Systematic study of the product $((E(2_2^+)/E(2_1^+))*B(\mathrm{E2})\uparrow)$ through the asymmetric rotor model

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Abstract: A systematic study of the product $((E(2_2^+)/E(2_1^+))*B(\mathrm{E2})\uparrow)$ is carried out in the major shell space $Z=50-82, N=82-126$ within the framework of the asymmetric rotor model where the asymmetry parameter γ_0 reflects change in the nuclear structure. A systematic study of the product $((E(2_2^+)/E(2_1^+))*B(\mathrm{E2})\uparrow)$ with neutron number N is also discussed. The product $((E(2_2^+)/E(2_1^+))*B(\mathrm{E2})\uparrow)$ provides a direct correlation with the asymmetry parameter γ_0. The effect of subshells is visible in Ba-Gd nuclei with $N>82$, but not in Hf-Pt nuclei with $N>104$. We study, for the first time, the dependency of the product $((E(2_2^+)/E(2_1^+))*B(\mathrm{E2})\uparrow)$ on the asymmetry parameter γ_0.

Keywords: nuclear structure, asymmetric rotor model, reduced electric quadrupole transition probability

PACS: 21.10.Re, 21.60.Ev, 27.60.+j **DOI:** 10.1088/1674-1137/40/9/094104

1 Introduction

The systematic study of nuclear structure with proton number Z or neutron number N gives a deeper understanding of the nuclear interactions involved. The collective nuclear structure of medium mass nuclei ($A=150$–200) changes from vibrator to rotor as we go away from the closed shell. It is preferred to reproduce these changes in the structure of nuclei with Z or N. Many microscopic and macroscopic methods have been introduced to study the nuclear spectra. In microscopic theory, like the shell model in spherical or deformed nuclei, the Z or N dependency of nucleon-nucleon interactions is observed to study the experimental data of nuclear spectra. In a phenomenological approach, the parameters of the Hamiltonian H are fitted to experimental data and the nuclear spectra are then studied. In empirical studies, the systematic variation of experimental data is observed with Z, N or A in order to understand the nucleon-nucleon interactions. The Interacting Boson Model (IBM) [1] has used all the above ways to study nucleon-nucleon interactions. Most of the studies have been done using the phenomenological IBM model (see Refs. [2–4]). The systematic study of the first excited state energy $E(2_1^+)$ and energy ratio $R_{4/2}=E(4_1^+)/E(2_1^+)$ with the total boson number $N_\mathrm{B}=N_\mathrm{p}+N_\mathrm{n}$ (where $N_\mathrm{p}=$ proton boson numbers and $N_\mathrm{n}=$ neutron boson numbers) and $N_\mathrm{p}N_\mathrm{n}$ product was studied by Casten [2]. In IBM-1, the coefficient parameter χ of the Hamiltonian in the quadrupole operator [5] plays an important role in determining the structural changes in nuclei. In IBM-2 [1], the structure of the nucleus is supposed to be a function of the $N_\mathrm{p}N_\mathrm{n}$ product.

The rigid triaxial Asymmetric Rotor Model (ARM) of Davydov and Filippov [6] explains the structure of transitional nuclei and the obtained results are better than the axially symmetric rotation model. The energy level spacing and transitional properties of excited states in even-even nuclei can be calculated using the ARM model. The changes in structure of nuclei are observed in terms of the asymmetry parameter γ_0 of the ARM model. A small correlation of $B(\mathrm{E2})$ ratios $(2_2^+\rightarrow 0_1^+/2_1^+)$, $(2_2^+\rightarrow 0/2_1^+\rightarrow 0)$, $(2_2^+\rightarrow 2_1^+/2_1^+\rightarrow 0)$ with $R_{2\gamma}=E(2_2^+)/E(2_1^+)$ and noticeable deviations in the curve was observed for low $R_{2\gamma}$ [7]. The results of the ARM model and Rotation-Vibration model [8] were compared by Davidson [9] and it was observed that both the models are equally good. The Davydov-Rostovsky model [10] was proposed to study the β-band and calculated the rotational level energies for spins $4,6,8$ in even-even nuclei. Gupta and Sharma [11] illustrated that the $B(\mathrm{E2},2_\gamma\rightarrow 0/2)$ and $B(\mathrm{E2},3_\gamma\rightarrow 2/4)$ ratios have some dependence on the asymmetry parameter γ_0. Mittal et al. [12] extended this search to study neutron-deficient light $\mathrm{Te}-\mathrm{Sm}$ nuclei for $N<82$ and predicted the same results. The correlation of $B(\mathrm{E2})\uparrow$ values with the asymmetry parameter γ_0 have been studied in Ref. [13].

