Research on α-decay for the superheavy nuclei with $Z = 118$–120*

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Abstract: The generalized liquid-drop model (GLDM) with the microscopic shell correction from relativistic Hartree-Fock (RHF) calculations is used to explore the α-decay of superheavy nuclei. The known nuclei with $Z = 106-118$ are chosen as examples for testing. The calculated half-lives of α-decay agree with the experimental data better than those from the GLDM with the shell correction in the Weizsäcker-Skyrme model. Moreover, the influence of the decay energy Q_α on α-decay is investigated. It is determined that the Q_α values obtained from the WS4 model with radial basis function (RBF) correction match the experimental data optimally. Owing to these advantages, the GLDM with the RHF shell correction and WS4+RBF Q_α values is adopted to predict the α-decay lifetime for the unknown superheavy nuclei with $Z = 118-120$. The trend of the available α-decay half-lives according to the neutron number is similar to the trends of the values from the GLDM calculation without shell correction as well as the universal decay law (UDL) formula. Comparably, the RHF shell correction depresses (raises) the α-decay lifetime for most nuclei with $N < 186$ ($N > 186$). In comparison with the half-lives of spontaneous fission, it can be concluded that the α-decay is dominant in the superheavy nuclei $^{281-304}118$, $^{284-306}119$, and $^{287-308}120$. These results are beneficial to the exploration of superheavy nuclei in experiments.

Keywords: generalized liquid-drop model, α-decay, superheavy nuclei, relativistic Hartree-Fock, WS4 model

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I. INTRODUCTION

Since physicists predicted that there exists a relatively stable region centered on the proton number $Z = 114$ and neutron number $N = 184$, called the island of stability [1, 2], superheavy nuclei have become one of the most prominent topics in nuclear physics. Several major scientific devices were involved in the synthesis of superheavy nuclei. In the Russian Dubna laboratory, physicists successfully carried out complete fusion reactions with the double magic nucleus ^{48}Ca beam and various actinide targets and also synthesized novel nuclides with $Z = 112-118$ [3-7]. To synthesize the nuclides beyond $Z = 118$, these laboratories, such as GSI in Germany [8-10], Dubna in Russia [11], and RIKEN in Japan [12], have continuously conducted experimental studies on the synthesis of elements $Z = 119$ and $Z = 120$ using thermal fusion reactions. Unfortunately, the novel nuclides exceeding $Z = 118$ are yet to be experimentally synthesized.

Owing to the experimental difficulties in synthesizing superheavy nuclei, theoretical studies on superheavy nuclei are necessary, which can provide a guide for experiments. Stability and decay are the two essential attributes of superheavy nuclei. α-decay is the main de-excitation mode of superheavy nuclei. From the α decay chains, physicists can identify superheavy nuclides [13]. The theoretical studies on α-decay can provide knowledge on the stability of superheavy nuclei and α-decay chains [14-16]. To study the α-decay of superheavy nuclei, physicists have developed several theoretical models, which include the generalized liquid drop model (GLDM) [17-19], density-dependent cluster model [20-22], Coulomb and proximity potential model [23-25], and fission-like model [26]. To better describe the α-decay using these models, physicists have proposed different interaction potentials between the α-particle and daughter nucleus, which include the proximity potential [27], Wood-Saxon potential [28], and the potential from the density functional theory [29]. For simplicity, physicists have also proposed several empirical formulas to describe α decay, such as the universal decay law (UDL) formula [30], Royer's formula [31], and Viola-Seaborg formula [32]. These formulas can satisfactorily reproduce the experi-

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mental half-lives of α decay.

Spontaneous fission is another important de-excitation method from superheavy nuclei to stable nuclei [33-35]. Because α-decay and spontaneous fission compete with each other in the process of superheavy nuclei de-excitation to stable nuclei, the study of spontaneous fission is also necessary. Compared with α-decay, the spontaneous fission of superheavy nuclei is more complicated and the theoretical description of spontaneous fission is very difficult. Consequently, Swiatecki proposed a semi-empirical formula for the half-lives of spontaneous fission [36]. Recently, based on richer experimental data, physicists have developed several novel empirical formulas for spontaneous fission half-lives [37-40].

In the α-decay and spontaneous fission processes of superheavy nuclei, the shell effect plays an important role [41]. As we know, more stable superheavy nuclei may exist near magic numbers. To consider the shell effect, Strutinsky proposed a method to quantitatively calculate the shell correction from single-particle levels [42]. The method has been widely applied to extract the shell correction based on different models [43, 44]. Recently, the shell correction is introduced in the generalized liquid-drop model, and it improves the description of the α-decay of superheavy nuclei [45, 46].

Although the introduction of the shell corrections can improve the theoretical description for the α-decay of superheavy nuclei, there are systematic deviations between the calculated half-lives and the experimental data, especially near the closed-shell $N = 126$. The novel relativistic Hartree-Fock (RHF) theory can describe self-consistently stable and exotic nuclei, and it is beneficial to evaluate the shell correction [43].

In addition to the shell corrections, the decay energy Q_α is another important parameter related to α-decay. The α-decay half-lives of superheavy nuclei are sensitive to the Q_α value. Several phenomenological and microscopic models are employed to fit the mass of the nuclei and extract the Q_α value. Comparably, the Weizsäcker-Skyrme-4 model (WS4) with the radial basis function (RBF) correction [47] is the model with the highest accuracy in describing nuclear mass.

Based on these considerations, we apply the RHF theory to calculate the single-particle levels and extract the shell correction. We adopt the Q_α values from the WS4 model with the RBF correction. Combining the RHF shell correction and the WS4+RBF Q_α values with the GLDM, we calculate the α-decay lifetimes for the known nuclei with $Z = 106-118$. In comparison with the experimental data, an excellent agreement is obtained. Owing to these advantages, we apply the GLDM with the RHF shell correction and WS4+RBF Q_α values to predict the α-decay lifetimes for the unknown superheavy nuclei with $Z = 118-120$. To determine the dominant decay mode from superheavy to stable nuclei, we also calculated the spontaneous fission lifetime for these superheavy nuclei using two empirical formulas. The related results are presented in the following sections.

The remainder of this paper is organized as follows. The theoretical framework is introduced in Sec. II. The numerical results and discussions are presented in Sec. III. Then, a summary is given in Sec. IV.

