= (\mathcal{P}_x, \mathcal{P}_y, \mathcal{P}_z), \\ \vec{\mathcal{R}} &= (\sin\beta \cos\alpha, \sin\beta \sin\alpha, \cos\beta), \\ \vec{\mathcal{S}} &= (\sin\alpha \cos\gamma - \cos\alpha \cos\beta \sin\gamma, \\ &\quad -\sin\alpha \cos\beta \sin\gamma - \cos\alpha \cos\gamma, \\ &\quad \sin\beta \sin\gamma), \\ \vec{\mathcal{T}} &= (\cos\alpha \cos\beta \cos\gamma + \sin\alpha \sin\gamma, \\ &\quad \sin\alpha \cos\beta \cos\gamma - \cos\alpha \sin\gamma, \\ &\quad -\sin\beta \cos\gamma). \end{aligned} \quad (17)$$

If the angle γ is integrated out, the angular distribution is reduced to

$$\mathcal{W}(\alpha, \beta) = \mathcal{A}_0 [1 + \mathcal{G}_0 \vec{\mathcal{P}} \cdot \vec{\mathcal{R}}]. \quad (18)$$

In the above equations, the parameters \mathcal{G}_0 , \mathcal{G}_1 , and \mathcal{G}_2 are determined by the intrinsic properties of the decays, in which the parity is not conserved in the production of intermediate states, such as $\Delta(1232)$, $\Lambda(1520)$ and $\bar{K}^*(892)$. If we observe the distribution of the angle

between, for example, $\vec{\mathcal{P}}$ and $\vec{\mathcal{R}}$, the slope of the asymmetry angular distribution is proportional to $\vec{\mathcal{G}}_0$. In this sense, labeling them as asymmetry parameters is a direct generalization for a two-body decay, for instance, $\Lambda \rightarrow p\pi^-$. If the parity is conserved, $F_{1/2,1/2} = -F_{-1/2,-1/2}$, $F_{-1/2,1/2} = F_{-1/2,-1/2}$, and $F_{-1/2,-1/2} = F_{1/2,-1/2}$. This leads to $\mathcal{G}_0 = 0$.

In experiments, Λ_c^+ polarization can be determined by measuring several moments, which are defined as the average of the polarization observables, formed with Euler angles. Some of the moments related to the polarization vector are listed below.

$$\begin{aligned} \mathcal{P}_x &= 3\langle \cos\alpha \sin\beta \rangle / \mathcal{G}_0, \quad \mathcal{P}_y = 3\langle \sin\alpha \sin\beta \rangle / \mathcal{G}_0, \\ \mathcal{P}_z &= 3\langle \cos\beta \rangle / \mathcal{G}_0. \end{aligned} \quad (19)$$

Here, we consider the factor $\mathcal{G}_0 = \mathcal{G}/\mathcal{A}_0$ to be the asymmetry parameter with the assumption $\mathcal{G} \neq 0$, which is intrinsically determined by the helicity amplitudes of the $\Lambda_c^+ \rightarrow pK^-\pi^+$ decay. Once it is determined in one experiment, it is applicable to other experiments for measurements of Λ_c^+ polarization.

IV. SENSITIVITY OF MEASUREMENT ON THE ASYMMETRY PARAMETER

The sensitivity of measurement on the asymmetry parameter, $\mathcal{G}_0 = \mathcal{G}/\mathcal{A}_0$, is determined by the number of Λ_c^+ events. We define the sensitivity as

$$\delta\mathcal{G}_0 = \frac{\sqrt{V(\mathcal{G}_0)}}{\mathcal{G}_0}, \quad (20)$$

where $V(\mathcal{G}_0)$ is the variance of \mathcal{G}_0 , which is estimated with the likelihood function. For a sample with N events, X_1, X_2, \dots, X_N , the likelihood function \mathcal{L} defined over the parameters \mathcal{G}_0 is given by

$$\mathcal{L}(X_1, X_2, \dots, X_N | \mathcal{G}_0) = \prod_{i=1}^N \widetilde{\mathcal{W}}(X_i | \mathcal{G}_0), \quad (21)$$

where $\widetilde{\mathcal{W}} = \mathcal{W}(\alpha, \beta) / \int \mathcal{W}(\alpha, \beta) \sin\beta d\beta d\alpha$ is the normalized function of the angular distribution (see Eq. (18)). With the maximum likelihood estimate, the variance of parameter \mathcal{G}_0 is defined by the expected value.

