Effect of positive Q-value neutron transfers on sub-barrier fusion reactions*

Pei-Wei Wen(温培威)[1,2] Zhao-Qing Feng(冯兆庆)[3] Fan Zhang(张凡)[1,2] Cheng Li(李成)[1,2]
Cheng-Jian Lin(林承键)[4] Feng-Shou Zhang(张丰收)[1,2,5;1)]

[1]The Key Laboratory of Beam Technology and Material Modification of Ministry of Education, College of Nuclear Science and Technology, Beijing Normal University, Beijing 100875, China
[2]Beijing Radiation Center, Beijing 100875, China
[3]Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China
[4]China Institute of Atomic Energy, Beijing 102413, China
[5] Center of Theoretical Nuclear Physics, National Laboratory of Heavy Ion Accelerator of Lanzhou, Lanzhou 730000, China

Abstract: The role of positive Q-value neutron transfers in sub-barrier fusion reactions has been studied with a modified quantum coupled channels model with all order couplings (CCFULL model). Neutron rearrangement related only to the dynamical matching condition with no free parameters is implemented in the model, which provides a way to understand especially the Q-value dependence of sub-barrier fusion reactions. The fusion cross sections of the collision systems ^{40}Ca+^{94,96}Zr have been calculated and analyzed. The general trend of experimental data can be reproduced well with additional channels for neutron rearrangement. We find that enhancement of sub-barrier fusion cross sections is closely related to the Q-value of the transferred neutrons, in particular for channels with sequential even number transferred neutrons.

Keywords: sub-barrier fusion, CCFULL model, neutron transfer

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1 Introduction

Heavy-ion fusion reactions have been extensively investigated for decades, both experimentally and theoretically, and are significant for understanding the mechanisms of nuclear reactions and the properties of nuclear structure [1–4]. However, there is still an increasing number of challenging open questions, such as the unexpected phenomenon of fusion hindrance far below the Coulomb barrier [5], enhancement or hindrance of the fusion process induced by weakly bound nuclei [2, 6, 7], the interplay of multinucleon transfer and fusion reactions [8–11], and so on.

In this paper special attention is devoted to the dynamics of neutron transfer mediated sub-barrier fusion reactions. By comparative measurements for the nickel isotopes ^{58}Ni + ^{58}Ni, ^{58}Ni + ^{64}Ni , and ^{64}Ni + ^{64}Ni, it has benn suggested that valence neutrons may directly and dynamically influence the fusion process in general and sub-barrier penetration in particular [12]. Afterwards, many systematic experimental studies for different isotope combinations have confirmed that positive Q-value neutron transfers will enhance the cross sections of sub-barrier fusion reaction. For example, ^{40}Ca + ^{44,48}Ca compared to ^{40}Ca + ^{40}Ca [13], ^{32}S + ^{94,96}Zr compared to ^{32}S + ^{90}Zr [14–16], ^{28}Si + ^{94}Zr compared to ^{28}Si + ^{90}Zr [17], ^{40}Ca+^{116,124,132}Sn [18, 19], and some other combinations have exhibited similar features [20–23]. Besides that, there are also some fusion systems with positive Q-values but showing no obvious sub-barrier enhancements, such as ^{18}O+^{92}Mo,^{118}Sn [24, 25], ^{36}S+^{58}Ni [26], and some others [27–29].

Microscopic simulated dynamic models can explain certain sub-barrier enhancement phenomena, These models include quantum molecular dynamics [30–32] and the time dependent Hartree-Fock model [33]. Including the nucleon transfer channels in a completely quantal coupled channel approach is still a challenging work [34]. There are some earlier attempts to take into account the main reaction channels in heavy-ion collisions, including elastic scattering, the inelastic excitation of low-lying surface modes, the transfer of one or a few nucleons and the complete fusion reaction with certain approximations

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1) E-mail: fszhang@bnu.edu.cn

[35–37]. Recently, a quantal coupled channels approach has been achieved for simultaneous description of the fusion cross sections and the transfer probabilities, considering transfer cross sections as part of quasi-elastic scattering at large angles [38].