II. THEORETICAL FRAMEWORK

A. *α*-decay

To study the α-decay of superheavy nuclei with the GLDM model, we sketch the theoretical formalism. In the GLDM model, the potential barrier is a function of the distance between the center of mass of two fragments; in addition, the penetration probability is calculated using the WKB approximation [18]. The decay constant λ is defined as

$$\lambda = P_\alpha \nu_0 P, \tag{1}$$

where P_α denotes the preformation factor. Owing to the lack of experimental data for superheavy nuclei, the preformation amplitude is considered the same as that of heavy nuclei. Based on Ref. [48], its value is fixed to $P_\alpha = 0.39$ for even-even nuclei, $P_\alpha = 0.25$ for odd-A nuclei, and $P_\alpha = 0.15$ for odd-odd nuclei via the comparison of theoretical results with experimental data. ν_0 represents the assault frequency of the α-particle in the parent nucleus, and can be calculated by the formula [49]

$$\nu_0 = \frac{1}{2R}\sqrt{\frac{2E_\alpha}{M}}, \tag{2}$$

with the radius of the parent nucleus denoted by R, α-particle energy corrected by recoil E_α, and mass of the α-particle denoted as M. P is the penetration probability and is calculated with the WKB approximation

$$P = \exp\left\{-\frac{2}{\hbar}\int_{r_{\rm in}}^{r_{\rm out}} \sqrt{2\mu[E(r)-Q_\alpha]}\mathrm{d}r\right\},$$

where $E(R_{\rm in}) = E(R_{\rm out}) = Q_\alpha$. μ, Q_α , and $E(r)$ denote the reduced mass, decay energy of the α -particle, and penetrating potential barrier of the α-particle, respectively. In the GLDM, $E(r)$ consists of five parts

$$E(r) = E_{\rm V} + E_{\rm S} + E_{\rm C} + E_{\rm prox} + E_{\rm mic}, \tag{3}$$

where $E_{\rm V}$, $E_{\rm S}$, $E_{\rm C}$, and $E_{\rm prox}$ represent the volume, surface, Coulomb, and proximity energies, respectively. More details can be found in Ref. [19]. $E_{\rm mic}$ represents the microcosmic corrections of α-decay, which mainly include the shell and pairing energy corrections. In the α-decay process of superheavy nuclei, the pairing energy is shape-dependent, and the detailed calculation can be found in Refs. [50, 51].

The shell correction is expressed as [52]:

$$E_{\rm shell} = E_{\rm shell}^{\rm sphere}(1-2.6\alpha^2){\rm e}^{-\alpha^2}, \quad (4)$$

where $\alpha^2 = (\delta\bar{R})^2/a^2$ represents the deviation of the deformed nucleus from the spherical nucleus. $a = 0.32r_0$ where r_0 denotes the nucleon radius, which is set to 1.2 fm, according to the droplet model of atomic nuclei [52]. $E_{\rm shell}^{\rm sphere}$ represents the shell correction energy of the spherical nucleus, and is calculated using the Strutinsky shell correction method [42]

$$E_{\rm shell}^{\rm sphere} = \sum_{i=1}^{N,Z}\varepsilon_i - \int_{-\infty}^{\tilde{\lambda}}\varepsilon\tilde{g}(\varepsilon){\rm d}\varepsilon, \quad (5)$$

where ε_i represent the energies of single-particle levels, $\tilde{\lambda}$ denotes the Fermi energy related to smoothed distribution, and $\tilde{g}(\varepsilon)$ is the density of levels. In Refs. [44, 46], the single-particle levels are obtained by solving the Schrö dinger equation with an axially deformed Woods-Saxon potential. In this research, the shell correction is extracted from the microscopic single-particle levels in the self-consistent RHF calculations [43]. More details on this can be found in Refs. [53-55]. With the available decay constant λ, the half-live is obtained using $T_{1/2} = \ln 2/\lambda$.

For comparison, we have also calculated the half-lives of the α-decay using the universal decay law (UDL) formula developed in Ref. [30]. The specific expression of UDL formula is

$$\log_{10}T_{1/2}({\rm s}) = aZ_1Z_2\sqrt{\frac{A_1A_2}{(A_1+A_2)Q_\alpha}} + b\sqrt{\frac{A_1A_2}{(A_1+A_2)}Z_1Z_2\left(A_1^{1/3}+A_2^{1/3}\right)} + c, \quad (6)$$

where the values of the parameters a, b, and c are determined by fitting experimental data.

B. spontaneous fission

Similar to the α-decay, the spontaneous fission is an essential mode from superheavy nuclei to stable nuclei. Owing to the complexity of spontaneous fission, the calculation of the spontaneous fission's lifetime is considerably difficult. Hence, physicists have proposed several semi-empirical formulas based on nuclear structure. In this study, we choose the two commonly empirical formulas to describe the spontaneous fission half-lives [37, 38], which can optimally reproduce the experimental data.

By considering the strong interaction, Coulomb interaction, and isospin effect, Xu *et al.* introduced a semi-empirical formula, calculating the half-lives of spontaneous fission for heavy and superheavy nuclei [38], which is expressed as

$$T_{1/2}({\rm s}) = \frac{\ln 2}{n\cdot P_{\rm sf}} = \exp\Big[2\pi\Big[c_0 + c_1A + c_2Z^2 + c_3Z^4 + c_4(N-Z)^2 - \Big(0.13323\frac{Z^2}{A^{1/3}} - 11.64\Big)\Big]\Big], \quad (7)$$

where $c_0 = -195.09227$, $c_1 = 3.10156$, $c_2 = -0.04386$, $c_3 = 1.40301\times10^{-6}$, and $c_4 = -0.03199$. n denotes the frequency factor chosen as a constant, and $P_{\rm sf}$ represents the penetration probability calculated by the WKB approximation. For convenience, in the following discussions, this formula is called the Xu formula.

Considering the isospin and shell correction, Santhosh *et al.* developed a generalized Swiatecki formula that calculates the half-life of spontaneous fission [37],

$$\log_{10}T_{1/2}({\rm yr}) = a\frac{Z^2}{A} + b\left(\frac{Z^2}{A}\right)^2 + c\left(\frac{N-Z}{N+Z}\right) + d\left(\frac{N-Z}{N+Z}\right)^2 + eE_{\rm shell} + f, \quad (8)$$

where $a = -43.25203$, $b = 0.49192$, $c = 3674.3927$, $d = -9360.6$, $e = 0.8930$, and $f = 578.56058$. $E_{\rm shell}$ denotes the shell correction energy obtained from the FRDM [56]. For convenience, in the following discussions, this formula is called KPS formula.

