

Microscopic study of higher-order deformation effects on the ground states of superheavy nuclei around $^{270}\text{Hs}^*$

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Abstract: We investigate the effects of higher-order deformations β_λ ($\lambda = 4, 6, 8$, and 10) on the ground state properties of superheavy nuclei (SHN) near the doubly magic deformed nucleus ^{270}Hs using the multidimensionally-constrained relativistic mean-field (MDC-RMF) model with five effective interactions: PC-PK1, PK1, NL3*, DD-ME2, and PKDD. The doubly magic properties of ^{270}Hs include large energy gaps at $N = 162$ and $Z = 108$ in the single-particle spectra. By investigating the binding energies and single-particle levels of ^{270}Hs in the multidimensional deformation space, we find that, among these higher-order deformations, the deformation β_6 has the greatest impact on the binding energy and influences the shell gaps considerably. Similar conclusions hold for other SHN near ^{270}Hs . Our calculations demonstrate that the deformation β_6 must be considered when studying SHN using MDC-RMF.

Keywords: density functional theory, superheavy nuclei, deformation effects

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I. INTRODUCTION

One of the challenges in modern nuclear physics is exploring the mass and charge limits of atomic nuclei [1–8]. The prediction of the existence of an “island of stability” of superheavy nuclei (SHN) was made in the 1960s [9–14]. Currently, only the elements with $Z \leq 118$ have been synthesized [15–17]. Various predictions of the center of the “island of stability” have been made [9–14, 18–20], but the position of this island is not well established. In contrast to the “island of stability,” the existence of a “shallow” of SHN has been well established theoretically and experimentally. This “shallow” connects the continent of stable nuclei to the “island of stability.” The center of this shallow is predicted to be around $Z = 108$ and $N = 162$ and consists of deformed SHN [21–24]. $^{270}_{108}\text{Hs}_{162}$ is a doubly magic deformed nucleus [25, 26] and offers a prototype for exploring the structure of SHN.

There are currently two kinds of theoretical approaches for studying the structures and properties of SHN, the macroscopic-microscopic method (MMM) and the microscopic method. Generally, the surface of a nucleus is parameterized as [27]

$$R(\theta, \varphi) = R_0 \left[1 + \beta_{00} + \sum_{\lambda=1}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \beta_{\lambda\mu}^* Y_{\lambda\mu}(\theta, \varphi) \right], \quad (1)$$

where $\beta_{\lambda\mu}$ is the deformation parameter and R_0 is the radius of a sphere with the same volume. However, there is an interesting consideration: how large should the dimension of the deformation space be when studying deformed SHN? In 1991, Patyk and Sobiczewski investigated the ground state properties of the heaviest even-even nuclei with proton numbers $Z = 90$ – 114 and neutron numbers $N = 136$ – 168 using the MMM and found that the β_6 degree of freedom is important for binding energies and the formation of deformed shells [23, 28]. Additionally, β_6 has a considerable influence on the moments of inertia [29, 30] and high- K isomers [31, 32]. The microscopic description of the structure of SHN can be achieved using density functional theories, with few studies investigating the influence of β_6 on the binding energy and shell structure of SHN to date.

Covariant density functional theory (CDFT) is one of the most successful self-consistent approaches and has been used to describe ground and excited states of nuclei throughout the nuclear chart [33–41]. To investigate the

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ground state properties, potential energy surfaces (PESs), and fission barriers of heavy nuclei and SHN, multidimensionally-constrained (MDC) CDFTs have been developed [41-44]. MDC-CDFTs have been applied to investigate hypernuclei [45-48], the fission barriers and the PESs of actinide nuclei [42, 43, 49], the ground state properties and PESs of ^{270}Hs [50], the nonaxial octupole Y_{32} correlations in $N = 150$ isotones [51] and Zr isotopes [52], octupole correlations in M χ D of ^{78}Br [53] and Ba isotopes [54], etc. In MDC-CDFTs, reflection and axial symmetry are both broken, and the shape degrees of freedom $\beta_{\lambda\mu}$, where μ is an even number, are self-consistently included, such as β_{20} , β_{22} , β_{30} , β_{32} , β_{40} , β_{42} , and β_{44} . Either the Bardeen-Cooper-Schrieffer (BCS) approach or the Bogoliubov transformation have been implemented to consider the pairing effects. With two different approaches to treat pairing correlations, there are two types of MDC-CDFTs: when using the BCS approach, the MDC relativistic mean-field (RMF) model is used, and for the Bogoliubov transformation, the MDC relativistic Hartree-Bogoliubov (RHB) theory is employed.

In this study, we use the MDC-RMF model to investigate the ground state properties of SHN around the doubly magic deformed nucleus ^{270}Hs and focus on the influence of the higher-order deformations. This paper is organized as follows. The MDC-CDFTs is introduced in Sec. II. In Sec. III, the results and discussions are presented. Finally, we summarize the study in Sec. IV.

