# Feynman rules for neutrinos and new neutralinos in the BLMSSM＊ 

Xing－Xing Dong（董幸幸 $)^{1 ; 1)} \quad$ Shu－Min Zhao（赵树民）$)^{1 ; 2)}$ Hai－Bin Zhang（张海斌）${ }^{1}$<br>Fang Wang（王芳）${ }^{3}$ Tai－Fu Feng（冯太傅 $)^{1,2}$<br>${ }^{1}$ Department of Physics and Technology，Hebei University，Baoding 071002，China<br>${ }^{2}$ State Key Laboratory of Theoretical Physics（KLTP），Institute of Theoretical Physics， Chinese Academy of Sciences， 100190 Beijing，China<br>${ }^{3}$ Department of Electronic and Information Engineering，Hebei University，Baoding 071002，China


#### Abstract

In a supersymmetric extension of the Standard Model where baryon and lepton numbers are local gauge symmetries（BLMSSM），we deduce the Feynman rules for neutrinos and new neutralinos．We briefly introduce the mass matrices for the particles and the related couplings in this work，which are very useful to research the neutrinos and new neutralinos．


Keywords：supersymmetry，Feynman rules，mass matrices
PACS：13．15．＋g，12．60．－y DOI：10．1088／1674－1137／40／9／093103

## 1 Introduction

In quantum field theory，the Standard Model（SM） is a theory concerning the electromagnetic weak and strong interactions．Though the lightest CP－even Higgs （ $m_{\mathrm{h}^{0}} \simeq 126 \mathrm{GeV}$ ）was detected by the LHC，the SM is unable to explain some phenomena and falls short of be－ ing a complete theory of fundamental interactions．In the neutrino sector，the observations of solar and atmo－ spheric neutrino oscillations［1－4］are not incorporated in the SM，which provides clear evidence for physics be－ yond the SM．Furthermore，the authors think that a well－ motivated dark matter candidate emerges from the neu－ trino sector［5－7］．

Physics beyond the SM has drawn physicists＇atten－ tion for a long time．One of the most appealing theories to describe physics at the TeV scale is the minimal super－ symmetric extension of the Standard Model（MSSM）［8－ 11］．The MSSM includes necessary additional new par－ ticles that are superparters of those in the SM．The right－handed neutrino superfields can extend the next－to－ minimal supersymmetric standard model（NMSSM），and these superfields only couple with the singlet Higgs［12－ 15］．In R－parity［16］conserved MSSM，the left－handed light neutrinos are still massless，leading to a failure to
explain the discovery from neutrino oscillations．There－ fore，theoretical physicists have extended the MSSM to account for the light neutrino masses and mixings．

As an extension of the MSSM which considers the local gauged baryon（B）and lepton（L）symmetries，the BLMSSM is spontaneously broken at the TeV scale［17－ 20］．In the BLMSSM，the lepton number is broken in an even number while baryon number can be changed by baryon number violating operators through one unit． The BLMSSM can not only account for the asymme－ try of matter－antimatter in the universe but also explain the data from neutrino oscillation experiments［21－23］． Compared with the MSSM，the BLMSSM includes many new fields such as new quarks，new leptons，new Higgs， and the superfields $\hat{X}$ and $\hat{X}^{\prime}$［24－26］．In this work，we mainly study the Feynman rules for the neutrino and new neutralinos in the BLMSSM．

In the BLMSSM，the light neutrinos get mass from the seesaw mechanism，and proton decay is forbid－ den［17－20］．Therefore，it is not necessary to build a large desert between the electroweak scale and grand unified scale．This is the main motivation for the BLMSSM． Many possible signals of the MSSM at the LHC have been studied by the experiments．However，with the broken B and L symmetries，the predictions and bounds

[^0]for the collider experiments should be changed. From the decays of right handed neutrinos [19, 20, 27], we can look for lepton number violation at the LHC. Similarly from the decays of squarks and gauginos, we can also detect baryon number violation at the LHC. For example, the channels with multi-tops and multi-bottoms may be caused by the baryon number violating decays of gluinos [19, 20].

After this introduction, we briefly summarize the main contents of the BLMSSM in Section 2. The mass matrices for the particles are collected in Section 3. Sections 4 and 5 are respectively devoted to the related couplings of neutralinos and neutrinos beyond the MSSM. We give some discussion and conclusions in Section 6.