III. DISCUSSION

Based on the preceding formalism, we calculate the half-lives of α -decay for the nuclei with $Z = 106-118$ using the GLDM with different shell corrections. The obtained results are presented in Table 1, where the first column labels the parent nuclei happening α-decay, and the second column presents the experimental Q_α values. The fifth, sixth, and seventh columns represent the calculated half-lives of α-decay by the GLDM without shell correction, GLDM with the WS shell correction, and GLDM with the RHF shell correction, respectively. For comparison, the half-lives of α-decay from the UDL formula are presented in the fourth column, and the experimental data are given in the third column.

Table 1 shows that the available half-lives of the α-decay in the four different calculations agree well with the experimental data for the nuclei with $Z = 106-118$.

Table 1. Half-lives of α-decay for the superheavy nuclei from Sg to Og isotopes. The results obtained from the calculations with the UDL, GLDM, and GLDM with the RHF shell correction and GLDM with the WS shell correction are presented in the forth, fifth, sixth, and seventh columns, respectively. The experimental data [45, 57-60] are displayed in the third column. The RMS of relative deviation between the calculated α-decay half-lives and the experimental data are presented in the last row.

Nucleus	Q_α(Exp)/MeV	$\log_{10}T_{1/2}$(Exp)/s	$\log_{10}T_{1/2}$ (UDL)/s	$\log_{10}T_{1/2}$(GLDM)/s	$\log_{10}T_{1/2}$(GLDM+WS)/s	$\log_{10}T_{1/2}$(GLDM+RHF)/s
^{269}Sg	8.54	2.924	2.063	1.396	2.202	2.963
^{271}Sg	8.67	1.982	1.573	0.931	1.611	2.348
^{270}Bh	9.06	1.785	0.683	0.341	1.040	1.881
^{271}Bh	9.42	0.176	−0.486	−0.942	−0.400	0.431
^{272}Bh	9.21	1.025	0.162	−0.160	0.415	1.155
^{274}Bh	8.94	1.643	1.019	0.594	1.143	1.886
^{273}Hs	9.67	−0.292	−0.888	−1.344	−0.911	−0.096
^{275}Hs	9.45	−0.699	−0.244	−0.783	−0.360	0.448
^{274}Mt	10.20	−0.357	−2.063	−2.167	−1.881	−0.989
^{275}Mt	10.48	−1.699	−2.851	−3.082	−2.807	−1.961
^{276}Mt	10.10	−0.284	−1.811	−1.957	−1.682	−0.802
^{278}Mt	9.58	0.653	−0.298	−0.629	−0.402	0.470
^{277}Ds	10.71	−2.456	−3.128	−3.342	−3.234	−2.274
^{279}Ds	9.85	−0.538	−0.743	−1.273	−1.235	−0.222
^{281}Ds	8.85	1.104	2.467	1.609	1.519	2.522
^{278}Rg	10.85	−2.377	−3.146	−3.124	−3.262	−2.160
^{279}Rg	10.53	−1.046	−2.307	−2.644	−2.792	−1.651
^{280}Rg	9.91	0.623	−0.550	−0.887	−1.018	0.069
^{281}Rg	9.41	1.230	0.989	0.250	0.157	1.148
^{282}Rg	9.16	2.000	1.799	1.205	1.129	1.994
^{281}Cn	10.45	−0.745	−1.745	−2.173	−2.301	−1.367
^{283}Cn	9.66	0.623	0.560	−0.160	−0.223	0.443
^{285}Cn	9.32	1.447	1.625	0.789	0.822	1.229
^{282}Nh	10.78	−1.137	−2.294	−2.428	−2.533	−1.748
^{283}Nh	10.38	−1.125	−1.205	−1.724	−1.799	−1.136
^{284}Nh	10.12	−0.013	−0.467	−0.858	−0.902	−0.392
^{285}Nh	10.01	0.623	−0.155	−0.814	−0.824	−0.516
^{286}Nh	9.79	0.978	0.500	−0.008	0.032	0.251
^{285}Fl	10.56	−1.000	−1.370	−1.879	−1.882	−1.587
^{286}Fl	10.35	−0.921	−0.789	−1.573	−1.545	−1.476
^{287}Fl	10.17	−0.319	−0.280	−0.945	−0.875	−0.950
^{288}Fl	10.07	−0.180	0.003	−0.904	−0.775	−1.019
^{289}Fl	9.98	0.279	0.259	−0.496	−0.292	−0.592
^{287}Mc	10.76	−1.432	−1.585	−2.091	−2.114	−2.152
^{288}Mc	10.65	−0.759	−1.297	−1.624	−1.635	−1.753
^{289}Mc	10.49	−0.481	−0.862	−1.477	−1.438	−1.600
^{290}Mc	10.41	−0.187	−0.648	−1.076	−0.996	−1.186
^{290}Lv	11.00	−2.081	−1.917	−2.596	−2.614	−2.722
^{291}Lv	10.89	−1.721	−1.637	−2.166	−2.154	−2.277
^{292}Lv	10.78	−1.886	−1.351	−2.126	−2.057	−2.205
^{293}Lv	10.71	−1.244	−1.173	−1.795	−1.683	−1.839
^{293}Ts	11.32	−1.658	−2.448	−2.880	−2.876	−2.971
^{294}Ts	11.18	−1.292	−2.098	−2.367	−2.317	−2.419
^{294}Og	11.82	−3.237	−3.370	−3.859	−3.869	−3.941
σ			0.76	1.03	0.92	0.66

Compared with the UDL formula, the GLDM calculations agree with the experimental data better, especially those with the RHF shell correction. To quantitatively evaluate the accuracy of different models, we calculate the relative deviation between the theoretical α-decay half-lives and experimental data with the formula

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left[\log_{10}\left(T_{1/2}^{\exp,i}/T_{1/2}^{\mathrm{cal},i}\right)\right]^2}, \tag{9}$$

where σ represents the root mean square (RMS) of relative deviation. The σ value of the GLDM calculation without shell correction is 1.03, while those with the WS and RHF shell corrections are 0.92 and 0.66, respectively. This indicates that the GLDM represents a good description of the α-decay of superheavy nuclei. The experimental half-lives of α-decay are reproduced well in the GLDM calculations with the shell corrections, especially with the RHF shell correction. Namely, the GLDM with the RHF shell correction is more precise in describing the α-decay of superheavy nuclei.