II. THEORETICAL FRAMEWORK

In the CDFT, nucleons interact with each other through the exchange of mesons and photons or point-coupling interactions. To obtain accurate saturation properties of nuclear matter, the non-linear coupling terms or the density dependence of the coupling constants are introduced. Subsequently, there are four kinds of covariant density functionals: either meson exchange (ME) or point-coupling (PC) combined with the non-linear (NL) or density dependent (DD) couplings. In this study, both the ME and PC density functionals are used. The main formulae of the MDC-CDFTs can be found in Refs. [41, 43, 50, 52]. For convenience, we only introduce the MDC-RMF with the NL-PC effective interactions briefly.

The NL-PC Lagrangian is

$$\mathcal{L} = \bar{\psi}(\gamma_\mu \partial^\mu - M)\psi - \mathcal{L}_{\text{lin}} - \mathcal{L}_{\text{nl}} - \mathcal{L}_{\text{der}} - \mathcal{L}_{\text{Cou}}, \quad (2)$$

where the linear, nonlinear, derivative couplings, and the Coulomb terms respectively are

$$\mathcal{L}_{\text{lin}} = \frac{1}{2}\alpha_S \rho_S^2 + \frac{1}{2}\alpha_V \rho_V^2 + \frac{1}{2}\alpha_{TS} \rho_{TS}^2 + \frac{1}{2}\alpha_{TV} \rho_{TV}^2, \quad (3)$$

$$\mathcal{L}_{\text{nl}} = \frac{1}{3}\beta_S \rho_S^3 + \frac{1}{4}\gamma_S \rho_S^4 + \frac{1}{4}\gamma_V [\rho_V^2]^2, \quad (4)$$

$$\mathcal{L}_{\text{der}} = \frac{1}{2}\delta_S [\partial_V \rho_S]^2 + \frac{1}{2}\delta_V [\partial_V \rho_V]^2 + \frac{1}{2}\delta_{TS} [\partial_V \rho_{TS}]^2 + \frac{1}{2}\delta_{TV} [\partial_V \rho_{TV}]^2, \quad (5)$$

$$\mathcal{L}_{\text{Cou}} = \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + e\frac{1-\tau_3}{2}A_0\rho_V, \quad (6)$$

where M represents the nucleon mass, e is the unit charge, and α_S , α_V , α_{TS} , α_{TV} , β_S , γ_S , γ_V , δ_S , δ_V , δ_{TS} , and δ_{TV} are coupling constants. The isoscalar density ρ_S , isovector density ρ_{TS} , the time-like components of isoscalar current ρ_V , and the time-like components of isovector currents ρ_{TV} are defined as

$$\rho_S = \bar{\psi}\psi, \rho_{TS} = \bar{\psi}\boldsymbol{\tau}\psi, \rho_V = \bar{\psi}\gamma^0\psi, \rho_{TV} = \bar{\psi}\boldsymbol{\tau}\gamma^0\psi. \quad (7)$$

The single particle wave function $\psi_k(\mathbf{r})$ with energy ϵ_k of a nucleon is obtained by solving the Dirac equation

$$\hat{h}\psi_k(\mathbf{r}) = \epsilon_k\psi_k(\mathbf{r}), \quad (8)$$

with the Dirac Hamiltonian

$$\hat{h} = \boldsymbol{\alpha} \cdot \mathbf{p} + \beta[M + S(\mathbf{r})] + V(\mathbf{r}), \quad (9)$$

where the scalar potential $S(\mathbf{r})$ and vector potential $V(\mathbf{r})$ are

$$\begin{aligned} S &= \alpha_S \rho_S + \alpha_{TS} \rho_{TS} \cdot \boldsymbol{\tau} + \beta_S \rho_S^2 + \gamma_S \rho_S^3 \\ &\quad + \delta_S \Delta \rho_S + \delta_{TS} \Delta \rho_{TS} \cdot \boldsymbol{\tau}, \\ V &= \alpha_V \rho_V + \alpha_{TV} \rho_{TV} \cdot \boldsymbol{\tau} + \gamma_V \rho_V^2 \rho_V \\ &\quad + \delta_V \Delta \rho_V + \delta_{TV} \Delta \rho_{TV} \cdot \boldsymbol{\tau} + e\frac{1-\tau_3}{2}A_0. \end{aligned} \quad (10)$$

In the MDC-CDFTs, the wave functions are expanded using the axially deformed harmonic oscillator (ADHO) basis [55, 56], which is obtained by solving the Schrödinger equation

$$\left[-\frac{\hbar^2}{2M} \nabla^2 + V_B(z, \rho) \right] \Phi_\alpha(\mathbf{r}\sigma) = E_\alpha \Phi_\alpha(\mathbf{r}\sigma), \quad (11)$$

where $\mathbf{r} = (z, \rho)$, where $\rho = \sqrt{x^2 + y^2}$, and

$$V_B(z, \rho) = \frac{1}{2}M(\omega_\rho^2 \rho^2 + \omega_z^2 z^2), \quad (12)$$

is the ADHO potential with the oscillator frequency, represented by ω_ρ (ω_z), perpendicular to (along) the z axis. More detailed formulae on the applications of ADHO in MDC-RMF can be found in Refs. [41, 43, 50, 52].

