Direct couplings of mimetic dark matter and their cosmological effects *

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Abstract: The original mimetic model was proposed to take the role of dark matter. In this paper we consider possible direct interactions of mimetic dark matter with other matter in the universe, especially standard model particles such as baryons and photons. By imposing shift symmetry, the mimetic dark matter field can only have derivative couplings. We discuss the possibilities of generating baryon number asymmetry and cosmic birefringence in the universe based on the derivative couplings of mimetic dark matter to baryons and photons.

Keywords: cosmology, dark matter, baryogenesis, cosmological CPT violation

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1 Introduction

The proposed mimetic model [1] was first considered as an extension of general relativity, in which the physical metric $g_{\mu\nu}$ is constructed in terms of an auxiliary metric $\tilde{g}_{\mu\nu}$ and a scalar field ϕ, as follows:

$$g_{\mu\nu}=\tilde{g}_{\mu\nu}\tilde{g}^{\alpha\beta}\phi_\alpha\phi_\beta/M_1^4 , \tag{1}$$

where $\phi_\alpha\equiv\nabla_\alpha\phi$ denotes the covariant derivative of the scalar field with respect to spacetime coordinates, and M_1 represents a certain mass scale so that the metric is dimensionless. Here we use the convention of most negative signature for the metrics. Usually when discussing the mimetic model in the literature, people use the convention of unit reduced Planck mass $M_p^2=1/(8\pi G_N)=1$ and only consider the case where $M_1=M_p=1$. In this paper we will show these mass scales explicitly and consider the general cases where M_1 is a different scale from M_p. In terms of this transform (1) the conformal mode of gravity is isolated to the scalar field in a covariant way and the physical metric is invariant under the Weyl rescalings of the auxiliary metric. By varying the Einstein-Hilbert action, which is constructed from the physical metric $g_{\mu\nu}$, with respect to the auxiliary metric and the scalar field, the resulting equations contain the Einstein equation and an equation of motion for an extra scalar mode which can mimic cold dark matter in the universe, so it is dubbed mimetic dark matter [1]. Later, as shown in Refs. [2–4], without introducing the auxiliary metric the mimetic matter can also be considered as a new scalar component with the constraint $\phi_\mu\phi^\mu=M_1^4$. Such a constraint can be realized by a Lagrange multiplier λ in the action, that is, the action can be written as,

$$S=\int d^4x\sqrt{g}\left[\frac{M_p^2}{2}R+\lambda(\phi_\mu\phi^\mu-M_1^4)\right]+S_m , \tag{2}$$

where $g=-\det|g_{\mu\nu}|$ and S_m is the action for other matter in the universe. These two points about the mimetic model are equivalent, at least classically. We will take the second point in this paper, as has been done in most papers on the mimetic model.

The mimetic model was generalized in Ref. [5] to include a potential $V(\phi)$, so that the mimetic matter obtains a pressure $p=-V(\phi)$. This is very similar to the generalization in Ref. [6] for the dusty fluid model. With a potential, the mimetic model has many applications in cosmology, e.g., it can provide inflation, bounce, dark energy, and so on. This has stimulated much interest in the literature. For instance, its relation with disformal transformations [7] was discussed in Refs. [8–10], the Hamiltonian analyses were given in Refs. [11], it has been applied in various modified gravity models [12–23], and has also been studied with extensive cosmological and astrophysical interests [24–35]. The instability problem of the cosmological perturbations of the mimetic model with higher derivatives was recently studied in Refs. [36–42]. Some other recent progress on this topic can be found in Refs. [43–45].

In this paper, we will consider the direct couplings of the mimetic field to other matter in the universe, especially the couplings to baryons and photons in the uni-

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verse. We will concentrate on the original mimetic model [1] where the mimetic field has the action in Eq. (2) and mimics the dark matter in the universe. This model has shift symmetry, i.e., the action is invariant under the field shift $\phi\to\phi+C$ by a constant C. We think that it is the shift symmetry that excludes the potential $V(\phi)$ and guarantees the mimetic field to be dark matter. In addition, the shift symmetry prevents any interactions other than derivative couplings of the mimetic dark matter to the rest of the world. In this paper we will study the possible derivative couplings mimetic matter can have to baryons and photons, and their corresponding effects in cosmology.