## 2 Main content of the BLMSSM

Extending the MSSM with local gauged baryon (B) and lepton (L) numbers, one obtains the BLMSSM, and at the TeV scale the local gauge symmetries are spontaneously broken. In this section, we briefly review some features of the BLMSSM. $S U(3)_{C} \otimes$ $S U(2)_{\mathrm{L}} \otimes U(1)_{Y} \otimes U(1)_{\mathrm{B}} \otimes U(1)_{\mathrm{L}}[19,20]$ is the basic gauge symmetry of the BLMSSM. The exotic leptons $\widehat{L}_{4} \sim\left(1,2,-1 / 2,0, L_{4}\right), \widehat{E}_{4}^{c} \sim\left(1,1,1,0,-L_{4}\right), \widehat{N}_{4}^{c} \sim$ $\left(1,1,0,0,-L_{4}\right), \widehat{L}_{5}^{c} \sim\left(1,2,1 / 2,0,-\left(3+L_{4}\right)\right), \widehat{E}_{5} \sim(1,1-$ $\left.1,0,3+L_{4}\right), \widehat{N}_{5} \sim\left(1,1,0,0,3+L_{4}\right)$ and the exotic quarks $\widehat{Q}_{4} \sim\left(3,2,1 / 6, B_{4}, 0\right), \widehat{U}_{4}^{c} \sim\left(\overline{3}, 1,-2 / 3,-B_{4}, 0\right)$, $\widehat{D}_{4}^{c} \sim\left(\overline{3}, 1,1 / 3,-B_{4}, 0\right), \widehat{Q}_{5}^{c} \sim\left(\overline{3}, 2,-1 / 6,-\left(1+B_{4}\right), 0\right)$, $\widehat{U}_{5} \sim\left(3,1,2 / 3,1+B_{4}, 0\right), \widehat{D}_{5} \sim\left(3,1-1 / 3,1+B_{4}, 0\right)$ are introduced to cancel L and B anomalies respectively. The exotic Higgs superfields $\widehat{\Phi}_{\mathrm{L}} \sim(1,1,0,0,-2), \widehat{\varphi}_{\mathrm{L}} \sim$ $(1,1,0,0,2)$ and $\widehat{\Phi}_{\mathrm{B}} \sim(1,1,0,1,0), \widehat{\varphi}_{\mathrm{B}} \sim(1,1,0,-1,0)$ are introduced respectively to break lepton number and baryon number spontaneously. The exotic Higgs superfields $\widehat{\Phi}_{\mathrm{L}}, \widehat{\varphi}_{\mathrm{L}}$ and $\widehat{\Phi}_{\mathrm{B}}, \widehat{\varphi}_{\mathrm{B}}$ acquire nonzero vacuum expectation values (VEVs), then the exotic leptons and exotic quarks obtain masses. The model also includes the superfields $\widehat{X} \sim\left(1,1,0,2 / 3+B_{4}, 0\right)$ and $\widehat{X}^{\prime} \sim(1,1,0,-(2 / 3+$ $\left.\left.B_{4}\right), 0\right)$ to make heavy exotic quarks unstable. Furthermore, the lightest mass eigenstate can be a dark matter candidate, while $\widehat{X}$ and $\widehat{X}^{\prime}$ mix together. Anomaly cancellation requires the emergence of new families. However there is no flavour violation at tree level since they do not mix with the SM fermions and there are no Landau poles at the low scale due to the new families.

The BLMSSM superpotential is given by [28]

$$
\begin{equation*}
\mathcal{W}_{\text {BLMSSM }}=\mathcal{W}_{\mathrm{MSSM}}+\mathcal{W}_{\mathrm{B}}+\mathcal{W}_{\mathrm{L}}+\mathcal{W}_{X} \tag{1}
\end{equation*}
$$

where $\mathcal{W}_{\text {MSSM }}$ represents the superpotential of the MSSM. The concrete forms of $\mathcal{W}_{\mathrm{B}}, \mathcal{W}_{\mathrm{L}}$ and $\mathcal{W}_{X}$ read as follows