To understand why the shell correction plays an important role in the α-decay of superheavy nuclei, we have plotted the potential barriers of α-decay in the GLDM with different shell corrections. In Fig. 1, the dashed line represents the α-decay barrier in the GLDM without shell correction; the dashed-dotted and solid lines are the α-decay barriers with the WS and RHF shell corrections, respectively. Compared with the results without shell correction, significant changes in the potential barriers in the GLDM with the WS and RHF shell corrections are in the range of $r < 10$ fm. In particular, with the RHF shell corrections, the change in the potential barrier is remarkable, which significantly influences the lifetime of α-decay. For the α-decay of ^{212}Po, the shell correction ensures that the potential of α-decay exhibits a potential barrier in the vicinity $r = 0$. Compared with the WS shell correction, the potential barrier modified by the RHF shell correction is more significant, which improves the prediction of the GLDM model on the half-lives of α-decay in comparison with the experimental data. Similarly, for the α-decay of ^{265}Rf, there appears a potential well near $r = 0$ after considering the shell correction. Compared with the WS shell correction, the potential changed by the RHF shell correction is more remarkable, which results in the calculated half lives of α-decay exhibiting a better agreement with the experimental data. These indicate that the shell correction changes the α-decay potential and improves the theoretical prediction of the α-decay lifetimes. The RHF shell correction changes the α-decay potential more significantly and enables the theoretical prediction to be superior to that of the WS shell correction.

To clarify the advantages of the RHF shell correction, we compare the α-decay half-lives with the different shell

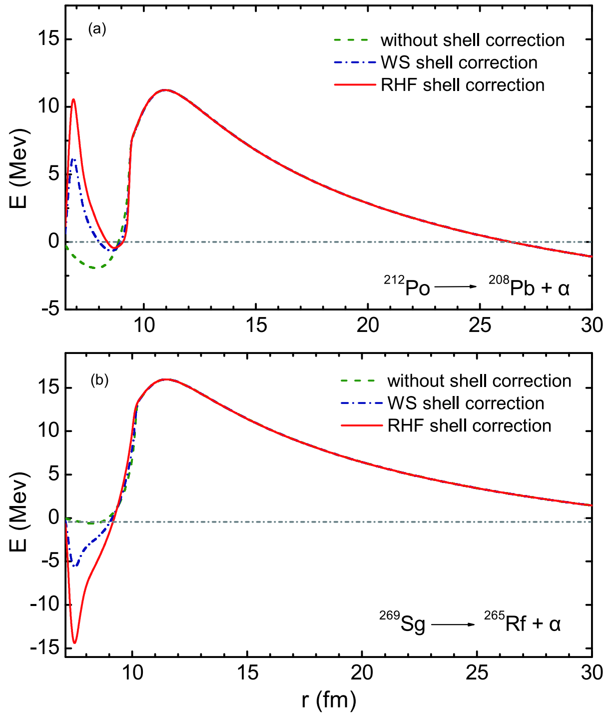

Fig. 1. (color online) Potential barriers of α-decay in the GLDM without shell correction, with WS shell correction, and with RHF shell correction. The results for ^{212}Po and ^{269}Sg are displayed in Fig. 1(a) and (b), respectively.

corrections. The differences between the calculated α-decay half-lives and the experiment data are plotted in Fig. 2, where the results obtained from the GLDM calculations with the WS and RHF shell corrections are presented in Fig. 2 (a) and (b) for the isotopes from Sg to Og. For the GLDM with the RHF shell correction, the differences between the calculated half-lives and experimental data lie in the interval (−0.5, 0.5). For those with the WS shell correction, the differences lie in the interval (−1.5, −0.5). This indicates that the half-lives of α-decay in the GLDM calculations with the RHF shell correction agree with the experimental data better than that with the WS shell correction. Because the RHF is a self-consistent microscopic theory and has gained considerable success in describing stable and exotic nuclei, it is more appropriate to extract the shell correction from the RHF in predicting the half-lives of α-decay using the GLDM.

The half-lives of α-decay are sensitive to the decay energy Q_α. In the recently presented WS4 model [47], by taking into account the surface diffusion correction of unstable nuclei and the radial basis function correction, the RMS deviation between the calculated mass and the experimental data for these 2353 known nuclei decreases to 237 keV. Considering that the WS4+RBF model accurately predicts nuclear mass, we extract the Q_α value from this model. With the Q_α value, we calculate the α-decay half-lives for these 45 isotopes from Sg-Og with the

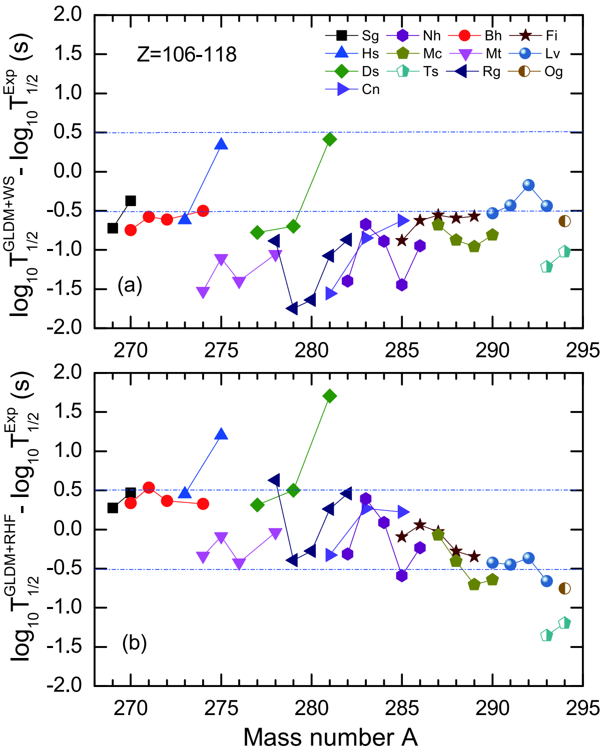

Fig. 2. (color online) Differences between the calculated half-lives of α-decay and the experimental data for the nuclei from Sg to Og. The GLDM calculations with the WS and the RHF shell corrections are presented in the upper and lower panels, respectively.