The global best fit equation for nuclear data was introduced by Grodzins [14]:

Received 16 December 2015

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$$(E(2_1^+)*B(\mathrm{E2})\uparrow)\sim \text{constant}, \quad (1)$$

using the relation between the reduced electric quadrupole transition probability, $B(\mathrm{E2};0_1^+\rightarrow 2_1^+)$ values ($=B(\mathrm{E2})\uparrow$ values) and the first excited state energy $E(2_1^+)$. Gupta [15] pointed out that the constancy of the Grodzins product breaks down in the combined effect of $Z=64$ subshell effect and transition at $N=88-90$. Recently, Kumari and Mittal [16] studied the correlation between the Grodzins product $(E(2_1^+)*B(\mathrm{E2})\uparrow)$ and $N_\mathrm{p}N_\mathrm{n}$ product, and further extended their research work to study the dependence of the Grodzins product $(E(2_1^+)*B(\mathrm{E2})\uparrow)$ on the asymmetry parameter γ_0 [17]. It would be interesting to replace the first excited state energy, $E(2_1^+)$ with energy ratio, $R_{2\gamma}$ because the asymmetry parameter γ_0 is sensitive to $R_{2\gamma}$. In this context, the equation becomes

$$(R_{2\gamma}*B(\mathrm{E2})\uparrow)\sim \text{constant}. \quad (2)$$

We present extensive experimental data for the product $(R_{2\gamma}*B(\mathrm{E2})\uparrow)$ and study it with the asymmetry parameter γ_0 for the first time.

2 Theory

The Hamiltonian of the ARM [6] can be written as:

$$H=\sum_{\lambda=1}^{3}\frac{AJ_\lambda^2}{2\sin^2\left(\gamma-\frac{2\pi}{3}\lambda\right)}, \quad (3)$$

where $A=\frac{\hbar^2}{4B\beta^2}$ has the dimensions of energy. The J_λ^2 are the angular momentum projection operators on the axes of the coordinate system related to the nucleus. The rotational level energies for 2,3,5 spins and the transitional probabilities between these energy levels have been obtained by treating the nucleus as a triaxial ellipsoid. The asymmetry parameter γ_0 of the ARM switches between 0° and 30°. It determines the deviation in shape of the nucleus from axial symmetry. When the asymmetry parameter γ_0 is zero, the energy spectrum is similar to an axially symmetric nucleus. Even though the increase of asymmetry parameter γ_0 affects the rotational energy spectrum of the nucleus very slightly, some new rotational energy states with spin 2, 3, 4, 5, 6 etc. appear (see Fig. 1 of Ref. [6]). This effect becomes large as the asymmetry parameter reaches $\gamma_0=20°$. The nucleus gets deformed with the increase in asymmetry parameter γ_0 and finally develops into a triaxial near $\gamma_0=30°$.