$$
\begin{align*}
\mathcal{W}_{\mathrm{B}}= & \lambda_{\mathrm{Q}} \hat{Q}_{4} \hat{Q}_{5}^{c} \hat{\Phi}_{\mathrm{B}}+\lambda_{U} \hat{U}_{4}^{c} \hat{U}_{5} \hat{\varphi}_{\mathrm{B}}+\lambda_{\mathrm{D}} \hat{D}_{4}^{c} \hat{D}_{5} \hat{\varphi}_{\mathrm{B}} \\
& +\mu_{\mathrm{B}} \hat{\Phi}_{\mathrm{B}} \hat{\varphi}_{\mathrm{B}}+Y_{u_{4}} \hat{Q}_{4} \hat{H}_{u} \hat{U}_{4}^{c}+Y_{d_{4}} \hat{Q}_{4} \hat{H}_{d} \hat{D}_{4}^{c} \\
& +Y_{u_{5}} \hat{Q}_{5}^{c} \hat{H}_{d} \hat{U}_{5}+Y_{d_{5}} \hat{Q}_{5}^{c} \hat{H}_{u} \hat{D}_{5}, \\
\mathcal{W}_{\mathrm{L}}= & Y_{e_{4}} \hat{L}_{4} \hat{H}_{d} \hat{E}_{4}^{c}+Y_{\nu_{4}} \hat{L}_{4} \hat{H}_{u} \hat{N}_{4}^{c}+Y_{e_{5}} \hat{L}_{5}^{c} \hat{H}_{u} \hat{E}_{5} \\
& +Y_{\nu_{5}} \hat{L}_{5}^{c} \hat{H}_{d} \hat{N}_{5}+Y_{\nu} \hat{L} \hat{H}_{u} \hat{N}^{c}+\lambda_{N^{c}} \hat{N}^{c} \hat{N}^{c} \hat{\varphi}_{\mathrm{L}} \\
& +\mu_{\mathrm{L}} \hat{\Phi}_{\mathrm{L}} \hat{\varphi}_{\mathrm{L}}, \\
\mathcal{W}_{X}= & \lambda_{1} \hat{Q} \hat{Q}_{5}^{c} \hat{X}+\lambda_{2} \hat{U}^{c} \hat{U}_{5} \hat{X}^{\prime} \\
& +\lambda_{3} \hat{D}^{c} \hat{D}_{5} \hat{X}^{\prime}+\mu_{X} \hat{X} \hat{X}^{\prime} . \tag{2}
\end{align*}
$$

The soft breaking terms $\mathcal{L}_{\text {soft }}$ of the BLMSSM are generally shown as $[17,18,28]$

