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An energy sum constraint of event mixing for Bose-Einstein correlations in reactions with $\pi\pi$X final states around 1 GeV *

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Abstract: We present a new event mixing technique for measuring two-pion Bose-Einstein correlations (BEC) in reactions with only two identical bosons among three final state particles. This new mixing method contains a missing mass consistency (MMC) cut and an energy sum order (ESO) cut. Unlike the previous proposed pion energy cut, which abandons nearly half the original events, the ESO cut does not eliminate any original events and hence improves the statistics of both original events and mixed events. Numerical tests using the $\gamma p \to \pi^0\pi^0 p$ events around 1 GeV are carried out to verify the validity of the ESO cut. This cut is able to reproduce the relative momentum distribution of the original events in the absence of BEC effects. In addition, its ability to observe BEC effects is tested by an event sample in the presence of BEC effects. Simulation results show the BEC effects can be observed clearly as an enhancement in the correlation function, and the BEC parameters extracted by this event mixing cut are consistent with the input BEC parameters.

Keywords: Bose-Einstein correlations, photoproduction, event mixing

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1 Introduction

In nuclear physics, the effect of Bose-Einstein correlations (BEC) has important applications for measuring the space-time properties of subatomic regions emitting identical bosons [1–4]. The BEC effect has been applied to investigations of the dynamics of heavy ion collisions with high multiplicity. However, in the case of low-multiplicity reactions, BEC measurements are still not fully satisfactory. The main reason for this is that the event mixing method [5, 6] used for BEC measurement is strongly disturbed by non-BEC factors such as global conservation laws and resonance decays [7, 8]. Unlike BEC measurements in high-energy elementary-particle collisions [9–19] and relativistic heavy-ion collisions [20–29] with large multiplicity, significant kinematical correlations of final state particles for exclusive reactions with only two identical bosons at low energies resulting from conservation laws complicate the BEC observation [30–32].

The BEC measurements for low-multiplicity reactions are also important. In contrast to inclusive reactions with large multiplicities, the kinematics of all ejectiles may be entirely determined in exclusive reactions and hence a kinematically complete measurement of final particles can provide complementary information. The authors of Ref. [7] tried to measure two-proton correlation functions in the reactions $pp \to pp+\eta$ and $pp \to pp$+pions in order to obtain complementary information which could shed light on the interpretation of the two-proton correlations observed in heavy ion reactions.

In addition, BEC can also be used as a tool to measure the spatial size of ultra-short lifetime nucleon resonances generally generated in two-meson productions at baryon resonance energies, such as $\gamma p \to \pi^0\pi^0 p$ at incident photon energies around 1 GeV. However, so far there are no BEC measurements for such reactions because BEC measurements in such exclusive reactions with low multiplicity are still extremely challenging, as indicated above. Another possible difficulty for BEC measurements in the exclusive reaction $\gamma p \to \pi^0\pi^0 p$ is that the BEC effect may disappear because of the coherent emission of two pions if they are from the same resonance. But in reality, the reaction $\gamma p \to \pi^0\pi^0 p$ is dominated by the sequential decay $\gamma p \to \pi^0\Delta \to \pi^0\pi^0 p$ around 1 GeV [33] and thus two pions may be emitted from different resonance decays. Therefore it is possible that a certain fraction of the two pions are emitted chaotically and their BEC effect can be measured.

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In order to establish a suitable event mixing method for such measurements, two mixing cuts were proposed for two-pion BEC measurement in the $\gamma p \to \pi^0\pi^0 p$ reaction at incident photon energies E_γ around 1 GeV (a non-perturbative QCD region) in Ref. [32]. The first cut, the missing mass consistency (MMC) cut, is introduced for the sake of the energy momentum conservation for mixed events. The second cut, the pion energy (PE) cut, is proposed in order to make a valid mixed event sample for extracting the correct BEC parameters. However, an obvious flaw of the PE cut is that a large proportion (about 40%) of the original events need to be excluded, leading to a sample population reduction and hence a worse analysis uncertainty.