This paper is organized as follows. In Section 2, we introduce briefly some properties of mimetic dark matter with a derivative coupling to the matter current. Then in Section 3, we will apply it to the baryogenesis model based on the coupling to baryons. The derivative couplings to photons and their implications in cosmology will be investigated in Section 4. Finally, we will conclude in Section 5.

2 Mimetic dark matter with derivative coupling

When considering the direct interaction between the mimetic field and other matter content, the original action (2) becomes

$$S = \int d^4x\sqrt{g}\left[\frac{M_p^2}{2}R+\lambda(g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi-M_1^4)\right] + \int d^4x\sqrt{g}\mathcal{L}_{int}+S_m \ . \tag{3}$$

As we mentioned in the previous section, the imposed shift symmetry requires that the interacting Lagrangian density $\mathcal{L}_{\rm int}$ can only depend on the derivatives of ϕ. Its simplest case is given by the following operator,

$$\mathcal{L}_{int}=\frac{1}{M_2}\nabla_\mu\phi J^\mu \ , \tag{4}$$

where J^μ is the matter current associated with a certain quantum number[1)]. The operator $\nabla_\mu\phi J^\mu$ has the mass dimension of five, so we need to introduce another mass scale M_2 to suppress it. The matter current is inversely proportional to the tensor density $\sqrt{g}$. For example, in a system of particles, the current can be defined as [46],

$$J^\mu=\frac{1}{\sqrt{g}}\sum_n q_n\int dx_n^\mu\delta^4(x-x_n) \ , \tag{5}$$

where q_n is the charge taken by the nth particle, x_n^μ is its coordinate, and $\delta^4(x-x_n)$ is the Dirac delta function. So the coupling term in the action is independent of the metric and has no contribution to the total energy-momentum tensor, which is the same as that of the original mimetic dark matter model [1]:

$$\tilde{T}^{\mu\nu}=2\lambda\nabla^\mu\phi\nabla^\nu\phi+T^{\mu\nu} \ , \tag{6}$$

where

$$T^{\mu\nu}=-\frac{2}{\sqrt{g}}\frac{\delta S_m}{\delta g_{\mu\nu}} \ , \tag{7}$$

which is the energy-momentum tensor of matter.

The gravitational field equation becomes

$$G_{\mu\nu}\equiv R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}=-\frac{1}{M_p^2}\tilde{T}_{\mu\nu} \ , \tag{8}$$

and its trace gives the Lagrange multiplier

$$2\lambda=-\frac{M_p^2G+T}{M_1^4} \ , \tag{9}$$

where G is the trace of the Einstein tensor and should not be confused with the Newton constant, and T is the trace of the energy-momentum tensor of matter. With this one can see that the energy-momentum tensor of the mimetic field has the form of a perfect fluid:

$$T_\phi^{\mu\nu}=-(M_p^2G+T)u^\mu u^\nu \ , \tag{10}$$

where the four velocity $u^\mu=\nabla^\mu\phi/M_1^2$, which is normalized as $g_{\mu\nu}u^\mu u^\nu=1$. This is given by the mimetic constraint, $g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi=M_1^4$. The energy-momentum tensor (10) shows that the mimetic matter is indeed dust-like, its pressure vanishes and its energy density is given by

$$\rho_\phi=-(M_p^2G+T) \ . \tag{11}$$

Hence, we see that the mimetic field can mimic the dark matter in the universe and the derivative coupling introduced in the action (3) does not change this consequence.

However, this derivative coupling will modify the equation of motion of the mimetic field to

$$2\lambda\Box\phi+2\nabla_\mu\lambda\nabla^\mu\phi+\frac{1}{M_2}\nabla_\mu J^\mu=0 \ , \tag{12}$$

where $\Box\equiv\nabla_\mu\nabla^\mu$. This shows that if the current is conserved, such a coupling vanishes and has no effect. However, when coupled to a non-conserved current, one can easily find that the above equation is equivalent to

$$\nabla_\mu T^{\mu\nu}=\frac{1}{M_2}\nabla_\mu J^\mu\nabla^\nu\phi \ , \tag{13}$$

which shows that there will be exchange of energy and momentum between mimetic dark matter and other matter. This effect is suppressed by M_2.