There are many approaches to calculate the asymmetry parameter γ_0. Firstly, Varshni and Bose [18] used $R_{4/2}$ to calculate the asymmetry parameter γ_0 and exclude nuclei with $R_{4/2}<8/3$. The first excited state $E(2_1^+)$ as well as the second excited state $E(4_1^+)$ were used to determine the asymmetry parameter γ_0 [19]. Gupta et al. [20] proposed another approach to calculate the value of the quadrupole moment Q using the sum of $B(\mathrm{E2})$ values of $2_1^+\rightarrow 0_1^+$ and $2_2^+\rightarrow 0_1^+$. The value of the asymmetry parameter γ_0 has been calculated but this approach was not so fruitful and had some shortcomings. Davydov et al. [6] used $R_{2\gamma}$ to determine the asymmetry parameter γ_0 and this approach was found to be valid [11]. We calculate the value of the asymmetry parameter γ_0 by using $R_{2\gamma}$ from the equation:

$$\gamma_0=\frac{1}{3}\sin^{-1}\left(\frac{9}{8}\left(1-\left(\frac{R_{2\gamma}-1}{R_{2\gamma}+1}\right)^2\right)\right)^{1/2}. \quad (4)$$

The experimental values of $E(2_2^+)$ and $E(2_1^+)$ are taken from the website of the National Nuclear Data Center, Brookhaven National Laboratory, USA [21]. The reduced electric quadrupole transition probability, $B(\mathrm{E2})\uparrow$ are obtained from Ref. [22] where the $B(\mathrm{E2})\uparrow$ values were compiled for the even-even nuclei of the $0\leqslant A\leqslant 260$ mass region.

3 Results and discussion

Gupta et al. [23] suggested splitting the major shell space $Z=50$–82, $N=82$–126 into four quadrants on the basis of valence-particles and valence-holes. Quadrant-I has particle-particle bosons, quadrant-II contains hole-particle bosons and quadrant-III consists of hole-hole bosons. Quadrant-IV contains particle-hole bosons and no nuclei lie in this region. A detailed discussion of the product $(R_{2\gamma}*B(\mathrm{E2})\uparrow)$ with the asymmetry parameter γ_0 in quadrants-I, II and III is presented in the following subsections.

3.1 Ba-Gd nuclei, $N>82$ region

This region contains particle-particle bosons of the $Z=50-82, N=82-126$ major shell region, named the $N>82$ region. The plot of product $(R_{2\gamma}*B(\mathrm{E2})\uparrow)$ against neutron number N for Ba-Gd nuclei is shown in Fig. 1. The sudden increase in the value of product $(R_{2\gamma}*B(\mathrm{E2})\uparrow)$ at $N=88$ is due to the onset of deformation at $Z=64$, as stated in many research papers, see Ref. [2, 24, 25]. The product $(R_{2\gamma}*B(\mathrm{E2})\uparrow)$ is plotted versus the asymmetry parameter γ_0 for quadrant-I in Fig. 2. This region is well known in the literature because of the presence of $Z=64$ subshell closure at $N=88-90$ isotones. The asymmetry parameter γ_0 is sensitive to $R_{2\gamma}$ for $\gamma_0=0°$ to 15° but it is much less sensitive in the range of $\gamma_0=20°$ to 30° (see Ref. [11]). On multiplying the energy ratio $R_{2\gamma}$ with $B(\mathrm{E2})\uparrow$ values, the asymmetry parameter γ_0 becomes less sensitive to the product ($R_{2\gamma}*B(\mathrm{E2})\uparrow$) for 20° to 30°. The effect of the $Z=64$ subshell is also visible in Fig. 2. The value of the product ($R_{2\gamma}*B(\mathrm{E2})\uparrow$) increases suddenly at $N=88-90$ isotones. Therefore, the nuclei Nd and Sm show a sharp increase in the product ($R_{2\gamma}*B(\mathrm{E2})\uparrow$) at $N=88-90$ ($\gamma_0\rightarrow 19°$ to 13°). However, the overall trend of the curve is smooth. The product ($R_{2\gamma}*B(\mathrm{E2})\uparrow$) shows more smoothness with the asymmetry parameter γ_0 as compared to the neutron number N for Ba-Gd nuclei in the $N>82$ region (see Figs. 1 and 2).