To improve the statistics, new event mixing cut conditions with no requirement on discarding original events are highly desirable. Therefore, in this work we propose a new cut condition, named the energy sum order (ESO) cut, to replace the PE cut. The ESO cut takes the two-boson energy sums of both original event and mixed event into consideration to constrain the mixing process for constructing a valid mixed event sample. Extensive numerical tests are performed to validate the ESO cut.

The paper is organized as follows. In Section 2, a description of the new event mixing method is given. In Section 3, we present details of the numerical tests which simulate the application of the new mixing cut to the $\gamma p \to \pi^0\pi^0 p$ reaction. The ability of this new method to observe BEC effects and to extract correct BEC parameters is investigated. In Section 4, a summary is given.

2 Event mixing method

BEC observations involve the measurement of a two-particle correlation function [5, 6], defined to be the ratio of the joint probability $P_{\rm BEC}(p_1p_2)$ of emitting two identical bosons with momenta p_1 and p_2 to the non-BEC probability $P_0(p_1,p_2)$ of the so-called "reference sample" free of BEC effects:

$$C_{\rm BEC}(p_1,p_2)=\frac{P_{\rm BEC}(p_1,p_2)}{P_0(p_1,p_2)}=N(1+\lambda_2 {\rm e}^{-r_0^2Q^2}). \tag{1}$$

The reference sample is generally constructed via the event mixing technique [5], selecting two bosons' momenta from different events. In general, the boson-emission source is assumed to be a sphere with a Gaussian density distribution and the correlation function has a dependence on the relative momentum Q ($Q^2=-(p_1-p_2)^2$) with two parameters r_0 and λ_2, where r_0 is the Gaussian radius of the boson-emitting volume, and λ_2 reflects the degree of emission chaoticity of the source ranging from 0 to 1 [3]. N is the normalization factor. Therefore, the two-boson correlation function is generally written as $C_2(Q)=N(1+\lambda_2 {\rm e}^{-r_0^2Q^2})$.

A valid reference sample should be identical to the real data in all aspects but free of BEC effects. This means an ideal event mixing method should make a reference sample having identical Q distribution to the original one and hence obtain a flat correlation function. However, in practice this ideal condition has not been achieved yet. Appropriate mixing constraints are used to bring the current mixing method closer to the ideal goal.

For applications in exclusive reactions with only two identical bosons, special mixing constraints are required to make a valid reference sample. In this work, the MMC cut [32] is still included in the mixing method, while a new mixing constraint, the energy sum order (ESO) cut, is introduced to replace the previously proposed PE cut [32]. The ESO cut requires that a mixed event should satisfy the relation:

$$\min(E_{\rm sum}^{\rm ori,1},E_{\rm sum}^{\rm ori,2})<E_{\rm sum}^{\rm mix}<\max(E_{\rm sum}^{\rm ori,1}E_{\rm sum}^{\rm ori,2}), \tag{2}$$

where $E_{\rm sum}^{\rm ori,1}$ and $E_{\rm sum}^{\rm ori,2}$ are the two-boson energy sums in the two original events, and $E_{\rm sum}^{\rm mix}$ is the same value in the mixed event.