Another way to include the coupling of neutron transfers in empirical and quantum coupled channels calculation of fusion reactions is to incorporate a Q-value dependent dynamical matching factor describing the mismatch of initial and final orbit, accompanied by a semi-classical transfer probability from transfer reaction [34, 39]. The GRAZING model is known to describe the transfer process well [40, 41], but there is no significant influence when nuclei are considered to describe the capture process in this model [42]. Coupling with the one-particle transfer form factor has also been used in quantum coupled channels calculation, but played a minor role compared with couplings to low-lying surface modes [36, 37]. The above works raise doubts about the role of transfer probability from the transfer reaction in neutron transfer mediated sub-barrier fusion reactions. The empirical coupled channels model (ECC) is often used to predict the penetration probability in the first step of superheavy nucleus synthesis [30, 43–45]. Besides, one neutron pair transfer channel related only to Q-value, included in the widths of the barrier distribution in the ECC model, has recently been used to achieve a global fit of fusion data [46]. After analysis with the above model, puzzling quadruple deformation parameters of ^{94}Zr and ^{96}Zr were thought to play a role in the related sub-barrier enhancement problem [47]. In this paper, the Q-value dependent dynamical matching factor with non-free parameters is considered in fusion reactions, aiming to study especially the Q-value dependence of sub-barrier fusion reactions.

The structure of this paper is as follows. The theoretical framework of the CCFULL approach is outlined in Section 2. Results and discussion are presented in Section 3. Finally, conclusions are drawn in Section 4.

2 Theoretical framework

The Q-value dependent dynamical matching factor is expressed as:

$$\alpha_x(E,l,Q)=N_x^{-1}\exp\left(-C_xQ^2\right), \tag{1}$$

where x refers to the x-th neutron transfer channel, and $C_x=R_B\mu_{12}/\mathcal{X}_x\hbar^2(2E-B)$ determines the variance of the distribution of neutron transfer probability with respect to Q, which is given by semiclassical first order perturbation theory for transfer and inelastic reactions between heavy ions [48]. If there are n_x sequential transfer neutrons in the x-th transfer channel, then $\mathcal{X}_x=\kappa(\varepsilon_1)+\kappa(\varepsilon_2)+...+\kappa(\varepsilon_{n_x})$ is responsible for the sequential transfer nature of this semiclassical approach, with $\kappa(\varepsilon_i)=\sqrt{2\mu_i\epsilon_i/\hbar^2}$, where ϵ_i is the separation energy of i-th transferred neutron. N_x is a normalization factor, namely

$$N_x=\int_{-E}^{Q_x}\exp(-C_xQ^2)\mathrm{d}Q, \tag{2}$$

where Q_x is the Q-value of the ground-to-ground x-th neutron transfer channel with n_x transferred neutrons.

By incorporating the neutron rearrangement factor as kind of barrier distribution, the total penetration probability, averaging over different neutron transfer channels, is written as

$$\begin{aligned}T_l(E)&=N_{\mathrm{tr}}^{-1}\int_{-E}^{Q_x}[\delta(Q)+\sum_{x=1}^{x_{\max}}\alpha_x(E,l,Q)]\\&\quad\times T_l^{\mathrm{CC}}(E+Q)\mathrm{d}Q\\&=N_{\mathrm{tr}}^{-1}\sum_{i=0}^{x_{\max}}T_i',\end{aligned} \tag{3}$$

where

$$T_0'=T_l^{\mathrm{CC}}(E), \tag{4}$$

$$\begin{aligned}T'_i&=\int_{-E}^{Q_i}T_l^{\mathrm{CC}}(E+Q)\alpha_i(E,l,Q)\mathrm{d}Q\\&=N_i^{-1}\int_{-E}^{Q_i}T_l^{CC}(E+Q)\exp\left(-C_iQ^2\right)\mathrm{d}Q.\end{aligned} \tag{5}$$

The no-transfer channel is denoted as T_0'. $x_{\max}$ is the maximal number of the included neutron transfer channels, which can be tuned according to the of Q-values and corresponding experimental data in this theoretical framework.