$$
\begin{align*}
& \mathcal{L}_{\text {soft }}=\mathcal{L}_{\text {soft }}^{\mathrm{MSSM}}-\left(m_{\tilde{\mathrm{v}}^{c}}^{2}\right)_{I J} \tilde{N}_{I}^{c *} \tilde{N}_{J}^{c}-m_{\tilde{Q}_{4}}^{2} \tilde{Q}_{4}^{\dagger} \tilde{Q}_{4} \\
& -m_{\tilde{U}_{4}}^{2} \tilde{U}_{4}^{c *} \tilde{U}_{4}^{c}-m_{\tilde{D}_{4}}^{2} \tilde{D}_{4}^{c *} \tilde{D}_{4}^{c}-m_{\tilde{Q}_{5}}^{2} \tilde{Q}_{5}^{c \dagger} \tilde{Q}_{5}^{c} \\
& -m_{\tilde{U}_{5}}^{2} \tilde{U}_{5}^{*} \tilde{U}_{5}-m_{\tilde{D}_{5}}^{2} \tilde{D}_{5}^{*} \tilde{D}_{5}-m_{\tilde{\mathrm{L}}_{4}}^{2} \tilde{\mathrm{~L}}_{4}^{\dagger} \tilde{L}_{4} \\
& -m_{\tilde{\nu}_{4}}^{2} \tilde{N}_{4}^{c *} \tilde{N}_{4}^{c}-m_{\tilde{e}_{4}}^{2} \tilde{E}_{4}^{c *} \tilde{E}_{4}^{c}-m_{\tilde{\mathrm{L}}_{5}}^{2} \tilde{L}_{5}^{c \dagger} \tilde{L}_{5}^{c} \\
& -m_{\tilde{\nu}_{5}}^{2} \tilde{N}_{5}^{*} \tilde{N}_{5}-m_{\tilde{e}_{5}}^{2} \tilde{E}_{5}^{*} \tilde{E}_{5}-m_{\Phi_{\mathrm{B}}}^{2} \Phi_{\mathrm{B}}^{*} \Phi_{\mathrm{B}} \\
& -m_{\varphi_{\mathrm{B}}}^{2} \varphi_{\mathrm{B}}^{*} \varphi_{\mathrm{B}}-m_{\Phi_{\mathrm{L}}}^{2} \Phi_{\mathrm{L}}^{*} \Phi_{\mathrm{L}}-m_{\varphi_{\mathrm{L}}}^{2} \varphi_{\mathrm{L}}^{*} \varphi_{\mathrm{L}} \\
& -\left(m_{\mathrm{B}} \lambda_{\mathrm{B}} \lambda_{\mathrm{B}}+m_{\mathrm{L}} \lambda_{\mathrm{L}} \lambda_{\mathrm{L}}+\text { h.c. }\right) \\
& +\left\{A_{u_{4}} Y_{u_{4}} \tilde{Q}_{4} H_{u} \tilde{U}_{4}^{c}+A_{d_{4}} Y_{d_{4}} \tilde{Q}_{4} H_{d} \tilde{D}_{4}^{c}\right. \\
& +A_{u_{5}} Y_{u_{5}} \tilde{Q}_{5}^{c} H_{d} \tilde{U}_{5}+A_{d_{5}} Y_{d_{5}} \tilde{Q}_{5}^{c} H_{u} \tilde{D}_{5} \\
& +A_{\mathrm{BQ}} \lambda_{\mathrm{Q}} \tilde{Q}_{4} \tilde{Q}_{5}^{c} \Phi_{\mathrm{B}}+A_{\mathrm{B} U} \lambda_{U} \tilde{U}_{4}^{c} \tilde{U}_{5} \varphi_{\mathrm{B}} \\
& \left.+A_{\mathrm{BD}} \lambda_{\mathrm{D}} \tilde{D}_{4}^{c} \tilde{D}_{5} \varphi_{\mathrm{B}}+B_{\mathrm{B}} \mu_{\mathrm{B}} \Phi_{\mathrm{B}} \varphi_{\mathrm{B}}+\text { h.c. }\right\} \\
& +\left\{A_{e_{4}} Y_{e_{4}} \tilde{L}_{4} H_{d} \tilde{E}_{4}^{c}+A_{\nu_{4}} Y_{\nu_{4}} \tilde{L}_{4} H_{u} \tilde{N}_{4}^{c}\right. \\
& +A_{e_{5}} Y_{e_{5}} \tilde{L}_{5}^{c} H_{u} \tilde{E}_{5}+A_{\nu_{5}} Y_{\nu_{5}} \tilde{L}_{5}^{c} H_{d} \tilde{N}_{5} \\
& +A_{v} Y_{v} \tilde{L} H_{u} \tilde{N}^{c}+A_{\nu^{c}} \lambda_{\nu c} \tilde{N}^{c} \tilde{N}^{c} \varphi_{\mathrm{L}} \\
& \left.+B_{\mathrm{L}} \mu_{\mathrm{L}} \Phi_{\mathrm{L}} \varphi_{\mathrm{L}}+\text { h.c. }\right\} \\
& +\left\{A_{1} \lambda_{1} \tilde{Q} \tilde{Q}_{5}^{c} X+A_{2} \lambda_{2} \tilde{U}^{c} \tilde{U}_{5} X^{\prime}\right. \\
& \left.+A_{3} \lambda_{3} \tilde{D}^{c} \tilde{D}_{5} X^{\prime}+B_{X} \mu_{X} X X^{\prime}+\text { h.c. }\right\}, \tag{3}
\end{align*}
$$

where $\mathcal{L}_{\text {soft }}^{\mathrm{MSSM}}$ represent the soft breaking terms of MSSM, and $\lambda_{\mathrm{B}}$ and $\lambda_{\mathrm{L}}$ are the gauginos of $U(1)_{\mathrm{B}}$ and $U(1)_{\mathrm{L}}$, respectively. The $S U(2)_{\mathrm{L}}$ doublets $H_{u}, H_{d}$ and $S U(2)_{\mathrm{L}}$ singlets $\Phi_{\mathrm{B}}, \varphi_{\mathrm{B}}, \Phi_{\mathrm{L}}, \varphi_{\mathrm{L}}$ acquire the nonzero VEVs $v_{u}, v_{d}$ and $v_{\mathrm{B}}, \bar{v}_{\mathrm{B}}, v_{\mathrm{L}}, \bar{v}_{\mathrm{L}}$ respectively,

$$
\begin{aligned}
& H_{u}=\binom{H_{u}^{+}}{\frac{1}{\sqrt{2}}\left(v_{u}+H_{u}^{0}+\mathrm{i} P_{u}^{0}\right)} \\
& H_{d}=\binom{\frac{1}{\sqrt{2}}\left(v_{d}+H_{d}^{0}+\mathrm{i} P_{d}^{0}\right)}{H_{d}^{-}},
\end{aligned}
$$