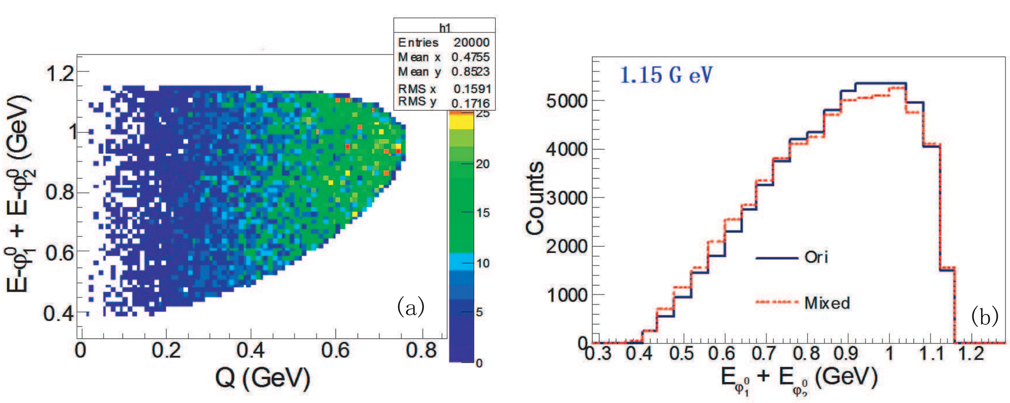

Fig. 1. (color online) (a) A 2-d scatter plot of the energy sum and the momentum difference invariant mass Q of two pions in the reaction $\gamma p \to \pi^0\pi^0 p$ at the incident photon energy of 1.15 GeV; (b) The energy sum of two pions both for the original sample and the mixed sample obtained by event mixing with the MMC cut alone.

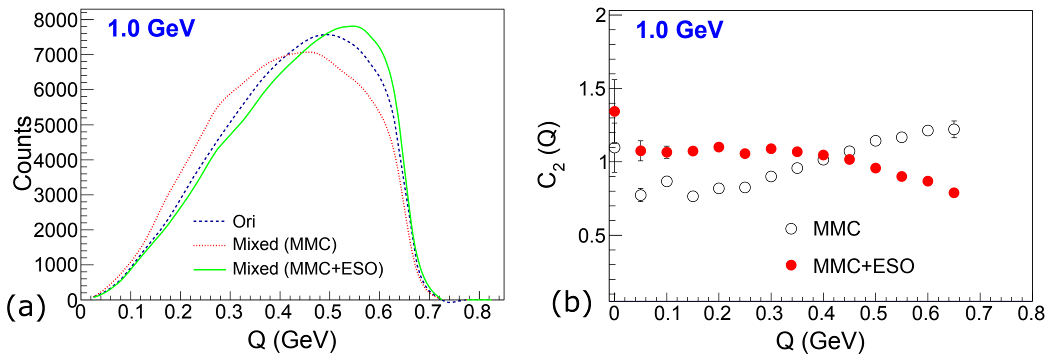

Fig. 2. (color online) A comparison of event mixing results from only the MMC cut with that from the combination cuts of MMC and ESO for the $\gamma p \rightarrow \pi^0\pi^0 p$ reaction at the incident photon energy of 1.0 GeV: (a) Q distribution, (b) correlation function.

The reason to adopt the two-pion energy sum to control the mixing process is that the two-pion momentum difference Q's upper limit has a sharp dependence on their energy sum, especially in the lower region, due to the energy-momentum conservation in the reaction $\gamma p \rightarrow \pi^0\pi^0 p$, as shown in Fig. 1(a). Without the ESO cut, the mixed sample with only the MMC cut shows that a mixed event with smaller energy sum of two pions survives more easily (Fig. 1(b)), indicating that more low-Q mixed events appear. This is also reflected in Fig. 2(a), showing the Q distribution of the mixed event with only the MMC cut is greater than that of the original sample in the low Q region. To correct this bias, the ESO cut is used to tune the Q distribution by limiting the mixed event's two-pion energy sum.

With the MMC and ESO cuts, the Q distribution shape is closer to the original one in the low Q region, as shown in Fig. 2(a), compared to that from the mixed sample using only the MMC cut. Because the BEC effects induce enhancement at low Q, this improvement made by the ESO cut is very helpful for effectively measuring BEC effects. Correspondingly, the MMC and ESO cuts produce a flat correlation function in the low Q range (see Fig. 2(b)).