The normalization constant in Eq. (3) is written as

$$\begin{aligned}N_{\mathrm{tr}}(E,l,Q)&=1+\int_{-E}^{Q_x}\alpha_{\mathrm{tr}}(E,l,Q)\mathrm{d}Q\\&=1+x_{\max}.\end{aligned} \tag{6}$$

$T_l^{\mathrm{CC}}(E)$ in the above equations is the penetration probability calculated by coupled channel calculation without neutron rearrangement, which can be predicted by the CCFULL approach with all order couplings [49] by solving the coupled channels Schrödinger equation

$$\begin{aligned}&\left[-\frac{\hbar^2}{2\mu}\frac{\mathrm{d}^2}{\mathrm{d}r^2}+\frac{l(l+1)\hbar^2}{2\mu r^2}+V_N^{(0)}(r)+\frac{Z_{\mathrm{P}}Z_{\mathrm{T}}e^2}{r}\right.\\&\left.+\epsilon_n-E\right]\psi_n(r)+\sum_m V_{nm}(r)\psi_m(r)=0,\end{aligned} \tag{7}$$

where the no-Coriolis approximation or isocentrifugal approximation is used. The solution of Eq. (7) is obtained under the incoming wave boundary condition The coupling potential is taken into account with full order in the CCFULL program. By summation over all possible intrinsic states, the fusion penetrability is given by

$$T_l^{\rm CC}(E)=\sum_n \frac{k_n(r_{\rm min})}{k_0}|T_n|^2. \quad (8)$$

Finally, the total fusion cross section is expressed as a sum over partial waves with angular momentum l at the centre-of-mass energy $E_{\rm c.m.}$, which is

$$\sigma_{\rm f}(E)=\sum_l \sigma_l(E)=\frac{\pi}{k_0^2}\sum_l (2l+1)T_l^{\rm CC}(E). \quad (9)$$

In this work, all calculations are carried out using the Woods-Saxon type nucleus-nucleus potential. The parameters are derived from the standard Akyüz-Winther parametrization [50], which is

$$U_{PT}=-16\pi\gamma a\frac{R_P R_T}{R_P+R_T}\frac{1}{1+\exp[(r-R_P-R_T)/a]}, \quad (10)$$

where

$$\frac{1}{a}=1.17[1+0.53(A_P^{-1/3}+A_T^{-1/3})], \quad (11)$$

$$R_i=1.2A_i^{1/3}-0.09, \quad (12)$$

$$\gamma=0.95(1-1.8\frac{(N_P-Z_P)(N_T-Z_T)}{A_P A_T}). \quad (13)$$

A_i, Z_i and N_i in above formulas are the mass, charge, and neutron numbers of i nucleus.

3 Results and discussion

For the reaction system ^{40}Ca+^{94,96}Zr, an octupole vibrational state (3^-) of ^{40}Ca and two quadrupole vibrational states (2^+) of ^{94,96}Zr are taken into account. The radius coupling parameter of the projectile used in the coupling Hamiltonian [49] is taken as 1.35 fm and that of the target is taken as 1.2 fm, to fit the experimental data of ^{40}Ca+^{94,96}Zr. Mutual excitations are included in the above coupled channel calculations for the two reaction systems. No rotational states are considered in this study. The excitation energies, parities λ^π and deformation parameters β_λ of the low-lying collective excited states for above nuclei are obtained from Refs. [51, 52]. The CCFULL model with the above parameters has been tested the for fusion reaction ^{40}Ca+^{90}Zr. When only vibration couplings are included, the experimental data can be described well by theoretical predictions.