$$
\begin{align*}
\Phi_{\mathrm{B}} & =\frac{1}{\sqrt{2}}\left(v_{\mathrm{B}}+\Phi_{\mathrm{B}}^{0}+\mathrm{i} P_{\mathrm{B}}^{0}\right), \\
\varphi_{\mathrm{B}} & =\frac{1}{\sqrt{2}}\left(\bar{v}_{\mathrm{B}}+\varphi_{\mathrm{B}}^{0}+\mathrm{i} \bar{P}_{\mathrm{B}}^{0}\right), \\
\Phi_{\mathrm{L}} & =\frac{1}{\sqrt{2}}\left(v_{\mathrm{L}}+\Phi_{\mathrm{L}}^{0}+\mathrm{i} P_{\mathrm{L}}^{0}\right), \\
\varphi_{\mathrm{L}} & =\frac{1}{\sqrt{2}}\left(\bar{v}_{\mathrm{L}}+\varphi_{\mathrm{L}}^{0}+\mathrm{i} \bar{P}_{\mathrm{L}}^{0}\right) \tag{4}
\end{align*}
$$

Therefore, the local gauge symmetry $S U(2)_{\mathrm{L}} \otimes U(1)_{Y} \otimes$ $U(1)_{\mathrm{B}} \otimes U(1)_{\mathrm{L}}$ is broken down to the electromagnetic symmetry $U(1)_{e}$. In Ref. [28], the mass matrices of exotic Higgs, exotic quarks and exotic scalar quarks are obtained. In the BLMSSM, because of the introduced superfields $\hat{N}^{C}$, the tiny masses of the light neutrinos are produced. Another result is six scalar neutrinos in the BLMSSM.

## 3 Particle mass matrices

Lepton neutralinos are made up of $\lambda_{\mathrm{L}}$ (the superpartner of the new lepton boson), $\psi_{\Phi_{\mathrm{L}}}$ and $\psi_{\varphi_{\mathrm{L}}}$ (the superpartners of the $S U(2)_{\mathrm{L}}$ singlets $\Phi_{\mathrm{L}}$ and $\left.\varphi_{\mathrm{L}}\right)$. The mass mixing matrix of lepton neutralinos is shown in the basis $\left(\mathrm{i} \lambda_{\mathrm{L}}, \psi_{\Phi_{\mathrm{L}}}, \psi_{\varphi_{\mathrm{L}}}\right)$ [29, 30]. Then 3 lepton neutralino masses are obtained from diagonalizing the mass mixing matrix $M_{\mathrm{L} N}$ by $Z_{N_{\mathrm{L}}}$,

$$
\begin{align*}
& M_{\mathrm{L} N}=\left(\begin{array}{ccc}
2 M_{\mathrm{L}} & 2 v_{\mathrm{L}} g_{\mathrm{L}} & -2 \bar{v}_{\mathrm{L}} g_{\mathrm{L}} \\
2 v_{\mathrm{L}} g_{\mathrm{L}} & 0 & -\mu_{\mathrm{L}} \\
-2 \bar{v}_{\mathrm{L}} g_{\mathrm{L}} & -\mu_{\mathrm{L}} & 0
\end{array}\right) \\
& \mathrm{i} \lambda_{\mathrm{L}}=Z_{N_{\mathrm{L}}}^{1 i} k_{\mathrm{L}_{i}}^{0}, \quad \psi_{\Phi_{\mathrm{L}}}=Z_{N_{\mathrm{L}}}^{2 i} k_{\mathrm{L}_{i}}^{0}, \\
& \psi_{\varphi_{\mathrm{L}}}=Z_{N_{\mathrm{L}}}^{3 i} k_{\mathrm{L}_{i}}^{0}, \quad \chi_{\mathrm{L}_{i}}^{0}=\binom{k_{\mathrm{L}_{i}}^{0}}{\bar{k}_{\mathrm{L}_{i}}^{0}} \tag{5}
\end{align*}
$$