3 Numerical tests

Numerical tests are performed to verify the proposed event mixing constraint. We take the $\gamma p \rightarrow \pi^0\pi^0 p$ reaction as an example to demonstrate this simulation. Both non-BEC (pure phase space) samples and BEC-existing samples are generated using a ROOT [34] utility named "TGenPhaseSpace" [35]. The pure phase space generation is governed by a weight based upon the phase-space integral R_N

$$R_N = \int \delta^4(P - \sum_{j=1}^{N} p_j) \prod_{i=1}^{N} \delta(p_i^2 - m_i^2) \mathrm{d}^4 p_i, \tag{3}$$

where P and p_i are the fourmomentum of the whole system and that of individual emitted particles, respectively. The BEC-existent samples are produced by filtering a pure phase space event sample based on Eq. (1) using the following procedure: (1) the relative momentum Q of two pions in each pure phase-space event is calculated; (2) the ratio $C_{\mathrm{BEC}}(Q)/C_{\mathrm{BEC}}^{\mathrm{max}}$ is compared with an uniform random number R ranging from 0 to 1, where $C_{\mathrm{BEC}}^{\mathrm{max}} = N(1+\lambda_2)$ is the maximum value of $C_{\mathrm{BEC}}(Q)$; (3) this event is accepted if $C_{\mathrm{BEC}}(Q)/C_{\mathrm{BEC}}^{\mathrm{max}} > R$.

The correlation functions of non-BEC samples are important benchmarks for BECexistent sample correlation function measurements. Thus, six phase space $\gamma p \rightarrow \pi^0\pi^0 p$ event samples free of BEC effects at typical incident photon energies from 1.0 GeV to 1.15 GeV with a step 0.03 GeV are generated and used to make mixed samples via the event mixing method with the ESO and the MMC cuts. The obtained correlation functions are shown in Fig. 3. Because they exhibit a Q^2 dependent pattern, the function $f(Q) = N(1+\alpha Q^2)$ is used as the fit function. The results show that the fit function can give a good description of the non-BEC correlation function. Correspondingly, the correlation function for BEC-existent samples should be fitted by a modified Eq. (1):

$$C_{\mathrm{BEC}}(Q) = N(1+\alpha Q^2)(1+\lambda_2 \mathrm{e}^{-r_0^2 Q^2}). \tag{4}$$

In order to verify the ability to observe BEC effects, we generated six BEC-present samples of $\gamma p \rightarrow \pi^0\pi^0 p$ events at incident photon energies of 1.0 GeV, 1.03 GeV, 1.06 GeV, 1.09 GeV, 1.12 GeV, and 1.15 GeV respectively. The BEC parameters are typically set to be $r_0 = 0.8$ fm and $\lambda_2 = 1.0$.

Figure 4 shows the Dalitz plots of the generated non-BEC and BEC samples of the $\gamma p \rightarrow \pi^0\pi^0 p$ events. The enhancement in Fig. 4(b) confirms that the BEC effects are included. Fig. 4(c) shows the Q distributions of both the non-BEC and BEC samples. As shown in Fig. 4(d), an obvious enhancement can be observed in the correlation function. By fitting Eq. (1) to this correlation function, the BEC parameters r_0 and λ_2 are found to be

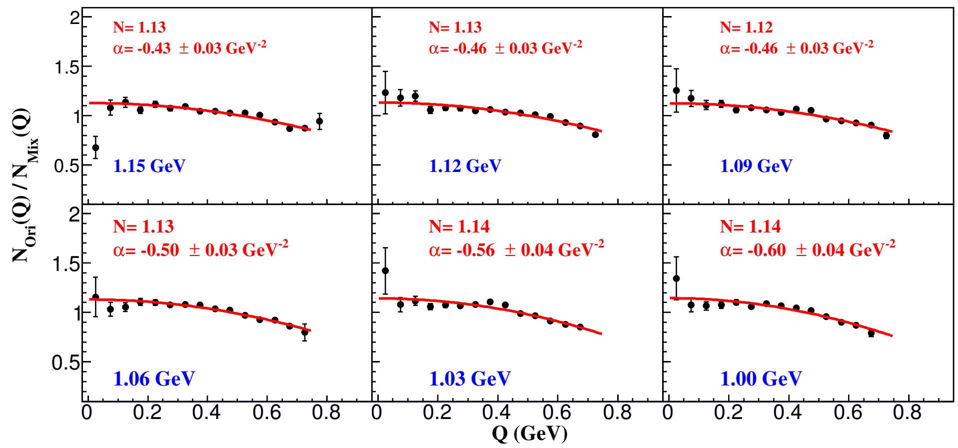

Fig. 3. (color online) Correlation functions obtained by the event mixing with the MMC cut and the ESO cut for the $\gamma p \to \pi^0\pi^0 p$ events at different incident photon energies as indicated in the plot. A Q^2 dependent function $f(Q)=N(1+\alpha Q^2)$ is used to fit the data.