3 Mimetic dark matter and baryogenesis

In this section we consider the case where the current in Eq. (4) is the baryon current, $J^\mu=J_B^\mu$. In the standard

1) A similar interaction has been proposed in Ref. [44]. There, $J^\mu=\rho u^\mu$ is the energy flux current of the matter and is different from the one considered here.

model of particle physics, baryon number is conserved at low energy scales or low temperature, but it is violated at the high energy scales which were present in the early universe. Considering the Friedmann-Robertson-Walker (FRW) universe in which the mimetic field is homogeneous and has non-zero time derivative, $\dot{\phi}=M_1^2$, the coupling $\nabla_\mu\phi J_B^\mu/M_2$ reduces to

$$\frac{1}{M_2}\nabla_\mu\phi J_B^\mu\rightarrow\frac{\dot{\phi}}{M_2}J_B^0=\frac{M_1^2}{M_2}n_B=\frac{M_1^2}{M_2}(n_b-n_{\bar{b}}) , \tag{14}$$

where n_B is the net baryon number density. In such a background with non-vanishing $\dot{\phi}$, the Lorentz symmetry (more exactly the boost symmetry) is broken and correspondingly the CPT symmetry in the baryon sector is violated. Such a CPT violation will make a difference between baryons and anti-baryons. In the early universe, when the baryon number violating processes were in thermal equilibrium, the derivative coupling induced an effective chemical potential for baryons and an opposite potential for anti-baryons [47], see also Refs. [48, 49],

$$\mu_b=\frac{\dot{\phi}}{M_2}=\frac{M_1^2}{M_2}=-\mu_{\bar{b}} , \tag{15}$$

so they have different thermal distributions and one will get a temperature dependent baryon number density [50],

$$n_B=n_b-n_{\bar{b}}=\frac{g_b\mu_bT^2}{6} , \tag{16}$$

where $g_b=2$ is the number of degrees of freedom of the baryon and T is the temperature. On the other hand, the entropy density of the universe is given by [50]

$$s=\frac{2\pi^2}{45}g_{*s}T^3 , \tag{17}$$

where g_{*s} counts the effective degrees of freedom of the species which contribute to the entropy of the universe, and the main contributions come from relativistic particles. So usually s is approximately of the same order of magnitude as the number density of radiation. With this we can obtain the baryon-to-entropy ratio

$$\frac{n_B}{s}=\frac{15g_b}{4\pi^2}\frac{\mu_b}{g_{*s}T}=\frac{15}{2\pi^2}\frac{M_1^2}{g_{*s}M_2T}\sim10^{-2}\frac{M_1^2}{M_2T} , \tag{18}$$

where we have considered the fact that $g_{*s}\sim100$ in the radiation dominated epoch well before the electroweak phase transition [50].

This provides a model for producing the baryon number asymmetry thermally. These kinds of models are different from the conventional baryogenesis models where three conditions must be satisfied as first proposed by Sakharov [51], one of which is the departure from thermal equilibrium. The key point leading to the baryon number asymmetry in the model considered here is CPT violation. Similar models were proposed in Ref. [47], where the scalar field was not determined, in Refs. [48, 49], where the scalar field was identified with the dark energy, and in Refs. [52, 53], where the scalar field was related to the curvature scalar. The baryogenesis model in Refs. [48, 49] gives a picture where the current accelerating expansion of the universe (driven by dark energy) and the generation of baryon number asymmetry can be described uniformly in the same framework. The model considered here gives another picture, where it is the dark matter (the mimetic field) that plays an important role in producing the baryon number asymmetry.

Equation (18) shows that the baryon number asymmetry was less at earlier times (higher temperature) and became larger at later times with the decreasing of the temperature. This asymmetry froze out at the temperature T_D when the baryon number violating interactions decoupled from the thermal bath,

$$\left(\frac{n_B}{s}\right)_D\sim10^{-2}\frac{M_1^2}{M_2T_D} . \tag{19}$$

After that, baryon number is conserved and the coupling in Eq. (4) will have no effect on baryons. The baryon number asymmetry in our current universe is about $(n_B/s)_D\sim10^{-10}$, as required by big bang nucleosynthesis and the observational data of the cosmic microwave background radiation (CMB) [54]. Furthermore, the decoupling temperature T_D of baryon number non-conservation is around 100 GeV as known from the standard model, so we have the relation

$$M_1\sim10^{-3}\sqrt{M_2\ \mathrm{GeV}} . \tag{20}$$

This shows that the scale M_1, which quantifies the time derivative of the mimetic field, cannot be very large. Even if the scale M_2 is as large as the Planck mass $M_p\sim10^{18}$ GeV, M_1 is around 10^6 GeV.