$\chi_{\mathrm{L}_{i}}^{0}(i=1,2,3)$ are the mass eigenstates of the lepton neutralinos.
$\lambda_{B}$ (the superpartner of the new baryon boson), $\psi_{\Phi_{\mathrm{B}}}$ and $\psi_{\varphi_{\mathrm{B}}}$ (the superpartners of the $S U(2)_{\mathrm{L}}$ singlets $\Phi_{\mathrm{B}}$ and $\varphi_{\mathrm{B}}$ ) mix together producing 3 baryon neutralinos. Using $Z_{N_{\mathrm{B}}}$ one can diagonalize the mass mixing matrix $M_{\mathrm{B} N}$, and obtain 3 baryon neutralino masses,

$$
\begin{aligned}
& M_{\mathrm{BN}}=\left(\begin{array}{ccc}
2 M_{\mathrm{B}} & -v_{\mathrm{B}} g_{\mathrm{B}} & \bar{v}_{\mathrm{B}} g_{\mathrm{B}} \\
-v_{\mathrm{B}} g_{\mathrm{B}} & 0 & -\mu_{\mathrm{B}} \\
\bar{v}_{\mathrm{B}} g_{\mathrm{B}} & -\mu_{\mathrm{B}} & 0
\end{array}\right) . \\
& \mathrm{i} \lambda_{\mathrm{B}}=Z_{N_{\mathrm{B}}}^{1 i} k_{\mathrm{B}_{i}}^{0}, \quad \psi_{\Phi_{\mathrm{B}}}=Z_{N_{\mathrm{B}}}^{2 i} k_{\mathrm{B}_{i}}^{0},
\end{aligned}
$$

$$
\begin{equation*}
\psi_{\varphi_{\mathrm{B}}}=Z_{N_{\mathrm{B}}}^{3 i} k_{\mathrm{B}_{i}}^{0}, \quad \chi_{\mathrm{B}_{i}}^{0}=\binom{k_{\mathrm{B}_{i}}^{0}}{\bar{k}_{\mathrm{B}_{i}}^{0}} \tag{6}
\end{equation*}
$$

The mass eigenstates of the baryon neutralinos are represented by $\chi_{\mathrm{B}_{i}}^{0}(i=1,2,3)$.

In this work, because neutrinos are Majorana particles, we can use the following expression. In the base $\left(\psi_{\nu_{\mathrm{L}}^{I I}}, \psi_{N_{R}^{c I}}\right)$, the formulas for neutrino mixing and mass matrix are shown as

$$
\begin{align*}
& Z_{N_{\nu}}^{T}\left(\begin{array}{cc}
0 & \frac{v_{u}}{\sqrt{2}}\left(Y_{v}\right)^{I J} \\
\frac{v_{u}}{\sqrt{2}}\left(Y_{v}^{T}\right)^{I J} & \frac{\bar{v}_{\mathrm{L}}}{\sqrt{2}}\left(\lambda_{N^{c}}\right)^{I J}
\end{array}\right) Z_{N_{\nu}} \\
& =\operatorname{diag}\left(m_{\nu^{\alpha}}\right), \quad \alpha=1, \ldots, 6 . \\
& \psi_{\nu_{\mathrm{L}}^{I}}=Z_{N_{\nu}}^{I \alpha} k_{N_{\alpha}}^{0}, \\
& \psi_{N_{R}^{c I}}=Z_{N_{v}}^{(I+3) \alpha} k_{N_{\alpha}}^{0},  \tag{7}\\
& \chi_{N_{\alpha}}^{0}=\binom{k_{N_{\alpha}}^{0}}{\bar{k}_{N_{\alpha}}^{0}} .
\end{align*}
$$

$\chi_{N_{\alpha}}^{0}$ denotes the mass eigenstates of the neutrino fields mixed by the left-handed and right-handed neutrinos.

The introduced super-fields $\hat{N}^{c}$ lead to six sneutrinos. From the superpotential and the soft breaking terms in Eqs. (2,3), we deduce the mass squared matrix of sneutrinos $\left(\mathcal{M}_{\tilde{n}}\right)$ in the base $\tilde{n}^{T}=\left(\tilde{v}, \tilde{N}^{c}\right)$. To obtain the mass eigenstates of sneutrinos, $Z_{\tilde{v}}$ is used for the rotation.