After obtaining the ADHO basis, the single-particle wave functions can be expanded using the basis

$$\psi_i(\mathbf{r}\sigma) = \left(\begin{array}{c} \sum_\alpha f_i^\alpha \Phi_\alpha(\mathbf{r}\sigma) \\ \sum_\alpha g_i^\alpha \Phi_\alpha(\mathbf{r}\sigma) \end{array} \right), \quad (13)$$

where α denotes a set of quantum numbers of the ADHO basis function, $\alpha \equiv \{n_z, n_\rho, m_l, m_s\}$, and f_i^α and g_i^α are the expansion coefficients. For the truncation of the ADHO basis, we follow Refs. [55, 57]. Finally, the wave functions are obtained by self-consistent iterations.

^{270}Hs is an axially deformed nucleus in the ground state [50, 58, 59]. Reflection-asymmetric deformations normally occur only for ultra-neutron-rich nuclei with $N \geq 182$ in the SHN region [59]. Thus, we only consider the axially symmetric deformations β_λ , where λ is an even number, in this study. To investigate the influence of each shape degree of freedom on the bulk properties of SHN, constraint calculations on mass multipole moments are performed [27]. In MDC-CDFTs, a modified linear-constraint method is implemented [42, 43] and the Routhian reads

$$E' = E_{\text{RMF}} + \sum_\lambda \frac{1}{2} C_\lambda Q_\lambda. \quad (14)$$

After the n th iteration, the variable $C_\lambda^{(n+1)}$ is determined by

$$C_\lambda^{(n+1)} = C_\lambda^{(n)} + k_\lambda (\beta_\lambda^{(n)} - \beta_\lambda), \quad (15)$$

where $C_\lambda^{(n)}$ is the value of the n th iteration, k_λ is a constant, and β_λ is the desired value of the deformation parameter.

The intrinsic multipole moments are calculated as

$$Q_{\lambda,\tau} = \int d^3r \rho_\tau(\mathbf{r}) r^\lambda Y_{\lambda 0}(\Omega), \quad (16)$$

where τ represents the nucleon (the neutron or the proton), and ρ_τ is the corresponding vector density. The deformation parameter $\beta_{\lambda,\tau}$ is given by

$$\beta_{\lambda,\tau} = \frac{4\pi}{3N_\tau R^\lambda} Q_{\lambda,\tau}, \quad (17)$$

where $R = r_0 A^{1/3}$, with $r_0 = 1.2$ fm, and N_τ represents the corresponding particle's number, A , N , or Z .

III. RESULTS AND DISCUSSIONS

To investigate the influence of higher-order deformations on the ground state properties of SHN, the doubly magic deformed nucleus ^{270}Hs , even-even Hs isotopes from ^{264}Hs to ^{276}Hs , and $N = 162$ isotones from ^{266}Rf to ^{272}Ds are analyzed. When investigating the ground state properties and the influence of higher-order deformations, the ADHO basis with $N_f = 20$ shells is adopted, leading to an accuracy of 0.1 MeV in total energy of ^{270}Hs [50] in the MDC-RMF calculations. In the particle-particle channel, a separable pairing force is adopted. Here, the strength and effective range of this force are taken to be the same as those in Ref. [50]: $G = 1.1G_0$, where $G_0 = 728$ MeV fm³, and $a = 0.644$ fm. The effective interactions PC-PK1 [60], PK1 [61], PKDD [61], DD-ME2 [62], and NL3* [63] are employed in the particle-hole channel.

The ground state properties, including deformation parameters β_λ ($\lambda = 2, 4, 6, 8$, and 10), radii, and binding energies of even-even Hs isotopes with the above-mentioned five effective interactions are given in Table 1, and even-even isotones with $N = 162$ are listed in Table 2. The binding energies of one nucleus with five effective interactions differ from each other, e.g., the largest binding energy of ^{270}Hs is 1973.77 MeV with PK1 and the smallest is 1967.41 MeV with PC-PK1. Such results are relatively close to the empirical values in AME2020, $E_B = 1969.65$ MeV [64–66], and are also comparable to the predictions of other models, such as MMM, with $E_B = 1969.20$ MeV [23], the Skyrme Hartree-Fock Bogoliubov mass model (HFB-24), with $E_B = 1968.45$ MeV [67], the Weizsäcker-Skyrme (WS) mass formula WS4, with $E_B = 1970.27$ MeV [68], the finite range droplet model (FRDM(2012)), with $E_B = 1971.48$ MeV [58], and several RMF calculations [69–73]. For other nuclei, similar conclusions can also be drawn. From these two tables, it is clear that all considered nuclei are deformed in the MDC-RMF calculations with five effective interactions. This is consistent with the results obtained from MMM calculations [23, 58, 59] and other global studies [74, 75]. Additionally, it has been shown that the inclusion of the rotational energy correction (REC) can improve the description of binding energies with PC-PK1 [60]. In this study, after considering RECs in PC-PK1 calculations, the binding energy of ^{270}Hs changes from 1967.45 to 1969.76 MeV, which is closer to the value provided in AME2020.