$$V^{-1}(\mathcal{G}_0) = E \left[-\frac{\partial^2 \ln \mathcal{L}(X_i | \mathcal{G}_0)}{\partial^2 \mathcal{G}_0} \right],$$

$$= N \int \frac{1}{\widetilde{\mathcal{W}}} \left[\frac{\partial \widetilde{\mathcal{W}}(X | \mathcal{G}_0)}{\partial \mathcal{G}_0} \right]^2 dX, \quad (22)$$

where $dX = \sin\beta d\beta d\alpha$.

We consider a special case with weak polarization and a small asymmetry parameter \mathcal{G}_0 , such that $\mathcal{G}_0 |\vec{\mathcal{P}}| \ll 1$. In particular, the Λ_c^+ events are accumulated at the e^+e^- collider near the $\Lambda_c^+ \bar{\Lambda}_c^-$ mass threshold in the BESIII experiment [10], with $\mathcal{P}_x = \mathcal{P}_z = 0$. Up to an accuracy of $O((\mathcal{G}_0 |\vec{\mathcal{P}}|)^5)$, we obtain

$$\delta \mathcal{G}_0 = \sqrt{\frac{105}{N}} \frac{1}{\mathcal{G}_0 \mathcal{P}_y} \sqrt{\frac{1}{35 + 21\mathcal{G}_0^2 \mathcal{P}_y^2 + 15\mathcal{G}_0^4 \mathcal{P}_y^4}}. \quad (23)$$

An inverse problem in the application of the spin polarimeter concerns the estimation of the polarization measurement sensitivity, which can be determined using the same procedure. We find that it has the same form as the \mathcal{G}_0 measurement because the polarization is solely dependent on the \mathcal{G}_0 measurement, as shown in Eq. (19), namely,

$$\frac{\Delta \mathcal{P}_y}{\mathcal{P}_y} = \sqrt{\frac{105}{N}} \frac{1}{\mathcal{G}_0 \mathcal{P}_y} \sqrt{\frac{1}{35 + 21\mathcal{G}_0^2 \mathcal{P}_y^2 + 15\mathcal{G}_0^4 \mathcal{P}_y^4}}. \quad (24)$$

Estimation of the sensitivity is dependent on the measurements of parameters \mathcal{P} and \mathcal{G}_0 . Figure 1 shows that the sensitivities of measurement $\delta \mathcal{G}_0$ varied with the

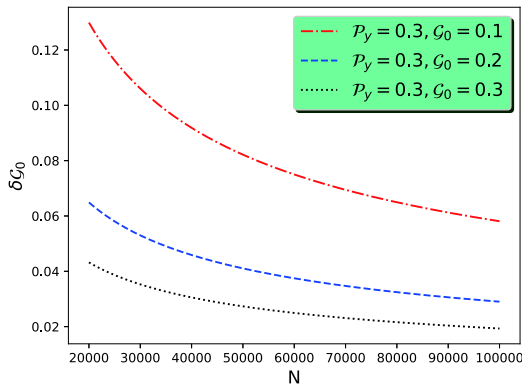


Fig. 1. (color online) Sensitivity of measurement $\delta \mathcal{G}_0$ for the different parameters \mathcal{P}_y and \mathcal{G}_0 with the decay $\Lambda_c^+ \rightarrow pK^-\pi^+$.

number of Λ_c^+ candidates for the different parameters, for instance, $\mathcal{P}_y = 0.3$ and $\mathcal{G}_0 = 0.1, 0.2, 0.3$. Here, we assume the use of an e^+e^- collider with unpolarized beams and $\mathcal{P}_x = \mathcal{P}_z = 0$, such as in the BESIII experiment. We can see that a sensitivity of a few percent can be achieved if 10^5 Λ_c^+ candidates survive from the event selection criteria.