In order to study the effect of positive Q-value on subbarrier enhancement for fusion reactions, the influence of different combinations of neutron transfer channels are investigated by closing or opening certain neutron transfer channels. The results are shown in Fig. 1(a) for ^{40}Ca+^{94}Zr and Fig. 1(b) for ^{40}Ca+^{96}Zr, denoted as wn$x$-$n_1n_2\dots n_x$, where x means there are x neutron transfer channels and n_i refers to the number of successive transferred neutrons in the i-th transfer channel. There are still large fusion cross section gaps between neutron transfer channels wn4-1234 and wn2-12 shown in Fig. 1(a) and Fig. 1(b) for ^{40}Ca+^{94,96}Zr, respectively. It can be seen that the gaps between sequential even number neutron transferred channels wn1-2 and wn2-12 are very small for these two fusion reactions. Similarly, the lines representing neutron transfer channels wn2-24 and wn4-1234 almost overlap. For both reaction systems, with vibration coupling and one neutron transfer channel added, the resulting fusion cross sections are not obviously changed with respect to the prediction with only vibration coupling. The fusion cross sections of wn2-24 and wn1-2 are even slightly larger than those of wn4-1234 and wn2-12. It is concluded that transfer channels with even transferred neutrons are dominant in these fusion systems.

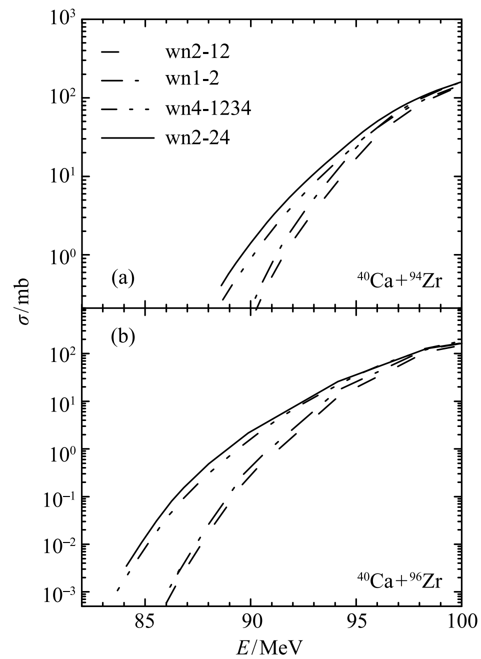

Fig. 1. Fusion cross sections for ^{40}Ca+ ^{94}Zr (a) and ^{40}Ca+^{96}Zr (b) with respect to incident energy in the center-of-mass frame. The theoretical calculations with vibration coupling and different neutron rearrangements are represented by different lines, which are denoted as wn$x$-$n_1n_2\cdots n_x$, where x means there are x neutron transfer channels and n_i refers to the number of successive transferred neutrons in the i-th transfer channel.

The above phenomenon can be explained from Eq. (3) and the corresponding Q-values of these reactions. Q-values for ground-to-ground neutron transfer of the nuclei mentioned above are listed in Table 1 for reference. For each neutron transfer channel T_i added, the denominator in Eq. (3) will be increased by one. Therefore, only when T_i is larger than the average of previous T, will clear enhancement be observed in the sub-barrier fusion reactions. Taking the Q-value as an example, this means that only when a large Q-value leap exists, will there be significant enhancement of sub-barrier fusion reactions. For example, Q_{1n} of ^{40}Ca+^{94}Zr is 0.143 MeV, which will not cause obvious enhancement, while Q_{2n} is 4.890 MeV, which will enhance the sub-barrier fusion cross sections a lot from this point of view. Similarly for do the transfers for Q_{3n} and Q_{4n}.

Table 1. Q-values of ground state to ground state neutron transfers from target to projectile (in MeV).

reaction	1n	2n	3n	4n
^{40}Ca+^{90}Zr	−3.606	−1.444	−5.865	−4.183
^{40}Ca+^{94}Zr	+0.143	+4.890	+4.188	+8.125
^{40}Ca+^{96}Zr	+0.508	+5.527	+5.241	+9.637