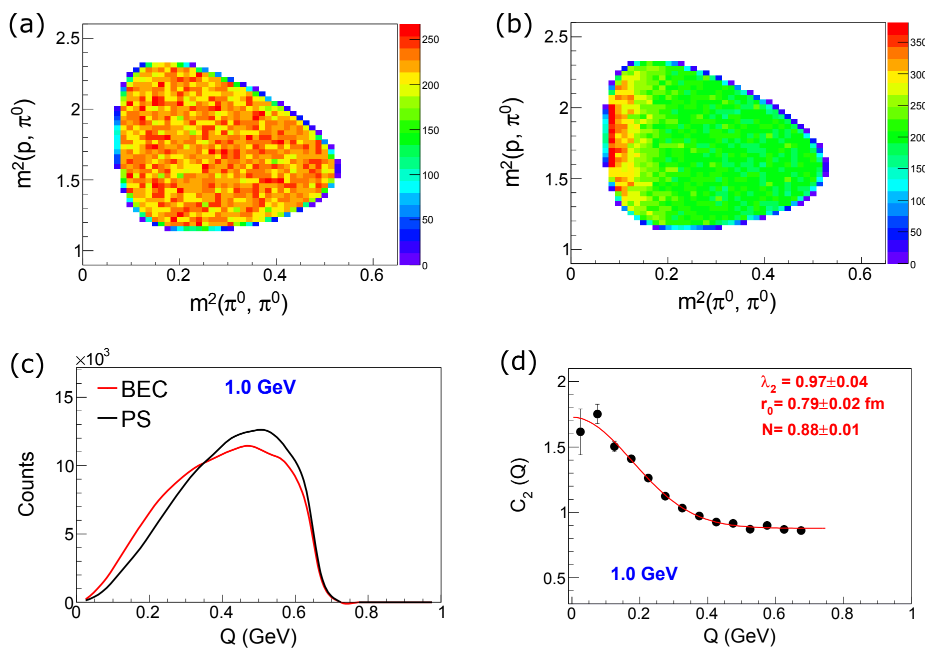

Fig. 4. (color online) Dalitz plot of $m^2(\pi^0 p)$ versus $m^2(\pi^0\pi^0)$ for the generated pure phase space $\gamma p \to \pi^0\pi^0 p$ sample free of BEC effects (a) and for the sample with BEC effects (b). The incident photon energy is 1.0 GeV. For the BEC sample, the BEC parameters are set to be $\lambda_2=1.0$ and $r_0=0.8$ fm. (c) Q spectra of the generated BEC samples. For comparison, the non-BEC sample's Q distribution (labeled as 'PS') is also presented. (d) Correlation functions of the BEC-sample calculated as the ratio of Q spectra of the BEC sample to that of the non-BEC sample. The fitted BEC parameters r_0 and λ_2 are presented as well, obtained by fitting Eq. (1) to the correlation functions.

consistent with the input values, confirmed the validity of the BEC-sample generating procedures.

Using the ESO cut and the MMC cut, the BEC effects can be successfully observed in the obtained correlation functions, as shown in Fig. 5. The BEC parameters r_0 and λ_2 are obtained by fitting Eq. (4) to the correlation function. Figure 6 compares the fit values with the input quantities that are obtained by fitting Eq. (1) to the ratio of Q spectrum of the BEC sample to that of the corresponding pure phase-space sample based on whichthe BEC sample is constructed. It is found that the fit values of r_0 are in good agreement with the input values at all energy points. The fitted λ_2 are just consistent with the input values at two energy points of 1.06 GeV and 1.12 GeV within error bars. All six fitted values of λ_2 are lower than the input values, indicating