V. CALIBRATION WITH $e^+e^- \rightarrow \Lambda_c^+ \bar{\Lambda}_c^-$ EVENTS

The advantages of calibrating the Λ_c^+ polarimeter with e^+e^- annihilation events originate from the parity conservation in this electromagnetic process. Also, Λ_c^+ polarization is well known. For the unpolarized e^+e^- beam experiment, a virtual photon is produced from e^+e^- annihilation with tensor polarization, while the longitudinal polarization is zero. Hence, the polarization of Λ_c^+ particles is only allowed along the direction normal to the Λ_c^+ production plane.

To calculate the Λ_c^+ SDM, we first define the Λ_c^+ helicity system (x - y - z), which is produced from the $e^+e^- \rightarrow \Lambda_c^+(\lambda_1) \bar{\Lambda}_c^-(\lambda_0)$ process, as shown in Fig. 2. The z axis is taken along the Λ_c^+ flying direction, the x axis is defined in the Λ_c^+ -production plane formed by the e^+ beam and Λ_c^+ momenta and is normal to the Λ_c^+ momentum, and the y axis is taken along the normal to the Λ_c^+ -production plane; therefore, x, y, z form a right-hand system. In the e^+e^- -center-of-mass (CM) system, the Λ_c^+ momentum is characterized by the polar angle θ . Then, the Λ_c^+ SDM is calculated using

$$\rho(\Lambda_c^+)_{\lambda_1, \lambda_1} = \sum_{m, \lambda_0} D_{m, \lambda_1 - \lambda_0}^{1*} (0, \theta, 0) \times D_{m, \lambda_1 - \lambda_0}^1 (0, \theta, 0) A_{\lambda_1, \lambda_0} A_{\lambda_1, \lambda_0}^*, \quad (25)$$

where $m = \pm 1$ is the spin projection of the virtual photon, and A_{λ_1, λ_2} denotes the helicity amplitudes. There are two independent amplitudes due to the constraints of parity conservation, that is, $A_{-1/2, -1/2} = A_{1/2, 1/2}$, $A_{-1/2, 1/2} = A_{1/2, -1/2}$. In the experiment, only the ratio of two amplitudes can be measured. The study on the Λ_c^+ angular distribution is related to the measurement of amplitude

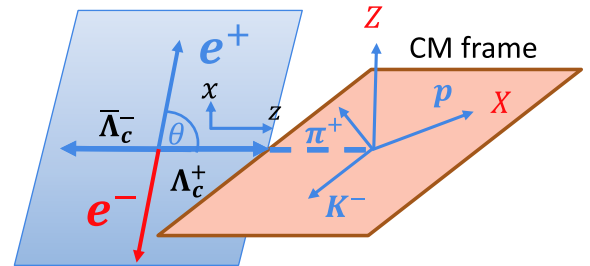


Fig. 2. (color online) Definition of the Λ_c^+ helicity system for its production from the $e^+e^- \rightarrow \Lambda_c^+ \bar{\Lambda}_c^-$, $\Lambda_c^+ \rightarrow pK^-\pi^+$ process.

ratio, namely, $\alpha_c = (|A_{1/2,-1/2}|^2 - 2|A_{1/2,1/2}|^2) / (|A_{1/2,-1/2}|^2 + 2|A_{1/2,1/2}|^2)$, where α_c is the angular distribution parameter for the Λ_c^+ particles. Using these relations, the Λ_c^+ polarization vector can be expressed as

$$\begin{aligned} \mathcal{P}_x &= \mathcal{P}_z = 0, \\ \mathcal{P}_y &= \frac{\sqrt{1-\alpha_c^2} \sin 2\theta \sin \Delta}{4(1+\alpha_c \cos^2 \theta)}, \end{aligned} \quad (26)$$