The consequence (19) also implies that the baryon isocurvature perturbation produced in this model is negligibly small. Baryon isocurvature perturbation is the spatial fluctuation of the baryon-to-entropy ratio. Equation (19) shows that this ratio is quite homogeneous, because it is only determined by the scales M_1, M_2 and the decoupling temperature T_D, which is further fixed by the parameters of the standard model.

We can also turn to the leptogenesis model in which we replace the baryon current in Eq. (4) with the $B-L$ current, where L denotes the lepton number. In this leptogenesis model, we will have the same result (19) except that T_D is the decoupling temperature of interactions which violate $B-L$ instead of the baryon number. From the viewpoint of particle physics, this decoupling temperature is usually much higher than the electroweak scale, so the scale M_1 can be relatively larger.

4 Cosmic birefringence induced by mimetic dark matter

In this section we will consider the derivative coupling of mimetic dark matter to photons via the term (4), in which J^μ constructed from the electromagnetic field is the Chern-Simons current,

$$J^\mu = A_\nu \widetilde{F}^{\mu\nu} \ , \tag{21}$$

where $\widetilde{F}^{\mu\nu} = (1/2)\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$ denotes the dual tensor of the electromagnetic tensor, which is defined as $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The divergence of this current is proportional to the Chern-Pontryagin density,

$$\nabla_\mu J^\mu = \frac{1}{2} F_{\alpha\beta}\widetilde{F}^{\alpha\beta} \ . \tag{22}$$

Such a coupling conserves gauge invariance but induces Lorentz and CPT violations in the photon sector with the non-trivial background of the mimetic field. The effect of the Chern-Simons term is to rotate the polarization directions of photons when they propagate within this backgound [55, 56]. For CMB, the rotation angle is [57]

$$\chi = \frac{1}{M_2}(\phi_{lss} - \phi_0) \ . \tag{23}$$

This phenomenon is dubbed cosmic birefringence in the literature. In the above equation, the subscript lss means the last scattering surface, at which CMB photons decouple with matter, and 0 represents the value at the present time. Usually this angle is anisotropic and its anisotropy depends on the distribution of ϕ_{lss} on the last scattering surface, which means the rotation angle is direction-dependent. However, at the background (at the leading order) the mimetic constraint leads to $\phi = M_1^2 t$, and we will have an isotropic rotation angle,

$$\chi = \frac{M_1^2}{M_2}(t_{lss} - t_0) \ . \tag{24}$$

More detailed discussions on the anisotropic rotation can be found in Refs. [57–59]. In the model considered here, the anisotropy of the rotation angle, which depends on the fluctuation of ϕ_{lss}, is negligibly small due to the mimetic constraint. Because t_{lss} is much smaller than $t_0 \sim 1/H_0$, where H_0 is the current Hubble rate, the rotation angle is about

$$\chi \sim -\frac{M_1^2}{M_2 H_0} \ . \tag{25}$$

Cosmic birefringence will change the power spectra of CMB polarization [60–62] because it will convert part of the E-mode polarization to the B-mode. This result makes it possible to measure the rotation angle in terms of the CMB observational data, as was first done in Ref. [62]. This has been developed into an important and precise method to test CPT symmetry. Currently, the upper limit on the rotation angle is about $\mathcal{O}(1^\circ)$ [63][1]. Using the radian measure, the constraint on the rotation angle is $|\chi| \lesssim 10^{-2}$. With Eq. (25), this requires

$$M_1 \lesssim 0.1\sqrt{M_2 H_0} \sim 10^{-22}\sqrt{M_2 \ \mathrm{GeV}} \ , \tag{26}$$

where we have considered $H_0 \sim 10^{-43}$ GeV. This result shows that M_1 should be extremely small. For example, if we set $M_2 = M_p$, $M_1 \lesssim 10^{-4}$ eV. Compared with the requirement of baryogenesis in the previous section, the scale M_1 should be much smaller, otherwise the rotation angle or the CPT violating signal would exceed the limit from the observational data.