GLDM. To clearly present the calculated results, these 45 isotopes are divided into four groups: even-even, even-odd , odd-even, and odd-odd nuclei. For comparison, the α-decay half-lives are also calculated by the GLDM with the Q_α extracted from the FRDM model. The differences between the calculated half-lives and the experimental data are presented in Fig. 3. Compared with the FRDM model, the deviations between the GLDM calculation with the WS4+RBF model and the experiment data are smaller for even-even, even-odd, odd-even, and odd-odd nuclei. Hence, in the following GLDM calculations, Q_α is extracted from the WS4+RBF model.

Previous studies demonstrate that the GLDM with the RHF shell correction is appropriate in describing the α-decay of superheavy nuclei, and the Q_α value extracted by the WS4+RBF calculations is remarkably consistent with the available data. Hence, it is fascinating to predict the half-lives of α-decay for the unknown superheavy nuclei using the GLDM with the RHF shell correction and the WS4+RBF Q_α values, which is significant in guiding the experimental detection of superheavy nuclei. In the following, we perform the GLDM calculations with the RHF shell correction and the WS4+RBF Q_α value for the superheavy nuclei with $Z = 118-120$. The calculated α-decay half-lives are presented in Fig. 4,

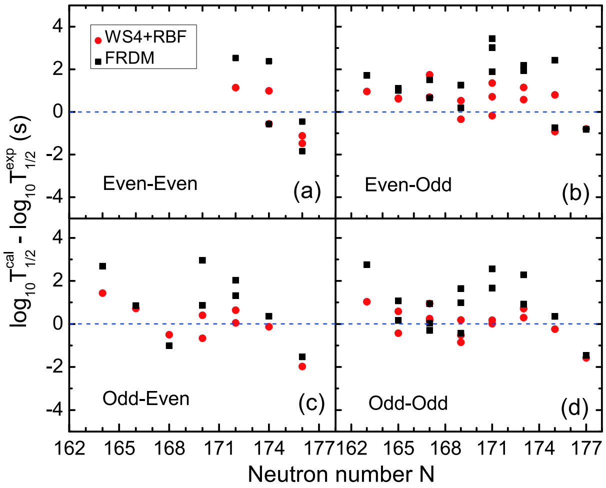

Fig. 3. (color online) Differences between the calculated half-lives of α-decay and the experimental data as a function of the neutron number for these nuclei from Sg to Og isotopes. The results for even-even, even-odd, odd-even, and odd-odd nuclei are presented in Fig. 3(a), (b), (c), and (d), respectively.

where the black circles and red hexagons represent the GLDM without and with the RHF shell correction, respectively. For comparison, the calculated half-lives of α-decay with the UDL formula are displayed as gray pentagons. In addition to the same trend of α-decay half-lives with the neutron number, the available half-lives of α-decay are comparable in the three calculations for the isotopes with $Z = 118-120$. A minimum value emerges at $N = 186$, which implies the important role of magic number in the α-decay. In addition, a sub-minimum vlaue appears at $N = 180$, which may imply a sub-magic number in these superheavy nuclei. Although the available α-decay half-lives are comparable, the impact of the shell correction on the half-lives of α-decay is not negligible in the GLDM calculations. In general, the RHF shell correction depresses the α-decay lifetimes for most of nuclei with $N < 186$ and increases the α-decay lifetimes for most of the nuclei with $N > 186$ in the GLDM calculations.

Because the results from the GLDM-RHF calculations for the known nuclei are more consistent with the experiment data, as presented in Fig. 2, the GLDM-RHF prediction on the α-decay half-life for these unknown superheavy nuclei should be more reliable. Although the deviations of the α-decay half-life in the calculations with and without shell corrections are quite small, considering that the more reliable the theoretical prediction, the more helpful it is in guiding the experimental detection of superheavy nuclei, the present improvement from the GLDM-RHF calculations is beneficial in exploring superheavy nuclei in experiments.

In addition to the α-decay, the spontaneous fission is one of the most important modes from superheavy to stable nuclei. Considering that the spontaneous fission and α-decay compete with each other, we have also cal-

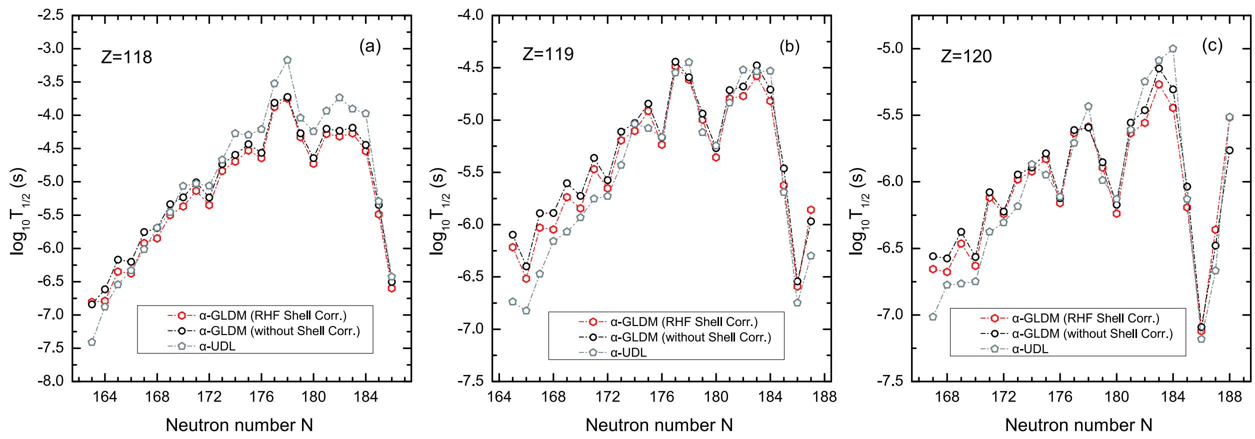

Fig. 4. (color online) Calculated α-decay half-lives for the isotopes with $Z = 118, 119, 120$. The red hexagons, black circles, and gray pentagons represent the results from the GLDM with the RHF shell correction, the GLDM without shell correction, and the UDL model, respectively. The α-decay half-lives for the isotopes with $Z = 118$, $Z = 119$, and $Z = 120$ are presented in Fig. 4(a), (b), and (c), respectively.