where Δ is the difference between the phase angles in the two independent helicity amplitudes. The nonzero $\sin \Delta$ allows us to observe the transverse polarization of Λ_c^+ production in e^+e^- collisions. Recent measurements have shown that Λ_c^+ TP is insignificant at energies near the $\Lambda_c^+ \bar{\Lambda}_c^-$ mass threshold, with measurements of $\alpha_c = -0.20 \pm 0.04 \pm 0.02$ [19] and $\sin \Delta = -0.28 \pm 0.13 \pm 0.03$ using BESIII data taken at energy point $\sqrt{s} = 4.6$ GeV [10]. Although the average of Λ_c^+ TP vanishes when an estimation is made over full detector coverage, the polarizations in the forward and backward hemisphere of the detector are nonvanishing and have a reverse sign. This allows us to measure the polarization constant $3\mathcal{A}_0/\mathcal{G}$ defined in the helicity system.

VI. MONTE-CARLO APPLICATION

We use the Monte-Carlo method to obtain the polarization constant. In a model-dependent analysis, the coupling constants f_i ($i = 1, 2, \dots, 10$) in the helicity amplitudes can be determined by fitting the angular distribution of Eq. (11) to data events by substituting the helicity amplitudes F_{λ_1, λ_2} of Eqs. (1)–(4). As an example, to show the simulation of Monte-Carlo event production, we only select the Λ_c^+ decay to $p\bar{K}^*(892)^0$.

In the calculation of the three Euler angles, we first boost the $p(p_1)K^-(p_2)\pi^+(p_3)$ final states to the Λ_c^+ rest frame so that $\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$, as shown in Fig. 2. We describe Λ_c^+ decay in the system $(X-Y-Z)$, which defines the orientation of the decay plane formed by the momenta of three $pK^-\pi^+$ final states. The X axis is taken along the proton flying direction, the Z axis is normal to the decay

plane [20, 21] and parallel to $\vec{p}_1 \times \vec{p}_2$, and the Y axis is chosen to form a right-hand system, as shown in Fig. 2. Moreover, Λ_c^+ polarization is described in the $(x-y-z)$ system (see Fig. 2), as defined in the previous section. For a given event, three successive Euler rotations must be performed so that the $(x-y-z)$ system is carried to overlap the $(X-Y-Z)$ frame, as shown in Fig. 3. First, angle α is rotated about the z -axis to align the y -axis along the nominal to the plane formed by $z \times \mathbf{Z}$. Then, angle β is rotated about the y -axis to align z to the \mathbf{Z} direction. Finally, angle γ is rotated about the \mathbf{Z} axis to overlap the two systems. The angles (β, α) correspond to the polar and azimuthal angles of the z axis in the $(X-Y-Z)$ system. The calculation of Euler angles is given in Appendix A.

In the decay plane, the momenta of the final states are denoted by p_1, p_2 , and p_3 , and their z components are zero. The direction of the proton is along the x axis; therefore, $p_{1x} = |\vec{p}_1|$, $p_{1y} = 0$, $p_{2x} = \vec{p}_1 \cdot \vec{p}_2 / |\vec{p}_1|$, $p_{3x} = \vec{p}_1 \cdot \vec{p}_3 / |\vec{p}_1|$, $p_{2y} = \vec{Y} \cdot \vec{p}_{2y}$, and $p_{3y} = \vec{Y} \cdot \vec{p}_{3y}$, where $\vec{Y} = (\vec{Z} \times \vec{p}_1) / |\vec{p}_1|$. Only the ratios of the f_i coupling constants are measured in the experiment, and we set $f_5 = f_6 = f_7 = f_8 = f_9 = f_{10} = 1 + i$ in the MC event genera-