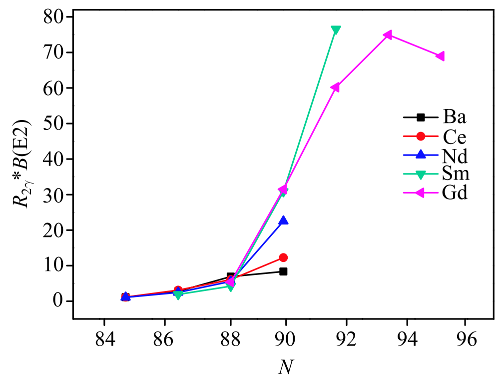

Fig. 1. Plots of the product ($R_{2\gamma}*B(\mathrm{E2})\uparrow$) vs. N for the $N>82$ region in Ba-Gd nuclei.

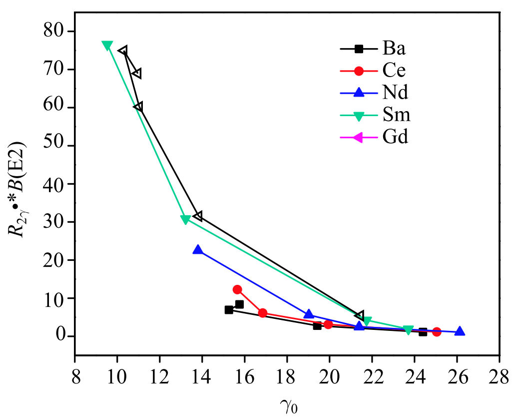

Fig. 2. Plots of the product ($R_{2\gamma}*B(\mathrm{E2})\uparrow$) *vs.* asymmetry parameter γ_0 for the $N>82$ region in Ba-Gd nuclei.

The product ($R_{2\gamma}*B(\mathrm{E2})\uparrow$) decreases smoothly with increase in the asymmetry parameter γ_0. The product ($R_{2\gamma}*B(\mathrm{E2})\uparrow$) shows more smoothness as compared to the Grodzins product ($E(2_1^+)*B(\mathrm{E2})\uparrow$) with asymmetry parameter γ_0 in quadrant-I (see Ref. [17]). The calculated values of asymmetry parameter γ_0 for all nuclei of quadrant-I are shown in Fig. 3. This can be taken as a reference for identifying the isotope numbers related to the data points in Fig. 2.

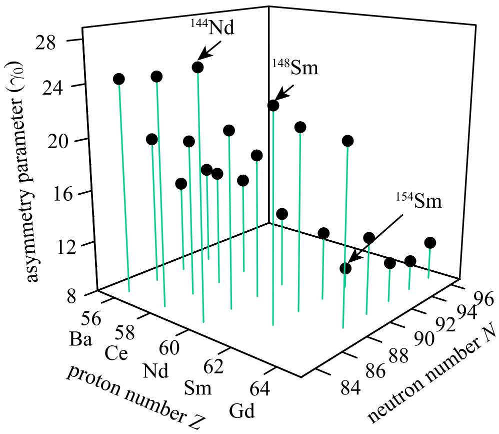

Fig. 3. The asymmetry parameter γ_0 as a function of proton number and neutron number obtained by using the Asymmetric Rotor Model for quadrant-I.

3.2 Dy-Hf nuclei, $N<104$ region

This is a proton-hole and neutron-particle boson subregion of the $Z=50-82, N=82-126$ major shell region known as quadrant-II. The plot of product ($R_{2\gamma}*B(\mathrm{E2})\uparrow$) against neutron number N for Dy-Hf nuclei is shown in Fig. 4. The product ($R_{2\gamma}*B(\mathrm{E2})\uparrow$) versus the asymmetry parameter γ_0 is plotted in Fig. 5 for the $N<104$ region. This quadrant-II is described as a transition region of $SU(3)$ to $U(5)$ or $O(6)$ region. The product ($R_{2\gamma}*B(\mathrm{E2})\uparrow$) shows more smoothness with the asymmetry parameter γ_0 as compared to the neutron number N for Dy-Hf nuclei in the $N<104$ region (see Figs. 4 and 5).