One can generalize these discussions to the cases of higher order couplings, i.e., coupling terms containing higher dimensional operators. We insist on discussing those interactions conserving the gauge invariance of the electromagnetic field and the shift symmetry of the mimetic dark matter. We also exclude coupling terms which explicitly break Lorentz covariance. The next order coupling satisfying these requirements is the following dimension-7 operator,

$$\mathcal{L}_{int} \ = \ \frac{\xi}{M_2^3}\phi^\rho(\nabla_\rho F_{\mu\nu})\widetilde{F}^{\mu\nu} \ , \tag{27}$$

where ξ is a dimensionless coupling constant which is usually thought to be of order one. This coupling term can be translated into the familiar form after some calculations, as shown below,

$$\begin{aligned}\mathcal{L}_{\rm int} \ &= \ \frac{\xi}{M_2^3}\phi^\rho(\nabla_\rho F_{\mu\nu})\frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta} \\ &= \ \frac{\xi}{M_2^3}\phi^\rho\nabla_\rho(F_{\mu\nu}\frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}) \\ &\quad -\frac{\xi}{M_2^3}\phi^\rho F_{\mu\nu}\frac{1}{2}\epsilon^{\mu\nu\alpha\beta}\nabla_\rho F_{\alpha\beta} \\ &= \ \frac{\xi}{M_2^3}\phi^\rho\nabla_\rho(F_{\mu\nu}\widetilde{F}^{\mu\nu}) - \mathcal{L}_{int} \ ,\end{aligned} \tag{28}$$

so that

$$\begin{aligned}\mathcal{L}_{\rm int} \ &= \ \frac{\xi}{2M_2^3}\phi^\rho\nabla_\rho(F_{\mu\nu}\widetilde{F}^{\mu\nu}) \to -\frac{\xi}{2M_2^3}\Box\phi F_{\mu\nu}\widetilde{F}^{\mu\nu} \\ &\to \frac{\xi}{M_2^3}(\nabla_\mu\Box\phi)A_\nu\widetilde{F}^{\mu\nu} \ ,\end{aligned} \tag{29}$$

where the arrows denote equivalences up to some total derivative terms. This interacting Lagrangian density together with $\sqrt{g}$ is not a topological term and will contribute to the energy-momentum tensor which sources the gravitational field. However, it is easy to see that this contribution is proportional to the Chern-Pontryagin density $F_{\mu\nu}\widetilde{F}^{\mu\nu}$ of the electromagnetic field. This is extremely small during the period from last scattering to now. This period is much later than the radiation dominated epoch, and we can safely neglect the back-reaction of this term to spacetime. What we focus on is the rotation angle of CMB polarization induced by this coupling. According to the discussions in the previous section, the rotation angle should be

$$\chi = \frac{\xi}{M_2^3}(\Box\phi_{lss} - \Box\phi_0) = 3\xi\frac{M_1^2}{M_2^3}(H_{lss} - H_0) \sim \frac{M_1^2}{M_2^3}H_{lss} \ , \tag{30}$$

1) In the future, the ground-based CMB experiment AliCPT, located in the Ali region of Tibet, China, should be able to detect a rotation angle as small as 0.01° [64, 65].

where we have considered the equation

$$\Box\phi=\ddot{\phi}+3H\dot{\phi}=3HM_1^2\ , \tag{31}$$

and neglected H_0 when compared with $H_{lss}\sim10^{-38}$ GeV. From the observational constraint on the rotation angle $|\chi|\lesssim10^{-2}$, we have

$$M_1\lesssim10^{18}M_2\sqrt{\frac{M_2}{\text{GeV}}}\ . \tag{32}$$

This is automatically satisfied as long as the scale M_2 is above 1 GeV, because M_1 should be less than M_p. However, if M_2 is much higher than 1 GeV, the rotation angle induced by the coupling in Eq. (27) would be too small to be detected.

Dimension-8 operators can be built as $\phi^\rho\phi_\rho F_{\mu\nu}\widetilde{F}^{\mu\nu}$ and $\phi^\rho\phi_\mu F_{\rho\nu}\widetilde{F}^{\mu\nu}$. The former has no effect because the mimetic constraint $\phi^\rho\phi_\rho=M_1^4$ renders this term to be the Chern-Pontryagin density (except a constant coefficient), which is a total derivative and can be dropped out from the total action. The latter operator is the same as the former up to a constant factor, as shown in the Appendix. Hence, both dimension-8 operators have null effect.