To determine the dimension of the deformation space when investigating the ground states of SHN using MDC-CDFTs, we calculate the binding energies of Hs isotopes and isotones with $N = 162$ in a different deformation space $\{\beta_\lambda; \lambda = 0, 2, \dots, \lambda_{\text{max}}\}$ with λ_{max} being the maximum order of deformation parameters, meaning all deformation parameters $\beta_\lambda \leq \beta_{\lambda_{\text{max}}}$ are considered self-consistent.

Table 1. Ground state properties, including quadrupole deformation parameters, of neutrons and protons ($\beta_{2,n}$ and $\beta_{2,p}$), deformation parameters β_2 , β_4 , β_6 , β_8 , and β_{10} , mass radius R_t , radii of protons and neutrons (R_p and R_n), charge radius R_c , and binding energy E_B of Hs isotopes using MDC-RMF with five effective interactions: PC-PK1, PK1, PKDD, DD-ME2, and NL3^{*}.

	$\beta_{2,n}$	$\beta_{2,p}$	β_2	β_4	β_6	β_8	β_{10}	R_n/fm	R_p/fm	R_t/fm	R_c/fm	E_B/MeV
PC-PK1												
²⁶⁴ Hs	0.270	0.280	0.274	-0.002	-0.060	-0.013	0.011	6.245	6.090	6.182	6.138	1924.415
²⁶⁶ Hs	0.266	0.276	0.271	-0.021	-0.063	-0.004	0.014	6.267	6.101	6.200	6.148	1939.205
²⁶⁸ Hs	0.262	0.273	0.266	-0.040	-0.063	0.004	0.015	6.288	6.111	6.217	6.158	1953.554
²⁷⁰ Hs	0.257	0.269	0.261	-0.057	-0.061	0.012	0.015	6.306	6.120	6.232	6.167	1967.408
²⁷² Hs	0.245	0.258	0.250	-0.060	-0.049	0.013	0.010	6.330	6.131	6.252	6.178	1979.303
²⁷⁴ Hs	0.216	0.228	0.221	-0.053	-0.038	0.009	0.006	6.344	6.135	6.263	6.182	1990.951
²⁷⁶ Hs	0.188	0.198	0.192	-0.049	-0.027	0.007	0.003	6.357	6.139	6.273	6.185	2002.778
PK1												
²⁶⁴ Hs	0.253	0.258	0.255	0.006	-0.058	-0.016	0.011	6.228	6.058	6.159	6.105	1934.074
²⁶⁶ Hs	0.253	0.258	0.255	-0.014	-0.065	-0.006	0.016	6.253	6.070	6.179	6.118	1947.952
²⁶⁸ Hs	0.256	0.261	0.258	-0.034	-0.070	0.005	0.019	6.278	6.084	6.201	6.131	1961.285
²⁷⁰ Hs	0.245	0.251	0.248	-0.044	-0.062	0.010	0.016	6.297	6.091	6.216	6.138	1973.766
²⁷² Hs	0.211	0.216	0.213	-0.029	-0.053	0.005	0.010	6.305	6.090	6.221	6.137	1985.924
²⁷⁴ Hs	0.194	0.198	0.195	-0.038	-0.040	0.006	0.010	6.322	6.097	6.234	6.144	1997.412
²⁷⁶ Hs	0.178	0.182	0.180	-0.047	-0.028	0.007	0.007	6.342	6.105	6.250	6.151	2008.356
PKDD												
²⁶⁴ Hs	0.250	0.255	0.252	0.001	-0.060	-0.015	0.011	6.207	6.053	6.145	6.101	1932.544
²⁶⁶ Hs	0.253	0.258	0.255	-0.020	-0.066	-0.004	0.016	6.233	6.067	6.166	6.115	1946.294
²⁶⁸ Hs	0.258	0.264	0.260	-0.041	-0.072	0.009	0.021	6.259	6.082	6.188	6.129	1959.686
²⁷⁰ Hs	0.252	0.261	0.256	-0.059	-0.062	0.017	0.017	6.278	6.091	6.204	6.138	1972.399
²⁷² Hs	0.211	0.217	0.213	-0.030	-0.056	0.006	0.019	6.282	6.087	6.205	6.134	1983.376
²⁷⁴ Hs	0.190	0.194	0.191	-0.039	-0.041	0.006	0.010	6.296	6.093	6.216	6.139	1994.504
²⁷⁶ Hs	0.174	0.179	0.176	-0.048	-0.028	0.007	0.007	6.316	6.100	6.233	6.147	2004.934
DD-ME2												
²⁶⁴ Hs	0.260	0.267	0.263	-0.001	-0.061	-0.012	0.014	6.178	6.073	6.136	6.121	1928.426
²⁶⁶ Hs	0.261	0.269	0.264	-0.023	-0.066	-0.001	0.019	6.200	6.086	6.154	6.133	1942.991
²⁶⁸ Hs	0.259	0.269	0.263	-0.042	-0.068	0.011	0.021	6.220	6.097	6.171	6.144	1957.270
²⁷⁰ Hs	0.252	0.264	0.257	-0.058	-0.060	0.017	0.017	6.236	6.105	6.184	6.152	1971.027
²⁷² Hs	0.213	0.222	0.216	-0.032	-0.054	0.007	0.012	6.240	6.101	6.185	6.148	1981.994
²⁷⁴ Hs	0.196	0.204	0.199	-0.039	-0.041	0.007	0.010	6.256	6.107	6.198	6.154	1993.391
²⁷⁶ Hs	0.178	0.186	0.181	-0.048	-0.027	0.008	0.007	6.271	6.113	6.210	6.160	2004.657
NL3 [*]												
²⁶⁴ Hs	0.265	0.271	0.267	0.004	-0.060	-0.014	0.013	6.260	6.079	6.186	6.126	1931.827
²⁶⁶ Hs	0.263	0.270	0.266	-0.017	-0.065	-0.004	0.017	6.284	6.090	6.206	6.137	1945.933
²⁶⁸ Hs	0.262	0.269	0.265	-0.036	-0.067	0.007	0.018	6.307	6.101	6.225	6.148	1959.582
²⁷⁰ Hs	0.256	0.265	0.260	-0.054	-0.061	0.015	0.017	6.326	6.109	6.240	6.156	1972.574
²⁷² Hs	0.233	0.241	0.236	-0.045	-0.051	0.010	0.012	6.344	6.115	6.254	6.162	1984.027
²⁷⁴ Hs	0.204	0.211	0.207	-0.039	-0.040	0.007	0.009	6.357	6.118	6.264	6.164	1995.465
²⁷⁶ Hs	0.185	0.193	0.188	-0.047	-0.028	0.008	0.006	6.376	6.125	6.279	6.171	2006.648