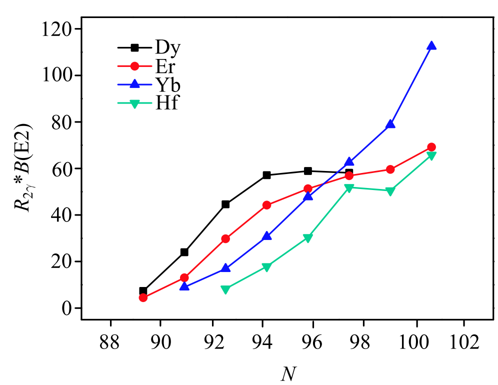

Fig. 4. Plots of the product ($R_{2\gamma}*B(\mathrm{E2})\uparrow$) vs. N in Dy-Hf nuclei.

The asymmetry parameter γ_0 is sensitive to $R_{2\gamma}$ for $\gamma_0=0°$ to 15°. This is also true for the product ($R_{2\gamma}*B(\mathrm{E2})\uparrow$) (see Fig. 5). However, the asymmetry parameter γ_0 ranging from 20° to 30° is less sensitive to $R_{2\gamma}$. This feature is seen in the product ($R_{2\gamma}*B(\mathrm{E2})\uparrow$). The Grodzins product ($E(2_1^+)*B(\mathrm{E2})\uparrow$) shows more constancy for deformed nuclei [15]. This is also true in the case of the product $(R_{2\gamma} * B(\mathrm{E2}) \uparrow)$ as quadrant-II consists of both well-deformed and transitional nuclei. The product $(R_{2\gamma} * B(\mathrm{E2}) \uparrow)$ smoothly decreases for all these nuclei. This shows the direct correlation of the product $(R_{2\gamma} * B(\mathrm{E2}) \uparrow)$ with the asymmetry parameter γ_0. The product $(R_{2\gamma} * B(\mathrm{E2}) \uparrow)$ shows smoother graphs than the Grodzins product $(E(2_1^+) * B(\mathrm{E2}) \uparrow)$ with asymmetry parameter γ_0 in quadrant-II (see Ref. [17]). The asymmetry parameter γ_0 as a function of N and Z for $N < 104$ in Dy-Hf nuclei is plotted in Fig. 6.

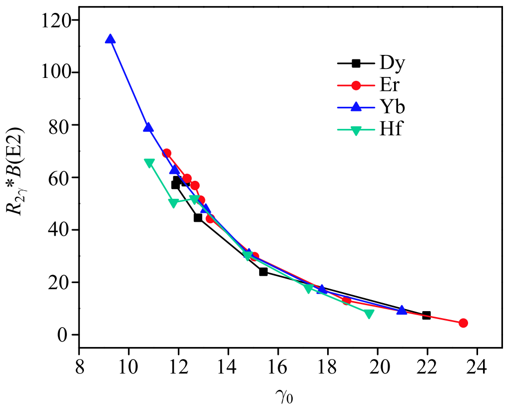

Fig. 5. Plots of the product $(R_{2\gamma} * B(\mathrm{E2}) \uparrow)$ vs. asymmetry parameter γ_0 for the $N < 104$ region in Dy-Hf nuclei.

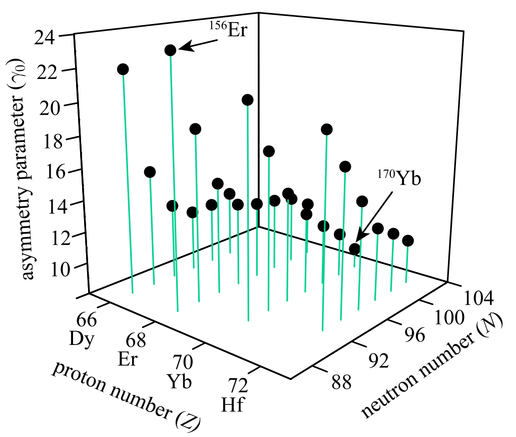

Fig. 6. The asymmetry parameter γ_0 as a function of proton number and neutron number obtained by using the Asymmetric Rotor Model for quadrant-II.