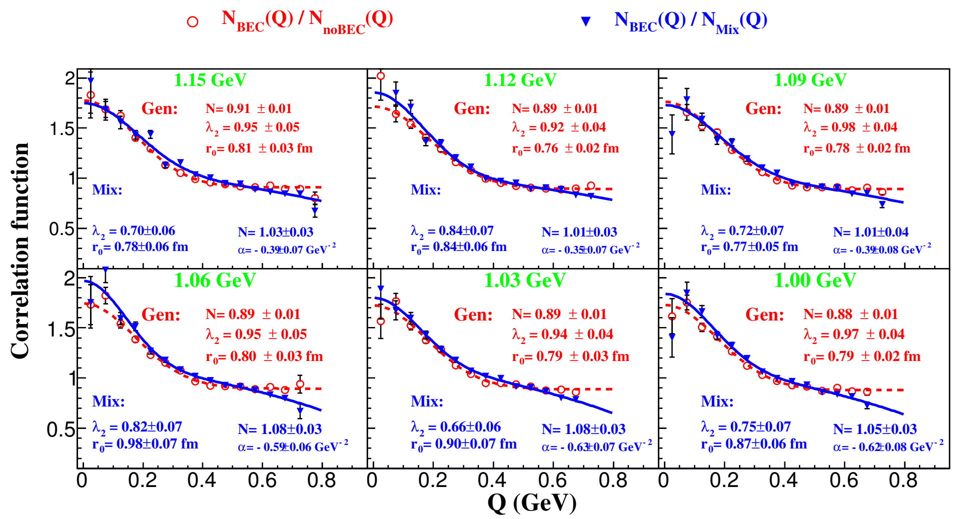

Fig. 5. (color online) Correlation functions at different incident photon energies for $\gamma p \to \pi^0\pi^0 p$ events in the presence of BEC effects (input BEC parameters: r_0=0.8 fm, λ_2=1.0). For comparison, the ratio of the Q spectrum of the generated BEC sample to that of the corresponding pure phase-space sample based on which the BEC sample is constructed is also shown.

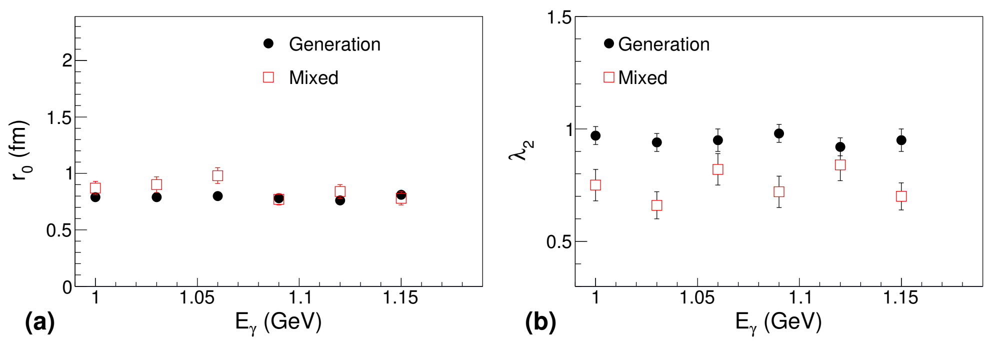

Fig. 6. (color online) Fitted values of r_0 (a) andλ_2 (b) obtained by event mixing with the MMC and ESO cuts at six incident photon energies E_γ=1.0, 1.03, 1.06, 1.09, 1.12, and 1.15 GeV for the $\gamma p \to \pi^0\pi^0 p$ events. For comparison, the values of r_0 and λ_2 for the generated sample with BEC effects are also shown.

Table 1. Comparison of the input BEC parameters (mean value over the six energy bins) with those from the event mixing.

	r_0/fm	λ_2	r_0/fm	λ_2	r_0/fm	λ_2
input	0.77±0.02	0.47±0.01	0.79±0.01	0.72±0.02	0.78±0.01	0.95±0.02
mixing	0.94±0.05	0.34±0.02	0.88±0.04	0.57±0.03	0.84±0.03	0.74±0.03

systematic bias estimation is needed.