Table 2. Ground state properties, including quadrupole deformation parameters, of neutrons and protons ($\beta_{2,n}$ and $\beta_{2,p}$), deformation parameters $\beta_2, \beta_4, \beta_6, \beta_8$ and β_{10} , mass radius R_t , radii of protons and neutrons (R_p and R_n), charge radius R_c , and binding energies E_B of $N = 162$ isotones using MDC-RMF with five effective interactions: PC-PK1, PK1, PKDD, DD-ME2, and NL3^{*}.

	$\beta_{2,n}$	$\beta_{2,p}$	β_2	β_4	β_6	β_8	β_{10}	R_n/fm	R_p/fm	R_t/fm	R_c/fm	E_B/MeV
PC-PK1												
²⁶⁶ Rf	0.261	0.274	0.266	-0.039	-0.060	0.005	0.013	6.303	6.083	6.218	6.129	1953.531
²⁶⁸ Sg	0.260	0.274	0.266	-0.048	-0.063	0.009	0.015	6.304	6.102	6.225	6.149	1961.448
²⁷⁰ Hs	0.257	0.269	0.261	-0.057	-0.061	0.012	0.015	6.306	6.120	6.232	6.167	1967.408
²⁷² Ds	0.249	0.258	0.253	-0.061	-0.055	0.015	0.012	6.308	6.139	6.240	6.186	1971.338
PK1												
²⁶⁶ Rf	0.256	0.265	0.260	-0.038	-0.062	0.007	0.017	6.296	6.053	6.202	6.100	1957.699
²⁶⁸ Sg	0.257	0.268	0.261	-0.046	-0.067	0.010	0.018	6.299	6.075	6.211	6.122	1966.814
²⁷⁰ Hs	0.245	0.251	0.248	-0.044	-0.062	0.010	0.016	6.297	6.091	6.216	6.138	1973.766
²⁷² Ds	0.225	0.228	0.226	-0.037	-0.054	0.007	0.012	6.292	6.104	6.217	6.151	1979.041
PKDD												
²⁶⁶ Rf	0.256	0.266	0.260	-0.043	-0.063	0.009	0.018	6.272	6.050	6.186	6.097	1955.230
²⁶⁸ Sg	0.259	0.272	0.264	-0.051	-0.070	0.013	0.020	6.277	6.073	6.197	6.120	1965.167
²⁷⁰ Hs	0.252	0.261	0.256	-0.059	-0.062	0.017	0.017	6.278	6.091	6.204	6.138	1972.399
²⁷² Ds	0.241	0.245	0.242	-0.060	-0.056	0.017	0.015	6.278	6.107	6.210	6.154	1977.775
DD-ME2												
²⁶⁶ Rf	0.255	0.268	0.260	-0.040	-0.061	0.008	0.017	6.224	6.062	6.161	6.109	1955.711
²⁶⁸ Sg	0.257	0.271	0.262	-0.048	-0.065	0.012	0.019	6.231	6.086	6.174	6.132	1964.571
²⁷⁰ Hs	0.252	0.264	0.257	-0.058	-0.060	0.017	0.017	6.236	6.105	6.184	6.152	1971.027
²⁷² Ds	0.242	0.249	0.245	-0.060	-0.054	0.017	0.015	6.241	6.123	6.193	6.170	1975.320
NL3 [*]												
²⁶⁶ Rf	0.261	0.272	0.266	-0.038	-0.062	0.007	0.017	6.325	6.070	6.226	6.116	1956.503
²⁶⁸ Sg	0.261	0.273	0.266	-0.047	-0.066	0.011	0.018	6.325	6.091	6.233	6.137	1965.582
²⁷⁰ Hs	0.256	0.265	0.260	-0.054	-0.061	0.015	0.017	6.326	6.109	6.240	6.156	1972.574
²⁷² Ds	0.248	0.253	0.250	-0.058	-0.055	0.016	0.014	6.326	6.127	6.247	6.174	1977.689