3.3 Hf-Pt nuclei, $N > 104$ region

These nuclei lie in quadrant-III of the major shell space $Z = 50-82, N = 82-126$ and contain the hole-hole subspace. Quadrant-III consists of nuclei which undergo transition from well deformed to γ-soft. The plot of product $(R_{2\gamma} * B(\mathrm{E2}) \uparrow)$ against neutron number N for Hf-Pt nuclei is shown in Fig. 7. The variation of the product $(R_{2\gamma} * B(\mathrm{E2}) \uparrow)$ versus the asymmetry parameter γ_0 shows a smoothly decreasing curve (see Fig. 8) for Hf-Pt nuclei. The product $(R_{2\gamma} * B(\mathrm{E2}) \uparrow)$ shows more smoothness with the asymmetry parameter γ_0 as compared to the neutron number N for Hf-Pt nuclei in the $N > 104$ region (see Figs. 7 and 8).

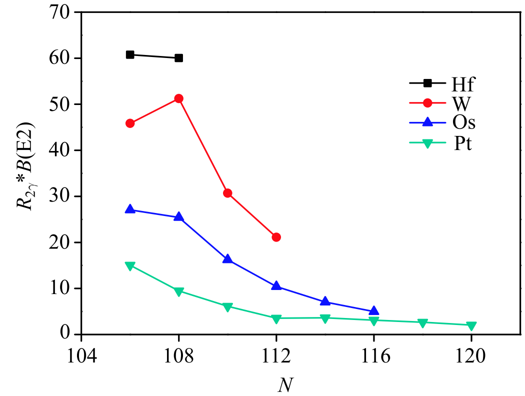

Fig. 7. Plots of the product $(R_{2\gamma} * B(\mathrm{E2}) \uparrow)$ vs. N in Hf-Pt nuclei.

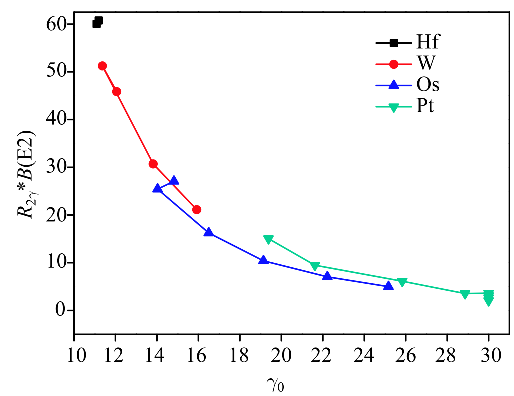

Fig. 8. Plots of the product $(R_{2\gamma} * B(\mathrm{E2}) \uparrow)$ vs. asymmetry parameter γ_0 for $N > 104$ region in Hf-Pt nuclei.

The $^{178-180}$Hf and $^{180-186}$W nuclei lie in the range $\gamma_0 = 10° - 16°$ and this region is sensitive to $R_{2\gamma}$. This shows that the product $(R_{2\gamma} * B(\mathrm{E2}) \uparrow)$ is also sensitive in the range of asymmetry parameter $\gamma_0 = 0°$ to $15°$ (as the product $(R_{2\gamma} * B(\mathrm{E2}) \uparrow)$ varies from 20 to 60 $\mathrm{e^2b^2}$). The $^{182-192}$Os nuclei vary from $\gamma_0 = 14°$ to $26°$, implying that they are shape phase transitional nuclei. The $^{184-196}$Pt nuclei lie in a less sensitive region of the asymmetry parameter γ_0. The product $(R_{2\gamma} * B(\mathrm{E2}) \uparrow)$ with asymmetry parameter γ_0 predicts that the $^{192-196}$Pt nuclei make a transition from γ-soft to rigid nuclei. A small neutron sub-shell gap is effective at $N = 114$ in quadrant-III (see Ref. [26]). The product $(R_{2\gamma} * B(\mathrm{E2}) \uparrow)$ is not affected due to this neutron shell gap at $N = 114$ because the maximum value of the asymmetry parameter γ_0 is 30°