5 Conclusion

In this paper, we have considered some direct interactions of mimetic dark matter to baryons and photons in the universe. With shift symmetry, the mimetic field can only couple to other matter derivatively. These couplings have null or negligible back reactions to the spacetime and will not spoil the successes of the standard cosmological model. However, the mimetic field has a non-trivial background with constant but non-vanishing time derivative. Within this background the Lorentz and CPT symmetries are broken spontaneously. In terms of this feature, we have constructed models to generate the baryon number asymmetry at thermal equilibrium and the cosmic birefringence in the CMB, and showed possible links between dark matter and other observable phenomena in our universe.

Appendix A

Here we will prove that the operator $\phi^\rho\phi_\mu F_{\rho\nu}\widetilde{F}^{\mu\nu}$ in four dimensional spacetime is equal to $\phi^\rho\phi_\rho F_{\mu\nu}\widetilde{F}^{\mu\nu}$ up to a constant factor. For this we expand it as

$$\begin{aligned}\phi^\rho\phi_\mu F_{\rho\nu}\widetilde{F}^{\mu\nu} &= \phi_i\phi^j F_{j0}\widetilde{F}^{i0}+\phi_0\phi^0F_{0k}\widetilde{F}^{0k}+\phi_0\phi^iF_{ik}\widetilde{F}^{0k}\\ &\quad+\phi_i\phi^0F_{0k}\widetilde{F}^{ik}+\phi_i\phi^jF_{jk}\widetilde{F}^{ik}\ .\end{aligned} \tag{A1}$$

First, we can see that the third and forth terms on the right-hand side of the above equation vanish. Using the electric and magnetic vector fields $\vec{E}$, $\vec{B}$ and the relations $F_{0k}\sim E_k$, $F_{ij}\sim\epsilon_{ijk}B^k$, $\widetilde{F}^{0k}\sim B^k$, $\widetilde{F}^{ij}\sim\epsilon^{ijk}E_k$, it is easy to show that the third term is roughly

$$\phi_0\phi^iF_{ik}\widetilde{F}^{0k}\sim\phi_0(\nabla\phi\times\vec{B})\cdot\vec{B}=0\ , \tag{A2}$$

and the fourth term is

$$\phi_i\phi^0F_{0k}\widetilde{F}^{ik}\sim\phi^0(\nabla\phi\times\vec{E})\cdot\vec{E}=0\ . \tag{A3}$$

The first and fifth terms on the right-hand side of Eq. (A1) can be combined as

$$\begin{aligned}&\phi_i\phi^jF_{j0}\widetilde{F}^{i0}+\phi_i\phi^jF_{jk}\widetilde{F}^{ik}=\phi_i\phi^j\epsilon^{0ikl}\left(\frac{1}{2}F_{0j}F_{kl}+F_{0l}F_{jk}\right)\\ &= \frac{1}{\sqrt{g}}(\phi^1\phi_1+\phi^2\phi_2+\phi^3\phi_3)(F_{01}F_{23}+F_{02}F_{31}+F_{03}F_{12})\\ &= \frac{1}{2}\phi^i\phi_i\epsilon^{0kjl}F_{0k}F_{jl}=\phi^i\phi_iF_{0k}\widetilde{F}^{0k}\ .\end{aligned} \tag{A4}$$

So, Eq. (A1) is

$$\begin{aligned}\phi^\rho\phi_\mu F_{\rho\nu}\widetilde{F}^{\mu\nu} &= \phi_0\phi^0F_{0k}\widetilde{F}^{0k}+\phi^i\phi_iF_{0k}\widetilde{F}^{0k}=\phi_\rho\phi^\rho F_{0k}\widetilde{F}^{0k}\\ &= \frac{1}{4}\phi_\rho\phi^\rho F_{\mu\nu}\widetilde{F}^{\mu\nu}\ ,\end{aligned} \tag{A5}$$

and at the last step we have considered

$$\begin{aligned}F_{\mu\nu}\widetilde{F}^{\mu\nu} &= F_{0k}\widetilde{F}^{0k}+F_{k0}\widetilde{F}^{k0}+F_{ij}\widetilde{F}^{ij}=2F_{0k}\widetilde{F}^{0k}+F_{ij}\epsilon^{ij0k}F_{0k}\\ &= 4F_{0k}\widetilde{F}^{0k}\ .\end{aligned} \tag{A6}$$

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