To investigate the systematic bias introduced by the event mixing method, the weighted mean fit values of both r_0 and λ_2 are compared to the weighted mean input values. It is found the mean fit value of r_0 (0.84±0.03) over the six energies is about 8% overestimated compared to the mean value of the inputs, 0.78±0.01. The mean value of the fit λ_2 is found to be 0.74±0.03, about 22% underestimated compared to the mean value of the inputs, 0.95±0.02. This requires that the BEC parameter $r_0$and λ_2 obtained by this mixing method should be corrected in practical applications.

The event mixing method is also tested with BEC samples of different input λ_2 values of 0.75 and 0.5. The results are listed in Table 1. When the input λ_2=0.5, the mixing obtained λ_2 value is about 28% underestimated and r_0 is about 21% overestimated. When the input λ_2=0.75, the mixing obtained λ_2 value is about 21% underestimated and r_0 is about 11% overestimated. This indicates that the systematic bias varies with the BEC parameter λ_2, and the correction should be estimated via simulation.

As long as the fit BEC parameters are considered, the ESO cut also has a large systematic error for BEC parameter measurement, especially for smaller input lambda. This is similar to the PE cut: typical deviations of r_0 and λ_2 are 12% and 20% respectively when input r_0=1.0 fm and $\lambda_2 = 1.0$ [32]. The pattern of the mixing-obtained correlation function for a pure phase space sample is critical for BEC parameter extraction. If it is not a perfectly flat line, additional fitting parameters should be introduced in the fitting function to improve the accuracy of the fitting. However, this may introduce additional systematic uncertainties.

An effective solution may be to find a self-adapting event mixing method, which can make a flat correlation for non-BEC samples by analyzing their original distribution. In other words, we need to add some parameters to delicately control the mixing process, so that the mixing process can be adjusted automatically in accordance with specific pattern of the original sample. To this end, special self-adapting procedures should be added in the current mixing method and the ESO cut may need to be modified correspondingly. This will be done in the future.

Another solution is to correct the correlation function via the double ratio method [36] as follows:

$$R(Q)=\frac{C_{\mathrm{BEC}}(p_1,p_2)}{C_{\mathrm{BEC,MC}}(p_1,p_2)}, \tag{5}$$

where the subscript "MC" refers to the corresponding distributions from the Monte Carlo simulated data generated without BEC effects. However, in the baryon resonance energy region, the intermediate processes for double-pion photoproduction are generally not well known, so accurate estimation of $C_{\mathrm{BEC,MC}}(p_1,p_2)$ is difficult. Therefore, a self-adapting event mixing method seems to be a promising solution.

4 Summary and discussion

A new event mixing method, containing two mixing cuts of MMC and energy sum order (ESO), is proposed to investigate two-pion BEC in reactions with only two identical pions among three final state particles. Compared to the previously proposed PE cut, which has to discard nearly 40% of original events, this cut has no requirement on event deletion and hence can improve the statistics of both original and mixed events. Extensive simulations have been carried out to test this new cut's ability to observe BEC effects. The simulation results validate its ability to reproduce the relative momentum distribution of the original events in the absence of BEC effects and to observe BEC effects for an event sample in the presence of BEC effects. For typical input BEC parameters r_0=0.8 fm and $\lambda_2 = 1.0$, the extracted mean value of r_0 with this cut is about 8% overestimated compared to the mean value of the inputs, and the extracted λ_2 is on average about 22% lower than the input value. Therefore, in real applications, the obtained r_0 and λ_2 value should be corrected accordingly. Although we here adopt the reaction $\gamma\mathrm{p}\to\pi^0\pi^0\mathrm{p}$ to demonstrate the event mixing method with the ESO and MMC cuts, it is also applicable to similar reactions with only two identical bosons among three final state particles. In reality, the $\gamma\mathrm{p}\to\pi^0\pi^0\mathrm{p}$ reaction is actually dominated by the delta-resonance process. Further studies are needed to establish a suitable corresponding event mixing method, taking into account the impact of resonances and limited transverse momenta of produced particles.

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