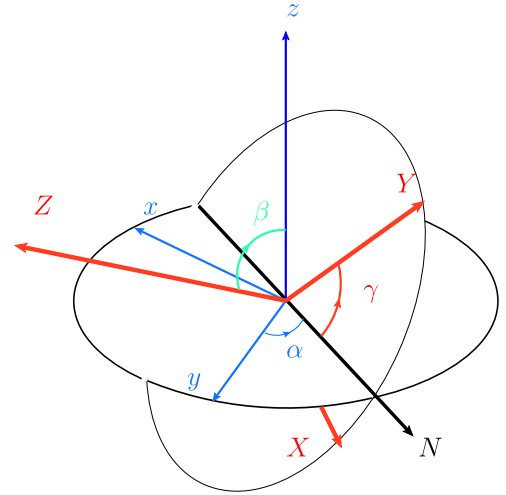


Fig. 3. (color online) Euler rotations to overlap the (x, y, z) system with the (X, Y, Z) system using angles (α, β, γ) for the decay $\Lambda_c^+ \rightarrow pK^-\pi^+$.

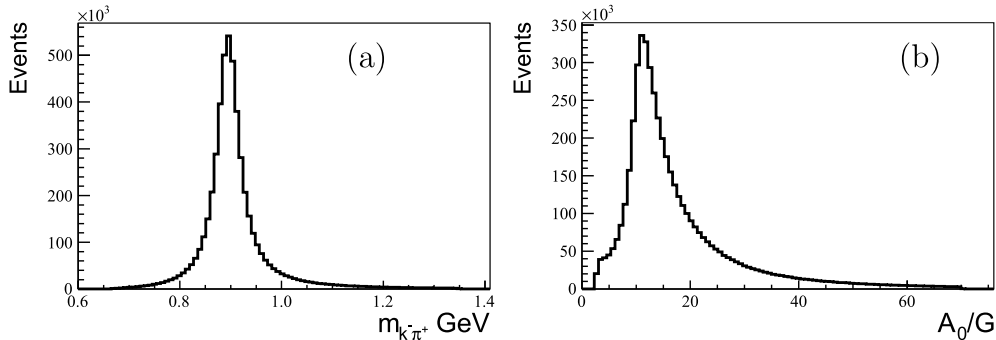


Fig. 4. m_{pK^-} and $\mathcal{A}_0/\mathcal{G}$ distribution in the generated Monte-Carlo events.

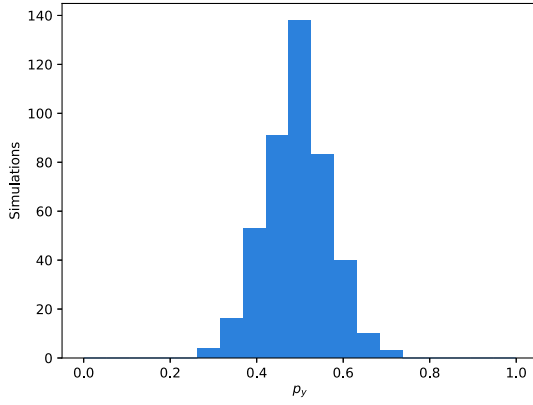


Fig. 5. (color online) Distribution of polarizations \mathcal{P}_y (left peak) and \mathcal{P}_z (right peak) for 438 sets of MC simulations for the decay $\Lambda_c^+ \rightarrow pK^-\pi^+$, with 200,000 events for each MC set.

tion. The mass and width of $\bar{K}^*(892)^0$ are taken as $m_{K^*} = 895.55$ MeV and $\Gamma_{K^*} = 47.3$ MeV, respectively. Figure 4 shows the events sampled using the acceptance-rejection method. $\bar{K}^*(892)$ is identified in the m_{pK^-} spectrum, and the calibration constant is distributed with a peak at approximately $\mathcal{A}_0/\mathcal{G} = 12.0$.