$$
\begin{align*}
\mathcal{M}_{\tilde{n}}^{2}\left(\tilde{\nu}_{I}^{*} \tilde{\nu}_{J}\right)= & \frac{g_{1}^{2}+g_{2}^{2}}{8}\left(v_{d}^{2}-v_{u}^{2}\right) \delta_{I J}+g_{\mathrm{L}}^{2}\left(\bar{v}_{\mathrm{L}}^{2}\right. \\
& \left.-v_{\mathrm{L}}^{2}\right) \delta_{I J}+\frac{v_{u}^{2}}{2}\left(Y_{v}^{\dagger} Y_{v}\right)_{I J}+\left(m_{\tilde{\mathrm{L}}}^{2}\right)_{I J} \\
\mathcal{M}_{\tilde{n}}^{2}\left(\tilde{N}_{I}^{c *} \tilde{N}_{J}^{c}\right)= & -g_{\mathrm{L}}^{2}\left(\bar{v}_{\mathrm{L}}^{2}-v_{\mathrm{L}}^{2}\right) \delta_{I J}+\frac{v_{u}^{2}}{2}\left(Y_{v}^{\dagger} Y_{\nu}\right)_{I J} \\
+ & 2 \bar{v}_{\mathrm{L}}^{2}\left(\lambda_{N^{c}}^{\dagger} \lambda_{N^{c}}\right)_{I J}+\left(m_{\tilde{N}^{c}}^{2}\right)_{I J} \\
+ & \mu_{\mathrm{L}} \frac{v_{\mathrm{L}}}{\sqrt{2}}\left(\lambda_{N^{c}}\right)_{I J}-\frac{\bar{v}_{\mathrm{L}}}{\sqrt{2}}\left(A_{N^{c}}\right)_{I J}\left(\lambda_{N^{c}}\right)_{I J} \\
\mathcal{M}_{\tilde{n}}^{2}\left(\tilde{\nu}_{I} \tilde{N}_{J}^{c}\right)= & \mu^{*} \frac{v_{d}}{\sqrt{2}}\left(Y_{v}\right)_{I J}-v_{u} \bar{v}_{\mathrm{L}}\left(Y_{v}^{\dagger} \lambda_{N^{c}}\right)_{I J} \\
& +\frac{v_{u}}{\sqrt{2}}\left(A_{N}\right)_{I J}\left(Y_{v}\right)_{I J} \\
Z_{\tilde{v}}^{\dagger} \mathcal{M}_{\tilde{n}} Z_{\tilde{v}}= & \left(m_{\tilde{N}^{1}}^{2}, m_{\tilde{N}^{2}}^{2}, m_{\tilde{N}^{3}}^{2}, m_{\tilde{N}^{4}}^{2}, m_{\tilde{N}^{5}}^{2}, m_{\tilde{N}^{6}}^{2}\right) . \tag{8}
\end{align*}
$$

The superfields $\Phi_{\mathrm{B}}^{0}$ and $\varphi_{\mathrm{B}}^{0}$ mix together, and their mass squared matrix is

$$
\mathcal{M}_{E \mathrm{~B}}^{2}=\left(\begin{array}{cc}
m_{Z_{\mathrm{B}}}^{2} \cos ^{2} \beta_{\mathrm{B}}+m_{A_{\mathrm{B}}^{0}}^{2} \sin ^{2} \beta_{\mathrm{B}}, & \left(m_{Z_{\mathrm{B}}}^{2}+m_{A_{\mathrm{B}}^{0}}^{2}\right) \cos \beta_{\mathrm{B}} \sin \beta_{\mathrm{B}} \\
\left(m_{Z_{\mathrm{B}}}^{2}+m_{A_{\mathrm{B}}^{0}}^{2}\right) \cos \beta_{\mathrm{B}} \sin \beta_{\mathrm{B}}, & m_{Z_{\mathrm{B}}}^{2} \sin ^{2} \beta_{\mathrm{B}}+m_{A_{\mathrm{B}}^{0}}^{2} \cos ^{2} \beta_{\mathrm{B}}
\end{array}\right),
$$

$$
\begin{equation*}
v_{\mathrm{B}_{t}}=\sqrt{v_{\mathrm{B}}^{2}+\bar{v}_{\mathrm{B}}^{2}}, \quad m_{Z_{\mathrm{B}}}=g_{\mathrm{B}} v_{\mathrm{B}_{t}}, \quad \Phi_{\mathrm{B}}^{0}=Z_{\phi_{\mathrm{B}}}^{1 i} H_{\mathrm{B}_{i}}^{0}, \quad \varphi_{\mathrm{B}}^{0}=Z_{\phi_{\mathrm{B}}}^{2 i} H_{\mathrm{B}_{i}}^{0}, \tag{9}
\end{equation*}
$$

with $m_{Z_{\mathrm{B}}}$ representing the mass of the neutral $U(1)_{\mathrm{B}}$ gauge boson $Z_{\mathrm{B}} . Z_{\phi_{\mathrm{B}}}$ is the rotation matrix to diagonalize the mass squared matrix $\mathcal{M}_{E \mathrm{~B}}^{2}$ and $H_{\mathrm{B}_{i}}^{0}(i=1,2)$
denotes the mass eigenstates of the baryon Higgs.
In the same way, we obtain the mass squared matrix for $\left(\Phi_{\mathrm{L}}^{0}, \varphi_{\mathrm{L}}^{0}\right)$