and for Pt nuclei, the curve merges at $\gamma_0 = 30°$. Therefore, we get the smooth decreasing curve for quadrant-III. This shows the direct dependency of the product $(R_{2\gamma} * B(\text{E2}) \uparrow)$ on the asymmetry parameter γ_0. The product $(R_{2\gamma} * B(\text{E2}) \uparrow)$ shows a smoother correlation with asymmetry parameter γ_0 as compared to the relation between the Grodzins product $(E(2_1^+) * B(\text{E2}) \uparrow)$ and the asymmetry parameter γ_0 in quadrant-III (see Ref. [17]). The asymmetry parameter γ_0 values in Hf-Pt nuclei for the $N > 104$ region are presented in Fig. 9 and these values can be considered as a reference to identify the isotope number related to the data points in Fig. 8.

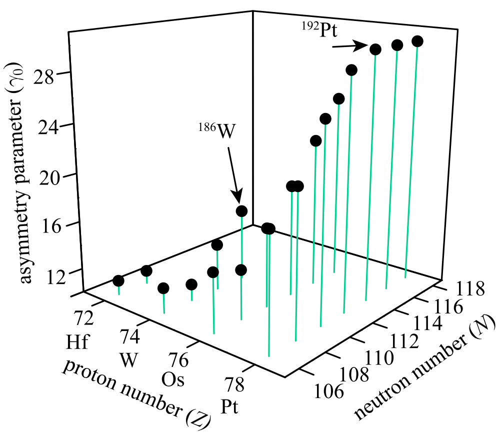

Fig. 9. The asymmetry parameter γ_0 as a function of proton number and neutron number obtained by using the Asymmetric Rotor Model for quadrant-III.

4 Conclusion

The product $(R_{2\gamma} * B(\text{E2}) \uparrow)$ provides a direct correlation with the asymmetry parameter γ_0. The systematics of the product $(R_{2\gamma} * B(\text{E2}) \uparrow)$ with neutron number N were also discussed. In all the above mentioned quadrants, the product $(R_{2\gamma} * B(\text{E2}) \uparrow)$ varies from 0 to 80 e^2b^2. In quadrant-I with $N > 82$, the product $(R_{2\gamma} * B(\text{E2}) \uparrow)$ decreases with increase in the asymmetry parameter γ_0. The effect of the $Z = 64$ subshell is also visible in Ba-Gd nuclei. In quadrant-II with $N < 104$, the product $(R_{2\gamma} * B(\text{E2}) \uparrow)$ decreases with increase in the asymmetry parameter γ_0. For well deformed nuclei, the product $(R_{2\gamma} * B(\text{E2}) \uparrow)$ shows a good correlation with the asymmetry parameter γ_0. The neutron subshell gap at $N = 114$ is also less effective in Pt nuclei in the product $(R_{2\gamma} * B(\text{E2}) \uparrow)$. The product $(R_{2\gamma} * B(\text{E2}) \uparrow)$ decreases with increase in the asymmetry parameter γ_0. The asymmetry parameter γ_0 is sensitive to the product $(R_{2\gamma} * B(\text{E2}) \uparrow)$ in range $\gamma_0 = 0°$ to $15°$. In the case of transitional nuclei, $(R_{2\gamma} * B(\text{E2}) \uparrow)$ lies in the range $\gamma_0 = 15°$ to $20°$. However, the product $(R_{2\gamma} * B(\text{E2}) \uparrow)$ shows somewhat better correlation with the asymmetry parameter γ_0 as compared to the Grodzins product $(E(2_1^+) * B(\text{E2}) \uparrow)$.

Parveen Kumari would like to gratefully acknowledge the MHRD for assistantship at NITJ. We are grateful to Dr. J.B. Gupta (Ramjas College, Delhi University, Delhi) for fruitful discussions on the manuscript.

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