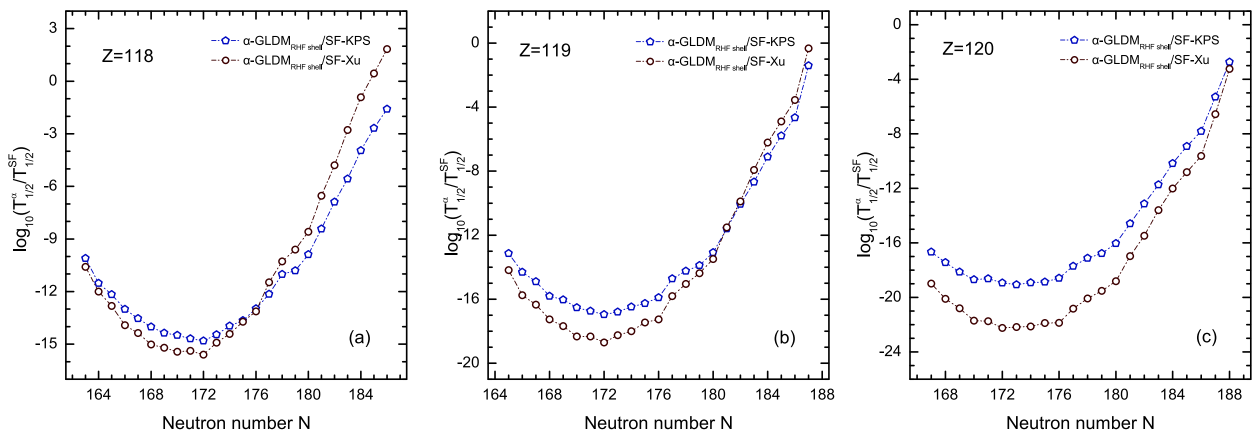

Fig. 5. (color online) Ratios of the α-decay lifetime to the spontaneous fission lifetime for the superheavy nuclei with $Z = 118, 119, 120$. The α-decay half-lives are calculated by the GLDM with RHF shell correction. The half-lives of spontaneous fission are calculated by the KPS and Xu formulas. The results for the superheavy nuclei with $Z = 118$, $Z = 119$, and $Z = 120$ are presented in Fig. 5(a), (b), and (c), respectively.

culated the half-lives of spontaneous fission by using the two commonly adopted empirical formulas: KPS and XU formulas. To save space, we have listed all the calculated half-lives of α-decay and spontaneous fission in the appendix A for the reader's convenience.

To ascertain the dominant process, we have calculated the ratios of the α-decay lifetimes to the spontaneous fission lifetimes for the superheavy nuclei with $Z = 118, 119, 120$, as plotted in Fig. 5. It can be observed that the ratios intially decrease up to the minimum value at approximately $N = 176$ and then increase with the neutron number. The ratios of the α-decay lifetime to the spontaneous fission lifetimes calculated with KPS formula significantly differ from those with the XU formula. These differences may originate from the shell correction because the shell correction has been considered in the KPS formula but not in the XU formula. This indicates that the shell exerts an important influence on the α-decay and spontaneous fission of superheavy nuclei. In addition, we have also observed the ratios of the α-decay lifetimes to the spontaneous fission lifetimes $\log_{10}(T_\alpha/T_{\rm sf})$ to be less than zero, with a few exceptions for the superheavy nuclei with $Z = 118, 119, 120$, which indicates that the α-decay is the main de-excitation mode from the superheavy to stable nuclei for the superheavy nuclei $^{281-304}118$, $^{284-306}119$, and $^{287-308}120$.

IV. SUMMARY

The generalized liquid-drop model with the microscopic shell correction from the relativistic Hartree-Fock calculations was adopted to explore the α-decay of superheavy nuclei. The known nuclei with $Z = 106 - 118$ were chosen as examples for testing. The calculated half-lives of α-decay agreed well with the experimental data. Compared with the UDL formula, GLDM calculations without shell correction, and GLDM calculations with WS shell correction, the GLDM with the RHF shell correction exhibited the best description for the α-decay of superheavy nuclei.

The shell corrections play an important role because the α -decay is clarified. Compared with the calculations without shell correction, the α-decay potential was significantly modified by shell corrections, especially for the RHF shell correction. For the α-decay of ^{212}Po (^{265}Rf), the shell correction ensured that the α -decay potential exhbited a barrier (well) in the vicinity $r = 0$. Compared with the WS shell correction, the potential barrier (well) modified by the RHF shell correction was sharper, which resulted in the theoretical prediction of the GLDM+RHF being superior to that of the WS shell correction.

In the GLDM calculations with the RHF shell correction, the available half-lives agreed with the experimental data better than those with the WS shell correction. The differences between the calculated half-lives of α -decay with the RHF shell correction and the experimental data for the isotopes from Sg to Og were evident in the interval (−0.5, 0.5), while those with the WS shell correction were evident in the interval (−1.5, −0.5). This indicates that the RHF shell correction is superior to the WS shell correction in describing the α-decay of superheavy nuclei.

The influence of the Q_α value on the α-decay was analyzed. It was determined that the Q_α values from the WS4 model with the radial basis function correction matched the experimental data better than the values from other models.

Combining these advantages of the RHF shell correction and the WS4+RBF, the GLDM+RHF with the WS4+RBF Q_α values was adopted to predict the α -decay lifetime for the unknown superheavy nuclei with $Z = 118 - 120$. The available half lives of α-decay according to the neutron number exhibited the same trend in comparison with those from the GLDM without shell correction and the UDL formula. A minimum value appeared at $N = 186$, which implied the important role of magic number in the α-decay, including a sub-minimum value at $N = 180$, which could be a sub-magic number in the superheavy nuclei. Compared with the GLDM calculations without shell correction, the RHF shell correction depressed the α-decay lifetime for most nuclei with $N < 186$ and increased the α-decay lifetime for most nuclei with $N > 186$. Compared with the half-lives of spontaneous fission, which are calculated with the two commonly used empirical formulas, it was determined that the α-decay is dominant in the superheavy nuclei $^{281-304}$118, $^{284-306}$119, and $^{287-308}$120. These results are beneficial in guiding the experimental detection of superheavy nuclei.

