

# Shell-model calculations for the semi-magic nucleus $^{85}\text{Br}$ and systematic features of the $N = 50$ odd- $A$ isotones\*

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**Abstract:** Level structures of  $^{85}\text{Br}$  have been investigated using the shell-model code nushellx within a large model space containing the neutron-core excitations across the  $N = 50$  closed shell. The calculated results have been compared with the available experimental data. Reasonable agreement between the experimental and calculated values is obtained, which indicates that the neutron-core excitations are essential to reproduce the level structures of  $^{85}\text{Br}$ . The systematic features of neutron-core excitations in the  $N = 50$  isotones are investigated.

**Keywords:** level structure, shell-model, neutron-core excitations

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## 1 Introduction

The structures of nuclei in the  $A \sim 80$  mass region have been the focus of extensive experimental and theoretical investigations in recent years. A number of interesting phenomena have been found, such as signature inversion [1], shape coexistence [2, 3], magnetic rotation [4, 5], chiral doublet bands [6], and multiple chiral doublet bands with octupole correlations [7]. In addition, as the breaking of the neutron-core has been systematically observed in the  $N = 50$  even-even [8–13] and odd- $A$  [14–17] isotones, investigation of the character of the  $N = 50$  shell gap, particularly its evolution with proton number, has also become a hot topic in this mass region [10, 18–21].

The systematic features of neutron-core excitation energies in the  $N = 50$  even-even isotones have been widely investigated in previous works [8–10, 18, 22]. However, relatively few studies have focused on the odd- $A$  isotones. This is mainly because of the limited or contradictory experimental results and the lack of specifically theoretical calculations containing the neutron-core excitations. For example, there exist three different level schemes for the semi-magic nucleus  $^{85}\text{Br}$  ( $Z = 35$ ,  $N = 50$ ), which were respectively reported in Refs. [14, 15, 23]. Furthermore, the identification of neutron-core excited states in  $^{85}\text{Br}$  remains controversial in the previous investigations [14, 23]. In Ref. [14], the  $19/2^+$  state at 4343 keV in  $^{85}\text{Br}$  was explained by the neutron-core excitation, based on the shell-model with the RITSSCHIL [24]

code. Then, Refs. [15, 23] extended and amended the level structures of  $^{85}\text{Br}$  by fusion-fission reactions, but did not perform any new theoretical calculation. In Ref. [23], the state at 4343 keV was reassigned as  $I^\pi = 17/2^-$ , and it was suggested that all the observed states up to the excitation energy of 5390 keV in  $^{85}\text{Br}$  can be described by various proton excitations.

Thus, it is interesting to investigate the origin of the levels and identify the role of the neutron-core excitations in  $^{85}\text{Br}$ . Based on the considerations above, we have performed shell-model calculations to investigate the level structures of  $^{85}\text{Br}$ . In order to further verify the calculated results, the evolution of the neutron-core excitations in the  $N = 50$  isotones is discussed.

## 2 Shell-model calculations

As mentioned above, the excited states of  $^{85}\text{Br}$  have been previously investigated in the framework of the shell-model with the RITSSCHIL code [14]. In their calculations, the model space contained the proton orbits  $1f_{5/2}$ ,  $2p_{3/2}$ ,  $2p_{1/2}$ ,  $1g_{9/2}$  and neutron orbits  $2p_{1/2}$ ,  $1g_{9/2}$ ,  $2d_{5/2}$  relative to an inert  $^{66}\text{Ni}$  ( $Z = 28$ ,  $N = 38$ ) core, and truncation of the occupation numbers in the  $2p_{1/2}$  and  $1g_{9/2}$  proton orbits was adopted. In the present work, the shell-model calculations have been performed using the NUSHELLX [25] code. The SNE model space is adopted with the residual interaction named SNET [26] in the code. The model space includes eight proton orbits ( $1f_{5/2}$ ,  $2p_{3/2}$ ,  $2p_{1/2}$ ,  $1g_{9/2}$ ,  $2d_{5/2}$ ,  $1g_{7/2}$ ,  $2d_{3/2}$ ,  $3s_{1/2}$ )

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and nine neutron orbits ( $1f_{5/2}$ ,  $2p_{3/2}$ ,  $2p_{1/2}$ ,  $1g_{9/2}$ ,  $2d_{5/2}$ ,  $1g_{7/2}$ ,  $2d_{3/2}$ ,  $3s_{1/2}$ ,  $1h_{11/2}$ ) relative to the  $^{56}\text{Ni}$  core.