ently, while other deformation parameters are constrained to zero. In Fig. 1, the binding energies of $N = 162$ isotones with five different effective interactions are plotted as a function of $\lambda_{\max}$. For convenience, we take ²⁷⁰Hs with the effective interaction PC-PK1 as an example to discuss the influence of each order of deformation on the binding energy in detail. When constraining ²⁷⁰Hs to be spherical, i.e., in the deformation space $\{\beta_\lambda; \lambda = 0\}$, the resulting binding energy is 1956.39 MeV, which is close to the prediction from the relativistic continuum Hartree-Bogoliubov theory, which is 1952.65 MeV [76], but not close to the value given in AME2020 (marked by a black square in Fig. 1). After considering the quadrupole deformation β_2 , the binding energy of ²⁷⁰Hs changes considerably (by approximately 8.43 MeV) and approaches that from AME2020. This result indicates the importance of the quadrupole deformation. The influence of the hexadecapole deformation β_4 on the

binding energy is smaller, with a change of only 0.68 MeV. Considering β_6 , the energy change is approximately 1.87 MeV, which is larger than that of β_4 , and E_B approaches the value provided in AME2020. Including β_8 and β_{10} does not affect the binding energy, which converges well at $\{\beta_\lambda; \lambda = 0, 2, \dots, 10\}$. From these results, we can conclude that to obtain a proper description of ²⁷⁰Hs, one should consider the β_6 deformation at least with respect to the binding energy. Calculated binding energy versus $\lambda_{\max}$ with other density functionals are also displayed in Fig. 1, and although the binding energies with five effective interactions differ from each other, the overall trend of E_B changing with $\lambda_{\max}$ are similar. The binding energy of ²⁷⁰Hs is predominantly changed by β_2 , then β_6 and β_4 . The influence of β_8 and β_{10} can be ignored. The RECs for ²⁷⁰Hs with PC-PK1 are 2.27, 2.03, 2.29, 2.31, and 2.31 MeV in deformation spaces $\{\beta_\lambda; \lambda = 0, \dots, \lambda_{\max}\}$, with $\lambda_{\max} = 2, 4, 6, 8$, and 10, respect-

ively. The values of RECs change slightly in different deformation spaces and only minimally influence the trends of binding energies with respect to $\lambda_{\max}$.

To check whether the above conclusion is valid for

other SHN, we perform similar calculations for even-even isotones with $N = 162$ and Hs isotopes, with the results also presented in Figs. 1 and 2. From these figures, we find that the binding energies of these nuclei are signi-

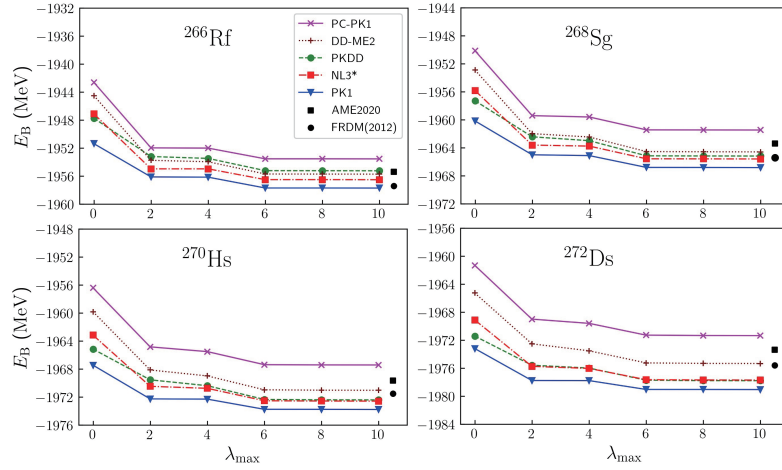


Fig. 1. (color online) Binding energies of $N = 162$ isotones for PC-PK1, DD-ME2, PKDD, NL3*, and PK1 as a function of $\lambda_{\max}$. The black square and black circle represent the values from AME2020 and FRMD(2012), respectively.