Using a large number of $\Lambda_c^+ \rightarrow pK^-\pi^+$ events, the averages of the helicity amplitudes squared in Eq. (14) are calculated to be $\mathcal{R}_1 + \mathcal{I}_1 i = -4.78 - 1.65i$, $\mathcal{R}_2 + \mathcal{I}_2 i = 106.14 + 4.96i$, and $\mathcal{G}_0 = 11.83/223.93$. We consider a general case and set the polarization vector $\vec{\mathcal{P}} = (0.0, 0.5, 0.0)$. Using an ensemble of generated events, the polarization can be calculated using the moments given in Eq. (19). The statistics of the polarizations are shown in Fig. 5. The two peaks are centered at the input polarization of the $\mathcal{P}_y, \mathcal{P}_z$ values and follow Gaussian distributions, with the standard deviation determining the statistical uncertainties. From these distributions, we obtain $\mathcal{P}_y = 0.494 \pm 0.074$. From these simulations, the sensitivity of \mathcal{P}_y , for example, is determined as 15.0%, which is dependent on the values of \mathcal{P}_x, N , and \mathcal{G}_0 . Furthermore, for a consistency check, we can also estimate the sensitivity of \mathcal{P}_y to be 14.7% using Eq. (24) by substituting in $\mathcal{P}_x = 0.5$, $N = 200000$, and $\mathcal{G}_0 = 11.83/223.93$ in the Monte-Carlo simulations.

VII. SUMMARY AND REMARKS

Measurement of the polarization transfer plays an important role in the search for new physics processes in charmed baryon decays. We show that the decay $\Lambda_c^+ \rightarrow pK^-\pi^+$ can be chosen as a spin polarimeter. A general description of the decay is developed using Euler angles, and the polarization parameters are derived using a helicity amplitude method. A relationship with parity violation is revealed using the phenomenological amplitude model. A Monte-Carlo simulation is performed, and the results show that charmed baryon polarization can be well separated using a set of Monte-Carlo events

with selected asymmetry parameters. The measurement of these asymmetry parameters in experiments is recommended.

APPENDIX A: CALCULATION OF EULER ANGLES

We take an intrinsic rotation starting with the Λ_c^+ -helicity system (x - y - z) to reach the target system (X - Y - Z). The unit vectors of the two systems are defined in the Λ_c^+ -CM system. The unit vectors \hat{X}, \hat{Y} , and \hat{Z} are defined by the CM momenta of the final states of the $\Lambda_c^+ \rightarrow p(p_1)K^-(p_2)\pi^+(p_3)$ decay, that is,

$$\hat{X} = \frac{\vec{p}_1}{|\vec{p}_1|}, \quad \hat{Z} = \frac{\vec{p}_1 \times \vec{p}_2}{|\vec{p}_1||\vec{p}_2|\sin\theta_{12}}, \quad \hat{Y} = \hat{Z} \times \hat{X}, \quad (\text{A1})$$

where p_i ($i = 1, 2, 3$) are defined in the CM system of Λ_c^+ , and θ_{12} is the angle spanned by the momenta \vec{p}_1 and \vec{p}_2 . The intersection of the plane xy and XY in Fig. 3 is expressed by

$$\hat{N} = \frac{\vec{z} \times \vec{Z}}{\sin\beta}, \quad (\text{A2})$$

where β is calculated with

$$\beta = \cos^{-1}(\hat{z} \cdot \hat{Z}). \quad (\text{A3})$$

We define two scalars, $Y_N = \hat{Z} \cdot (\hat{N} \times \hat{Y})$ and $y_N = \hat{z} \cdot (\hat{y} \times \hat{N})$, then α and γ are calculated using

$$\alpha = \begin{cases} \cos^{-1}(\hat{y} \cdot \hat{N}), & \text{if } y_N \geq 0, \\ 2\pi - \cos^{-1}(\hat{y} \cdot \hat{N}), & \text{if } y_N < 0, \end{cases} \quad (\text{A4})$$

$$\gamma = \begin{cases} \cos^{-1}(\hat{Y} \cdot \hat{N}), & \text{if } Y_N \geq 0, \\ 2\pi - \cos^{-1}(\hat{Y} \cdot \hat{N}), & \text{if } Y_N < 0. \end{cases} \quad (\text{A5})$$