We have performed two sets of shell-model calculations SM1 and SM2. For SM1, the valence protons are allowed to move freely among the  $1f_{5/2}$ ,  $2p_{3/2}$ ,  $2p_{1/2}$  and  $1g_{9/2}$  orbits, while the neutron shells  $1f_{5/2}$ ,  $2p_{3/2}$ ,  $2p_{1/2}$  and  $1g_{9/2}$  are kept fully occupied, i.e., only the proton excitations are taken into consideration. SM2 has the same proton configuration space as SM1. Besides, SM2 allows a single  $1g_{9/2}$  neutron across the  $N = 50$  closed shell into the  $2d_{5/2}$  orbit ( $\nu 1g_{9/2}^{-1}2d_{5/2}$ ), which leads to an expansion of the configuration space relative to SM1 and the shell-model calculations in Ref. [14].

### 3 Results and discussion

Comparisons of the experimental excitation energies for the positive- and negative-parity states of  $^{85}\text{Br}$  [27] with the predictions of the present shell-model calculations are shown in Figs. 1 and 2, respectively. The dominant wave functions of states within the SM2 configuration space are listed in Table 1. As shown in Fig. 1, the calculated states for both SM1 and SM2 configuration spaces are in reasonable agreement with the experimental data from the ground state  $3/2^-$  to  $11/2^-$  state. Obvious discrepancies (about 2 MeV) between the SM1 and SM2 appear from states  $13/2^-$  to  $19/2^-$  and only the shell-model calculations within the SM2 configuration space are in good agreement with the experimental states of  $^{85}\text{Br}$ . This indicates that the neutron-core excitations should be taken into consideration to interpret the negative-parity high-spin states of  $^{85}\text{Br}$ . As shown in Table 1, the main configurations of the states from  $3/2^-$  to  $11/2^-$  are  $\pi(1f_{5/2}2p_{3/2})^7$  or  $\pi(1f_{5/2}2p_{3/2})^62p_{1/2}$ . For the states from  $13/2^-$  to  $19/2^-$ , the main configurations are  $\pi(1f_{5/2}2p_{3/2})^7 \otimes \nu 1g_{9/2}^{-1}2d_{5/2}$  or  $\pi 1f_{5/2}^5 2p_{1/2}^2 \otimes \nu 1g_{9/2}^{-1}2d_{5/2}$ . It is clearly shown that the states from  $13/2^-$  to  $19/2^-$  come from neutron-core excitations across the  $N = 50$  closed shell.

As shown in Fig. 2, quantitative agreement is also obtained when comparison is performed between the positive-parity states [27] with the results of shell-model calculations. It should be noted that the observed ( $21/2$ ) state at 5390 keV has no parity assignment in previous work. Considering this state only decays to the ( $19/2^+$ ) state [23] and shows agreement with the calculated  $21/2^+$  state in SM2, we assume positive parity for this state. In Fig. 2, the calculations containing the neutron-core excitations (SM2) provide an improved description for the  $19/2^+$  and  $21/2^+$  levels in comparison with the pure proton excitations (SM1). In Table 1, the calculated main configuration of the  $19/2^+$  and  $21/2^+$  states is  $\pi(1f_{5/2}2p_{3/2})^6 1g_{9/2} \otimes \nu 1g_{9/2}^{-1}2d_{5/2}$ , which indicates that the  $19/2^+$  and  $21/2^+$  states originate from the neutron-core

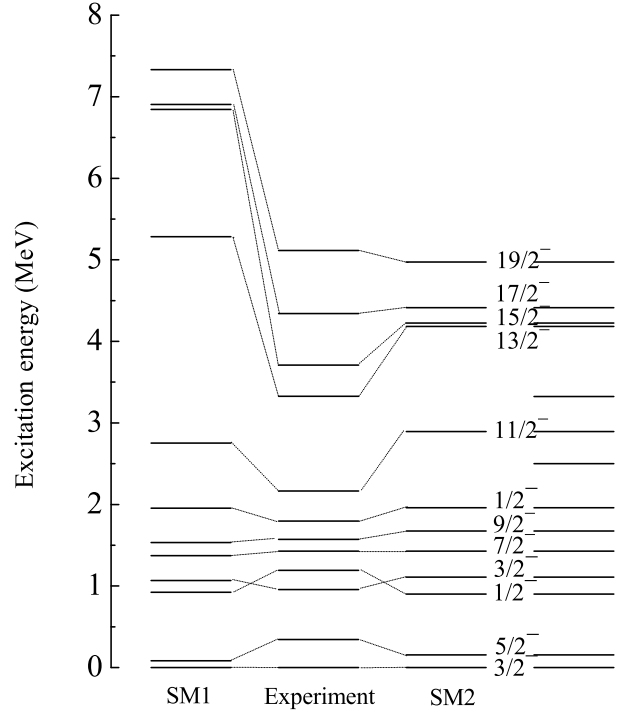


Fig. 1. Comparison of the negative-parity states from Ref. [27] in  $^{85}\text{Br}$  with the results of shell-model calculations. The calculations are performed using both the pure proton excitations (SM1) and also allowing neutron-core excitations across the  $N = 50$  shell gap (SM2).