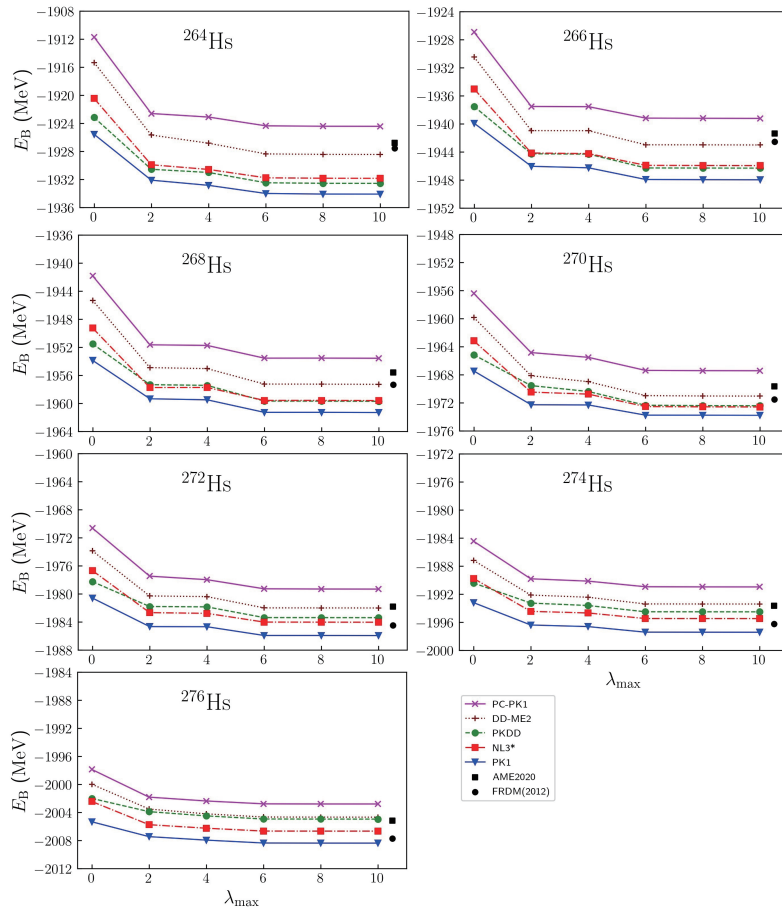


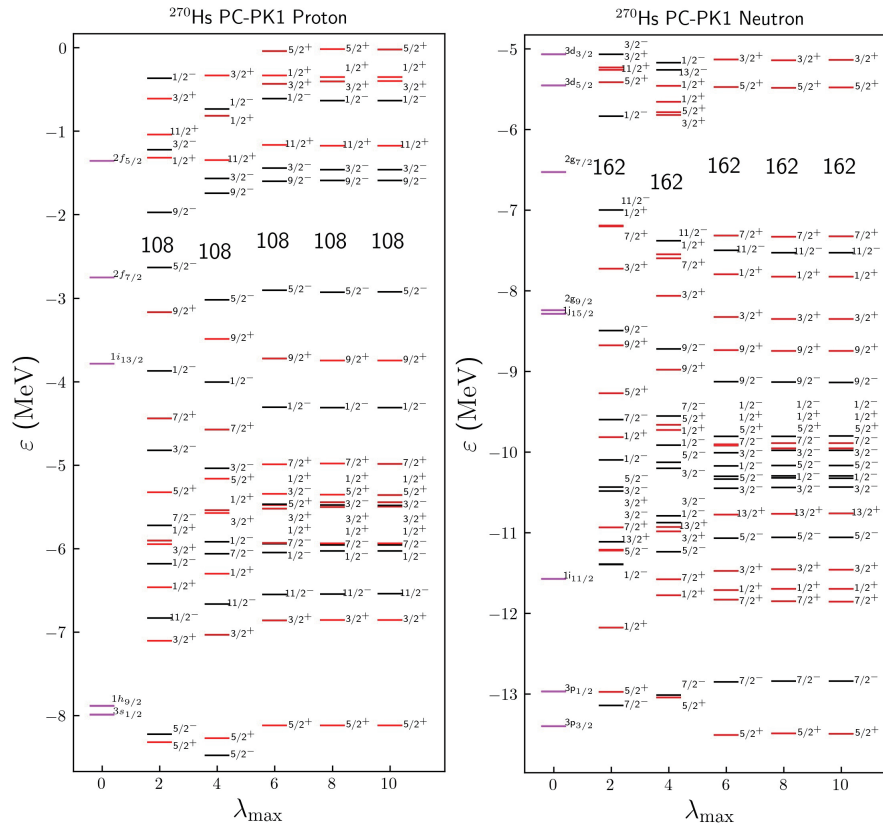
Fig. 2. (color online) Binding energies of Hs isotopes for PC-PK1, DD-ME2, PKDD, NL3*, and PK1 as a function of $\lambda_{\max}$. The black square and black circle represent the values from AME2020 and FRMD(2012), respectively.