APPENDIX A

In this section, all the calculated α-decay half-lives are presented in Table A1, where the α-decay half-lives from the GLDM with the RHF shell correction are listed in the third column. For comparison, the α-decay half-lives calculated with the UDL formula are presented in the fourth column. The half-lives of spontaneous fission calculated with the KPS and Xu formulas are presented in the fifth and sixth columns of Table A1, respectively.

Table A1. Calculated half-lives of α-decay and spontaneous fission for the superheavy nuclei with Z = 118–120. The half-lives of α-decay are calculated by the GLDM with the RHF shell correction and the UDL formula. The half-lives of spontaneous fission are calculated by the two commonly used empirical formulas. The Q_α of α-decay are obtained from the WS4 model with the RBF corrections.

A	Q_α/MeV	$\log_{10}T^{\alpha}_{1/2}$/s		$\log_{10}T^{\rm sf}_{1/2}$/s	
	WS4+RBF	GLDM+RHF	UDL	KPS	Xu
		Z=118			
281	13.77	−6.805	−7.411	3.294	3.782
282	13.49	−6.788	−6.877	4.739	5.216
283	13.32	−6.348	−6.543	5.819	6.472
284	13.21	−6.376	−6.333	6.636	7.548
285	13.05	−5.921	−6.015	7.612	8.446
286	12.89	−5.850	−5.691	8.161	9.165
287	12.77	−5.497	−5.457	8.866	9.705
288	12.59	−5.368	−5.064	9.122	10.067
289	12.56	−5.135	−5.026	9.557	10.250
290	12.57	−5.349	−5.062	9.468	10.254

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Table A1-continued from the previous page

A	Q_α/MeV	$\log_{10} T^{\alpha}_{1/2}$/s		$\log_{10} T^{\rm sf}_{1/2}$/s	
	WS4+RBF	GLDM+RHF	UDL	KPS	Xu
291	12.39	−4.835	−4.671	9.625	10.079
292	12.21	−4.694	−4.274	9.273	9.726
293	12.21	−4.528	−4.295	9.118	9.194
294	12.17	−4.643	−4.210	8.333	8.484
295	11.88	−3.875	−3.523	8.269	7.596
296	11.73	−3.749	−3.171	7.266	6.528
297	12.08	−4.327	−4.039	6.480	5.283
298	12.16	−4.728	−4.243	5.161	3.859
299	12.02	−4.280	−3.935	4.143	2.256
300	11.93	−4.316	−3.737	2.570	0.476
301	12.00	−4.271	−3.904	1.302	−1.483
302	12.02	−4.538	−3.975	−0.587	−3.621
303	12.58	−5.489	−5.293	−2.812	−5.936
304	13.10	−6.598	−6.428	−5.011	−8.430
305	12.89	−5.856	−5.987	−7.621	−11.102
306	12.46	−5.176	−5.061	−10.006	−13.952
307	11.90	−3.770	−3.778	−12.244	−16.981
308	11.18	−2.285	−1.967	−14.478	−20.187
309	10.70	−0.842	−0.652	−16.552	−23.572
310	10.41	−0.250	0.162	−19.019	−27.135
311	10.08	0.892	1.150	−21.208	−30.875
312	9.74	1.699	2.221	−23.804	−34.794
313	8.63	5.603	6.180	−26.146	−38.891
314	8.37	6.379	7.236	−28.820	−43.165
315	8.28	6.870	7.570	−28.066	−47.618
316	8.60	5.390	6.257	−30.687	−52.249
317	8.53	5.953	6.536	−33.048	−57.057
318	8.40	6.333	7.054	−38.137	−62.043
319	8.28	7.032	7.523	−40.641	−67.207
320	9.15	3.693	4.142	−43.424	−72.549
321	9.00	4.390	4.659	−46.226	−78.069
322	8.89	4.588	5.075	−49.404	−83.767
323	8.71	5.373	5.720	−53.598	−89.642
324	8.47	6.036	6.666	−56.699	−95.695
325	8.24	7.061	7.611	−59.663	−101.926
326	8.12	7.264	8.095	−62.890	−108.334
327	7.91	8.234	8.979	−65.145	−114.920
328	7.60	9.349	10.425	−68.661	−121.684
329	7.31	10.790	11.803	−71.633	−128.625
330	7.15	11.301	12.618	−74.978	−135.744

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Table A1-continued from the previous page

A	Q_α/MeV	$\log_{10} T^{\alpha}_{1/2}$/s		$\log_{10} T^{sf}_{1/2}$/s	
	WS4+RBF	GLDM+RHF	UDL	KPS	Xu
331	6.94	12.547	13.732	−77.125	−143.041
332	6.76	13.572	14.717	−80.670	−150.515
333	6.77	13.891	14.652	−84.032	−158.166
334	6.57	14.966	15.764	−87.702	−165.996
335	6.26	17.119	17.655	−91.213	−174.002
336	6.07	18.202	18.875	−94.690	−182.186
337	5.72	20.879	21.295	−97.846	−190.548
338	5.35	23.553	24.109	−101.251	−199.087
339	5.25	24.643	24.927	−104.503	−207.803
		Z=119			
284	13.56	−6.215	−6.738	6.928	7.969
285	13.60	−6.516	−6.825	7.797	9.231
286	13.42	−6.028	−6.472	8.870	10.315
287	13.26	−6.047	−6.159	9.764	11.219
288	13.20	−5.737	−6.069	10.305	11.945
289	13.13	−5.844	−5.932	10.682	12.492
290	13.04	−5.469	−5.753	11.273	12.860
291	13.02	−5.653	−5.729	11.304	13.050
292	12.87	−5.194	−5.431	11.608	13.061
293	12.69	−5.105	−5.036	11.375	12.893
294	12.70	−4.913	−5.079	11.356	12.547
295	12.73	−5.236	−5.166	10.662	12.022
296	12.45	−4.488	−4.549	10.232	11.319
297	12.40	−4.613	−4.448	9.632	10.437
298	12.69	−4.997	−5.118	8.889	9.376
299	12.74	−5.357	−5.247	7.721	8.138
300	12.55	−4.794	−4.834	6.800	6.720
301	12.40	−4.771	−4.520	5.287	5.125
302	12.40	−4.579	−4.536	4.097	3.351
303	12.39	−4.818	−4.531	2.292	1.399
304	12.91	−5.625	−5.690	0.171	−0.732
305	13.40	−6.591	−6.749	−1.935	−3.041
306	13.18	−5.857	−6.298	−4.452	−5.528
307	12.76	−5.241	−5.412	−6.352	−8.193
308	12.04	−3.526	−3.772	−8.115	−11.037
309	11.35	−2.161	−2.053	−10.249	−14.058
310	10.86	−0.710	−0.748	−12.215	−17.258
311	10.76	−0.624	−0.475	−14.593	−20.636
312	10.56	0.131	0.075	−16.702	−24.192
313	9.37	3.407	3.865	−19.192	−27.926