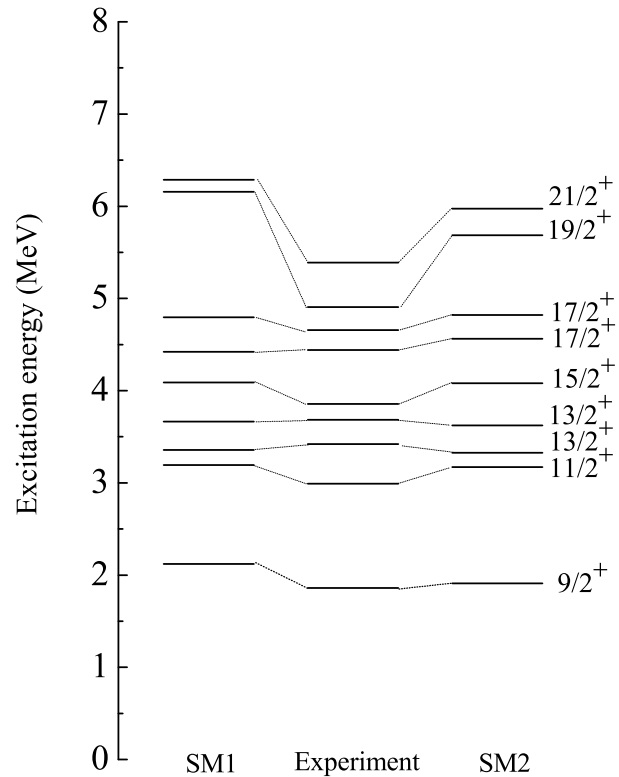


Fig. 2. Similar to Fig. 1 but for the positive-parity states.

Table 1. The dominant components of the wave functions for the positive- and negative-parity states within the SM2 configuration space.

$J^\pi$ ( $\hbar$ )	protons				neutron		partitions
	$1f_{5/2}$	$2p_{3/2}$	$2p_{1/2}$	$1g_{9/2}$	$1g_{9/2}$	$2d_{5/2}$	%
9/2 <sup>+</sup>	4	2	0	1	10	0	37.19
11/2 <sup>+</sup>	5	1	0	1	10	0	28.35
	4	2	0	1	10	0	28.51
13/2 <sup>+</sup>	4	2	0	1	10	0	48.90
13/2 <sub>2</sub> <sup>+</sup>	3	3	0	1	10	0	32.86
	4	2	0	1	10	0	20.18
15/2 <sup>+</sup>	5	1	0	1	10	0	26.69
	3	3	0	1	10	0	29.64
17/2 <sup>+</sup>	4	2	0	1	10	0	49.94
17/2 <sub>2</sub> <sup>+</sup>	4	2	0	1	10	0	53.92
19/2 <sup>+</sup>	4	2	0	1	9	1	48.45
21/2 <sup>+</sup>	4	2	0	1	9	1	85.34
1/2 <sup>-</sup>	5	2	0	0	10	0	31.94
	4	3	0	0	10	0	25.18
1/2 <sub>2</sub> <sup>-</sup>	4	2	1	0	10	0	48.33
	6	0	1	0	10	0	21.45
3/2 <sup>-</sup>	6	1	0	0	10	0	44.77
	4	3	0	0	10	0	21.77
3/2 <sub>2</sub> <sup>-</sup>	4	3	0	0	10	0	37.23
	5	2	0	0	10	0	19.64
5/2 <sup>-</sup>	5	2	0	0	10	0	58.81
7/2 <sup>-</sup>	5	2	0	0	10	0	34.30
	4	3	0	0	10	0	29.44
9/2 <sup>-</sup>	5	2	0	0	10	0	58.24
	5	1	1	0	10	0	16.89
11/2 <sup>-</sup>	4	3	0	0	10	0	67.63
13/2 <sup>-</sup>	4	3	0	0	9	1	59.45
15/2 <sup>-</sup>	4	3	0	0	9	1	62.45
17/2 <sup>-</sup>	5	0	2	0	9	1	50.96
19/2 <sup>-</sup>	5	2	0	0	9	1	52.88

excitations. The calculated configuration for the two states can be supported by the following systematic investigation for the  $N = 50$  isotones.

Based on the overall considerations of the positive- and negative-parity calculations, the present results indicate that a single  $1g_{9/2}$  neutron across the  $N = 50$  closed shell into the  $2d_{5/2}$  orbit is essential to reproduce the observed level structures in  $^{85}\text{Br}$ , especially for the negative-parity states.