ificantly changed by β_2 . The influence of β_4 and β_6 should also be considered, and the contribution to the total energy from β_6 is larger than that from β_4 . For Hs isotopes, with decreasing neutron number, the value of β_2 increases substantially and the differences of the total energies between the spherical case and in the ground states increase, which can be observed in Fig. 2. For isotones with $N = 162$, the value of β_2 only minimally changes with proton number. For these nuclei, the relationship between binding energies and $\beta_{\lambda_{\max}}$ are the same as for ^{270}Hs .

It is well known that shell structure is particularly important for SHN and very sensitive to the deformation of the nucleus [28]. Using ^{270}Hs as an example, we explore how the deformations influence the shell gaps at $Z = 108$ and $N = 162$ by investigating the structure of single-particle levels (SPLs) in different deformation spaces. In Fig. 3, we illustrate the SPLs for protons and neutrons of ^{270}Hs versus $\lambda_{\max}$, calculated with PC-PK1. When $\lambda_{\max} = 10$, i.e., for the ground state, the energy gaps at $Z = 108$ and $N = 162$ are approximately 1.34 MeV and 1.85 MeV, which are considerably large for such a heavy nucleus [77] and result in deformed shells.

In the spherical limit, $\lambda_{\max} = 0$, each single particle

state is labelled by $|nlj\rangle$, where n , l , and j denote the radial quantum number, orbital angular momentum, and total angular momentum, respectively. It is obvious that there are no shell gaps at $Z = 108$ and $N = 162$. After including β_2 , a spherical orbital $|nlj\rangle$ with a degeneracy of $2j+1$ splits into $(2j+1)/2$ levels, with each one represented by Ω^π with the projection Ω of total angular momentum on the symmetry axis and the parity π . It is found that due to quadrupole correlations, the shell gaps at $Z = 108$ and $N = 162$ appear, with values of 0.66 MeV and 1.17 MeV, respectively. When including β_4 into the deformation space, the order of SPLs around the two gaps changes and the shell gaps at $Z = 108$ (up to approximately 1.28 MeV) and $N = 162$ (up to approximately 1.56 MeV) increase considerably. The impact of β_6 on the shell gap at $Z = 108$ is smaller, only 0.02 MeV. Although for neutrons, the shell gap at $N = 162$ increases by approximately 0.26 MeV. The inclusion of β_8 and β_{10} produces only extremely minimal changes in the shell gaps and the order of SPLs. From this, one can conclude that β_2 plays a vital role in the formation of the shell closures $Z = 108$ and $N = 162$, which are further enhanced by β_4 . The influence of β_6 is relatively small and the effects of β_8 and β_{10} can be considered negligible. However, there remains a



question: where do the Y_{60} correlations come from? By checking the SPLs, we find that two proton levels $1/2^+$ originating from the spherical orbitals $3s_{1/2}$ and $1i_{13/2}$ are very close to each other and the mixing of these two spherical orbitals in the deformed SPLs results in Y_{60} correlations. For neutrons, these correlations originate from the mixing of the spherical orbitals $3p_{3/2}$ and $1j_{15/2}$ in the levels $3/2^-$, close to the neutron Fermi energy.

IV. SUMMARY

In this study, we investigate the ground state properties of SHN around ^{270}Hs in multidimensional deformation spaces using the MDC-RMF model with five density functionals. The influence of higher-order deformation parameters on the ground state of nuclei near ^{270}Hs are investigated, including the binding energies and SPLs. We show that the binding energies of deformed SHN around ^{270}Hs are significantly affected by higher-order deformations. In particular, the influence of β_6 on binding energy is larger than that of β_4 . For the doubly magic nucleus ^{270}Hs , the deformed shell gaps at $Z = 108$ and $N = 162$ are mainly determined by quadrupole correlations and are enhanced by the inclusion of β_4 . In conclusion, at least the β_6 degree of freedom should be considered in the investigation of SHN using CDFTs. It is also very in-

teresting to investigate how the higher-order deformations influence other properties of SHN, such as the moment of inertia and energy spectra, by using density functional theories. Additionally, we would like to mention that the calculations performed in this study can also be performed using the deformation relativistic Hartree-Bogoliubov (DRHBc) theory [78-84], in which the scalar potential and densities are expanded in terms of the Legendre polynomials, though the DRHBc theory is much more time consuming than the MDC-RMF model. Very recently, the influence of higher-order deformations on possible bound nuclei beyond the drip line has also been investigated in the transfermium region from No ($Z = 102$) to Ds ($Z = 110$) using the DRHBc theory [85], and the development of a nuclear mass table using the DRHBc theory is in progress [86-89].

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