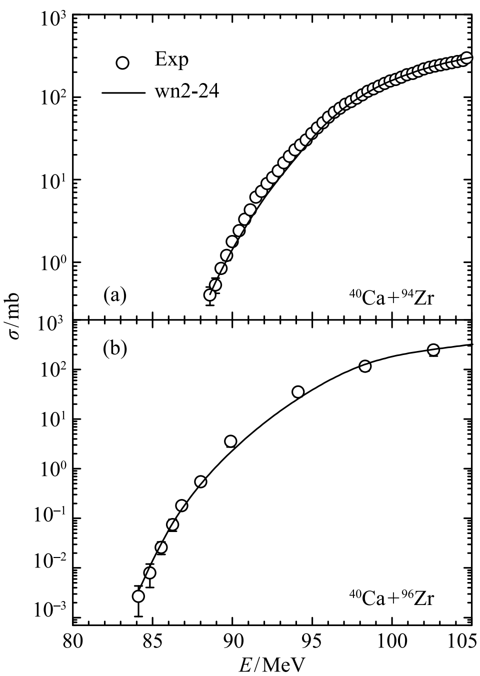

Fig. 2. Experimental and calculated fusion cross sections for ^{40}Ca+^{94}Zr (a) and ^{40}Ca+^{96}Zr (b) with respect to incident energy in the center-of-mass frame. Experimental data, denoted by open circles, are obtained from Ref. [53] for ^{40}Ca+^{94}Zr and Ref. [54] for ^{40}Ca+^{96}Zr. Denotations of lines are the same as in Fig. 1.

For the fusion reaction ^{40}Ca+ ^{94}Zr, with two more neutron transfers the fusion cross section will not be enhanced according to the above analysis, because Q_{5n} (3.57 MeV) and Q_{6n} (4.65 MeV) are all smaller than Q_{4n} (8.12 MeV). To make the theory consistent for both ^{40}Ca+ ^{94}Zr and ^{40}Ca+ ^{96}Zr, four neutron transfer channels at most are considered in this study. In Fig.2, comparisons of experimental and theoretical results of fusion reactions for ^{40}Ca+ ^{94}Zr (a) and ^{40}Ca+ ^{96}Zr (b) are displayed. Theoretical fusion cross sections are predicted by the CCFULL model implemented with vibration couplings and neutron rearrangement coupling. It can be clearly seen from Fig. 2(a) and Fig. 2(b) that the experimental fusion cross sections can all be reproduced well when two neutron transfer channels (wn2-24) are included for ^{40}Ca+ ^{94,96}Zr. The differences between theoretical calculation (wn4-1234) and experimental data can be reflected from the slight gaps between the solid lines and open circles. For ^{40}Ca+ ^{94}Zr, the calculated results are a little lower than the experimental data in the sub-barrier energy region, which will result in the peak of barrier distribution of this solid line being lower than the one extracted from experimental data.

4 Conclusions

In conclusion, the effect of positive Q-value neutron transfers on sub-barrier fusion reactions has been studied in detail by combining the modified quantum coupled channels model with all order couplings (the CCFULL model). The model takes into account neutron rearrangement related only to the dynamical matching condition, which is used to study especially the Q-value dependence of sub-barrier fusion reactions. No more free parameters have been introduced by incorporating neutron rearrangement. The fusion cross sections of the collision systems ^{40}Ca+^{94,96}Zr have been reproduced well by the CCFULL approach with standard Akyüz-Winther parametrization. The influence of different combinations of neutron transfer channels were investigated by closing or opening certain neutron transfer channels. By comparing different isotope combinations, it has been found that fusion cross sections can be significantly enhanced by neutron transfers with positive Q-values. Moreover, analysis of the results of successive neutron transfers shows that enhancement of sub-barrier fusion cross sections is not directly proportional to the Q-value, and transfer channels with even transferred neutrons play a vital role in the reaction systems ^{40}Ca+^{94,96}Zr.

The nature of this method is successive transfer, but transfer channels with even transferred neutrons are dominant for the reaction systems ^{40}Ca+^{94,96}Zr, which could be compared with the pair transfer in understanding pair correlation. These results suggest that a large amount of information has been included in the Q-value itself, such as pairing effects and rearrangement effects, which should be considered by further development of the fully quantal coupled channels method.

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