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Table A1-continued from the previous page

A	Q_α/MeV	$\log_{10} T^{\alpha}_{1/2}$/s		$\log_{10} T^{\mathrm{sf}}_{1/2}$/s	
	WS4+RBF	GLDM+RHF	UDL	KPS	Xu
314	8.97	4.940	5.281	−21.446	−31.838
315	8.65	5.861	6.504	−24.069	−35.928
316	8.67	5.978	6.419	−26.488	−40.196
317	9.20	3.807	4.394	−29.167	−44.642
318	9.13	4.375	4.661	−31.728	−49.266
319	8.98	4.746	5.202	−34.545	−54.068
320	8.83	5.545	5.747	−36.947	−59.047
321	8.68	5.901	6.317	−39.861	−64.205
322	8.50	6.780	6.988	−42.509	−69.540
323	7.93	8.866	9.406	−45.533	−75.053
324	8.60	6.362	6.577	−48.306	−80.744
325	8.77	5.409	5.877	−51.416	−86.613
326	8.79	5.523	5.820	−54.246	−92.659
327	8.47	6.390	7.045	−57.482	−98.884
328	8.23	7.458	8.024	−60.588	−105.285
329	7.87	8.650	9.604	−63.231	−111.865
330	7.67	9.668	10.540	−66.232	−118.622
331	7.65	9.382	10.582	−68.196	−125.557
332	7.57	10.062	10.964	−71.338	−132.669
333	7.44	10.720	11.594	−74.825	−139.959
334	7.29	11.864	12.339	−78.085	−147.427
335	7.07	12.908	13.451	−81.653	−155.072
336	7.10	13.047	13.310	−85.010	−162.894
337	6.34	17.211	17.656	−88.645	−170.894
338	6.23	18.210	18.368	−91.853	−179.072
339	5.89	20.325	20.645	−95.186	−187.427
		Z=120			
287	13.84	−6.655	−7.016	10.012	12.336
288	13.71	−6.678	−6.775	10.766	13.426
289	13.69	−6.464	−6.765	11.675	14.337
290	13.68	−6.630	−6.750	12.063	15.070
291	13.48	−6.119	−6.376	12.504	15.624
292	13.44	−6.241	−6.306	12.687	15.999
293	13.37	−5.981	−6.184	13.080	16.195
294	13.22	−5.922	−5.869	12.997	16.213
295	13.25	−5.828	−5.948	13.040	16.052
296	13.32	−6.160	−6.112	12.425	15.713
297	13.12	−5.632	−5.709	12.073	15.195
298	12.98	−5.585	−5.434	11.532	14.498
299	13.23	−5.893	−5.989	10.884	13.623

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Table A1-continued from the previous page

A	Q_α/MeV	$\log_{10} T^{\alpha}_{1/2}$/s		$\log_{10} T^{\rm sf}_{1/2}$/s	
	WS4+RBF	GLDM+RHF	UDL	KPS	Xu
300	13.29	−6.238	−6.130	9.801	12.569
301	13.04	−5.635	−5.608	8.947	11.337
302	12.87	−5.557	−5.247	7.572	9.927
303	12.79	−5.268	−5.088	6.465	8.338
304	12.74	−5.444	−5.000	4.734	6.571
305	13.26	−6.192	−6.129	2.722	4.625
306	13.77	−7.121	−7.182	0.689	2.501
307	13.50	−6.360	−6.667	−1.060	0.199
308	12.95	−5.516	−5.513	−2.782	−2.281
309	12.14	−3.678	−3.692	−4.438	−4.940
310	11.48	−2.367	−2.059	−6.484	−7.776
311	11.18	−1.400	−1.270	−8.371	−10.791
312	11.20	−1.612	−1.342	−10.653	−13.984
313	11.01	−0.931	−0.835	−12.649	−17.356
314	10.74	−0.480	−0.099	−15.062	−20.905
315	9.41	3.573	4.088	−17.195	−24.632
316	9.17	4.151	4.933	−19.689	−28.538
317	9.91	1.949	2.373	−22.025	−32.621
318	9.91	1.703	2.365	−24.646	−36.882
319	9.83	2.239	2.627	−27.107	−41.322
320	9.66	2.623	3.168	−29.815	−45.939
321	9.51	3.326	3.661	−32.207	−50.734
322	9.35	3.676	4.208	−35.049	−55.707
323	9.10	4.728	5.096	−37.697	−60.858
324	7.98	8.857	9.653	−40.775	−66.186
325	7.70	10.202	10.916	−43.335	−71.693
326	9.27	3.792	4.424	−46.474	−77.377
327	8.67	6.069	6.701	−49.146	−83.239
328	8.50	6.407	7.346	−52.340	−89.279
329	8.60	6.091	6.919	−55.181	−95.497
330	8.39	6.572	7.763	−58.337	−101.892
331	8.09	7.861	9.044	−59.574	−108.465
332	7.99	7.991	9.478	−62.830	−115.216
333	7.95	8.627	9.648	−65.879	−122.144
334	7.91	8.810	9.806	−69.275	−129.250
335	7.74	9.932	10.564	−72.470	−136.533
336	6.53	16.236	16.994	−75.974	−143.994
337	6.29	17.967	18.475	−79.232	−151.633
338	6.35	17.494	18.124	−82.786	−159.449
339	7.55	11.161	11.441	−86.137	−167.442

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