#### 4 Energy evolution of the $N = 50$ neutron-core excitation

To further confirm the existing of the neutron-core excitations in  $^{85}\text{Br}$  and investigate the evolution in energy of the  $N = 50$  neutron-core excitation, we have performed a systematic investigation of the neutron-core excitations. Plots of the excitation energies of the states from the neutron excitations as a function of proton number  $Z$  in the  $N = 50$  even-even and odd- $A$  isotones are

presented in Figs. 3(a) and 3(b), respectively. The data are taken from Refs. [8, 10–12, 14–16, 23, 28–36].

The systematic behavior of the  $N = 50$  even-even isotones from  $^{82}\text{Ge}$  to  $^{90}\text{Zr}$  is shown in Fig. 3(a). The negative-parity states from neutron-core excitations are not included in the present systematic investigation due to the lack of experimental data. As shown in Fig. 3(a), the energies of the positive-parity states from neutron-core excitations increase with increasing  $Z$  number, which implies that the  $N = 50$  shell gap increases with increasing  $Z$  number. This has been explained by the attractive tensor part of the  $\pi f_{5/2}-\nu g_{9/2}$  and  $\pi p_{1/2}-\nu g_{9/2}$  interactions [18, 37] as the  $\pi f_{5/2}$  and  $\pi p_{1/2}$  orbits are filled from  $Z = 32$  to 40.

Figure 3(b) shows the evolution of the excitation energies relative to the  $9/2^+$  state of the  $N = 50$  odd- $A$  isotones from  $^{85}\text{Br}$  to  $^{91}\text{Nb}$ . One can see from Fig. 3(b) that the neutron-core excitation energies for both positive- and negative-parity states increase with increasing  $Z$  number, which is similar to the  $N = 50$  even-even

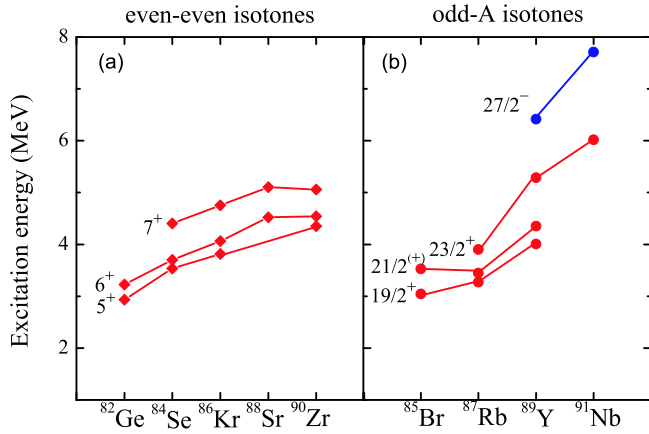


Fig. 3. (color online) Excitation energies for the states from the neutron core excitations are compared for the even-even (a) and odd-A (b) isotones. The energies of the states in the left (a) and right (b) parts of the figure are relative to the  $0^+$  and  $9/2^+$  states, respectively. Positive-parity excited states are drawn with red symbols, and negative-parity excited states are drawn with blue symbols.

isotones. It should be noted that the  $19/2^+$  and  $21/2^+$  states in  $^{85}\text{Br}$  fit the systematic behavior of the  $N = 50$  odd-A isotones, which further supports the parity assignment of the  $21/2^+$  state, and also supports the  $19/2^+$  and  $21/2^+$  states coming from the neutron-core excitation. In addition, we note that the increases of neutron-core excitation energies with  $Z$  for the  $N = 50$  odd-A isotones

have larger slopes than those for the  $N = 50$  even-even isotones. This might be explained by the unpaired proton in the  $N = 50$  odd-A isotones. It is an interesting phenomenon, and needs further theoretical and experimental studies.

## 5 Conclusion

Shell-model calculations containing the neutron-core excitations across the  $N = 50$  closed shell have been performed to investigate the level structures of  $^{85}\text{Br}$ . The evolution of the neutron-core excitation energies in the  $N = 50$  isotones has been discussed. The following conclusions have been drawn from the present work:

(I) The present calculations indicate that the neutron-core excitations are essential to reproduce high-spin level structures of  $^{85}\text{Br}$ . Based on the present calculations, the  $19/2^+$ ,  $21/2^+$ ,  $13/2^-$ ,  $15/2^-$ ,  $17/2^-$ , and  $19/2^-$  states in  $^{85}\text{Br}$  come from the neutron-core excitation across the  $N = 50$  gap.

(II) Increasing level energies of the states from the neutron-core excitations as the proton number increases is found in the  $N = 50$  odd-A isotones. The phenomenon is similar to that in the  $N = 50$  even-even isotones. However, the neutron-core excitation energies for the  $N = 50$  odd-A isotones increase faster than those of the  $N = 50$  even-even isotones with increasing  $Z$  number. This could be attributed to the unpaired proton in the  $N = 50$  odd-A isotones. This is still an open question that needs further research.

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