APPENDIX B: PROPAGATORS FOR SPIN 3/2 AND 1 RESONANCES

The propagator for the spin 1 resonance is taken as

$$S_{\mu\nu}^1(k, m, \Gamma) = \frac{\tilde{g}_{\mu,\nu}(k)}{k^2 - m^2 + im\Gamma} \quad \text{with} \quad (\text{B1})$$

$$\tilde{g}_{\mu,\nu}(k) = -g_{\mu,\nu} + \frac{k_\mu k_\nu}{k^2}. \quad (\text{B2})$$

The propagator for the spin 3/2 resonance is taken as

$$S_{\mu\nu}^{3/2}(k, m, \Gamma) = \frac{1}{k^2 - m^2 + im\Gamma} \frac{2}{5} (-\gamma_{\mu_1} \not{k} \gamma_{\mu_2} + m\gamma_{\mu_1} \gamma_{\mu_2}) \left\{ \frac{1}{2} [\tilde{g}_{\mu_1\mu_2}(k)\tilde{g}_{\mu_1\nu}(k) + \tilde{g}_{\mu_1\nu}(k)\tilde{g}_{\mu_1\mu_2}(k)] - \frac{1}{3} \tilde{g}_{\mu_2\nu}(k)\tilde{g}_{\mu_1\mu}(k) \right\}. \quad (\text{B3})$$

References

- [1] M. Galanti, A. Giammanco, Y. Grossman *et al.*, JHEP **11**, 067 (2015)
- [2] B. Konig, J. G. Korner, and M. Kramer, Phys. Rev. D **49**, 2363-2368 (1994)
- [3] V. G. Baryshevsky, arXiv: 1504.06702
- [4] A. S. Fomin, A. Y. Korchin, A. Stocchi *et al.*, J. High Energy Phys. **08**, 120 (2017)
- [5] E. Bagli, L. Bandiera, G. Cavoto *et al.*, Eur. Phys. J. C **77**, 828 (2020)
- [6] V. G. Baryshevsky, Eur. Phys. J. C **79**, 350 (2019)
- [7] S. Aiola, L. Bandiera, G. Cavoto *et al.*, Phys. Rev. D **103**(7), 072003 (2021)
- [8] P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020) and 2021 update
- [9] R. Aaij *et al.* (LHCb), JHEP **03**, 043 (2018)
- [10] M. Ablikim *et al.* (BESIII), Phys. Rev. D **100**, 072004 (2019)
- [11] D. Marangotto, Amplitude analysis and polarisation measurement of the Λ_c^+ baryon in $pK^-\pi^+$ final state for electromagnetic dipole moment experiment, [Ph.D. thesis], Universit degli studi di Milano, 2020, <https://cds.cern.ch/record/2713231>
- [12] M. Jezabek, K. Rybicki, and R. Rylko, Phys. Lett. B **286**, 175 (1992)
- [13] M. Ablikim *et al.* (BESIII), Phys. Rev. D **103**, L091101 (2021)
- [14] M. Ablikim *et al.* (BESIII), Phys. Rev. Lett. **116**, 052001 (2016)
- [15] S. B. Yang *et al.* (Belle), Phys. Rev. Lett. **117**, 011801 (2016)
- [16] E. M. Aitala *et al.* (E791), Phys. Lett. B **471**, 449-459 (2000)
- [17] D. Marangotto, Adv. High Energy Phys. **2020**, 7463073 (2020)
- [18] M. Mikhasenko *et al.* (JPAC), Phys. Rev. D **101**, 034033 (2020)
- [19] M. Ablikim, M. N. Achasov, P. Adlarson *et al.* (BESIII Collaboration), Phys. Rev. Lett. **120**, 132001 (2018)
- [20] S. U. Chung, SPIN FORMALISMS, CERN Yellow Reports: Monographs, doi: 10.5170/CERN-1971-008
- [21] S. M. Berman and M. Jacobi, Phys. Rev. **139**, B1023 (1965)