$$
\begin{align*}
& \mathcal{M}_{E \mathrm{~L}}^{2}=\left(\begin{array}{cc}
m_{Z_{\mathrm{L}}}^{2} \cos ^{2} \beta_{\mathrm{L}}+m_{A_{\mathrm{L}}^{0}}^{2} \sin ^{2} \beta_{\mathrm{L}}, & \left(m_{Z_{\mathrm{L}}}^{2}+m_{A_{\mathrm{L}}^{0}}^{2}\right) \cos \beta_{\mathrm{L}} \sin \beta_{\mathrm{L}} \\
\left(m_{Z_{\mathrm{L}}}^{2}+m_{A_{\mathrm{L}}^{0}}^{2}\right) \cos \beta_{\mathrm{L}} \sin \beta_{\mathrm{L}}, & m_{Z_{\mathrm{L}}}^{2} \sin ^{2} \beta_{\mathrm{L}}+m_{A_{\mathrm{L}}^{0}}^{2} \cos ^{2} \beta_{\mathrm{L}}
\end{array}\right), \\
& v_{\mathrm{L}_{t}}=\sqrt{v_{\mathrm{L}}^{2}+\bar{v}_{\mathrm{L}}^{2}}, \quad m_{Z_{\mathrm{L}}}=2 g_{\mathrm{L}} v_{\mathrm{L}_{t}}, \quad \Phi_{\mathrm{L}}^{0}=Z_{\phi_{\mathrm{L}}}^{1 i} H_{\mathrm{L}_{i}}^{0}, \quad \varphi_{\mathrm{L}}^{0}=Z_{\phi_{\mathrm{L}}}^{2 i} H_{\mathrm{L}_{i}}^{0} . \tag{10}
\end{align*}
$$

Here, $m_{Z_{\mathrm{L}}}$ is the mass of the neutral $U(1)_{\mathrm{L}}$ gauge boson $Z_{\mathrm{L}} . Z_{\phi_{\mathrm{L}}}$ is used to obtain mass eigenvalues for the matrix $\mathcal{M}_{E \mathrm{~L}}^{2} . \quad H_{\mathrm{L}_{i}}^{0}(i=1,2)$ are the lepton Higgs mass eigenstates.

## 4 Couplings of neutralinos beyond the MSSM

### 4.1 New MSSM neutralino couplings

From the superpotential $\mathcal{W}_{\mathrm{L}}$ in Eq. (2) and the interactions of gauge and matter multiplets $i g \sqrt{2} T_{i j}^{a}\left(\lambda^{a} \psi_{j} A_{i}^{*}-\right.$ $\bar{\lambda}^{a} \bar{\psi}_{i} A_{j}$ ), we deduce the couplings of MSSM neutralinoexotic lepton-exotic sleptons:

$$
\begin{aligned}
\mathcal{L}( & \left.\chi^{0} l^{\prime} \tilde{l}^{\prime}\right)=\sum_{i, k=1}^{2} \sum_{j=1}^{4}\left\{\overline { \chi } _ { j } ^ { 0 } \left[\left(\frac { 1 } { \sqrt { 2 } } \left(\frac{e}{s} Z_{N}^{2 j}\right.\right.\right.\right. \\
& \left.\left.+\frac{e}{c} Z_{N}^{1 j}\right) U_{\mathrm{L}}^{1 i} Z_{\tilde{e}_{4}}^{1 k *}+Y_{e_{4}} U_{\mathrm{L}}^{1 i} Z_{N}^{3 j} Z_{\tilde{e}_{4}}^{2 k *}\right) P_{\mathrm{L}} \\
& +\left(Y_{e_{4}}^{*} Z_{\tilde{e}_{4}}^{1 k *} Z_{N}^{3 j *} W_{\mathrm{L}}^{2 i}\right. \\
& \left.\left.-\sqrt{2} \frac{e}{c} Z_{N}^{1 j *} W_{\mathrm{L}}^{2 i} Z_{\tilde{e}_{4}}^{2 k *}\right) P_{R}\right] L_{i+3}^{\prime} \tilde{E}_{4}^{k *} \\
& -\bar{\chi}_{j}^{0}\left[\left(Y_{v_{4}} U_{N}^{1 i} Z_{N}^{4 j} Z_{\tilde{v}_{4}}^{2 k *}\right.\right. \\
& \left.+\frac{1}{\sqrt{2}}\left(\frac{e}{s} Z_{N}^{2 j}-\frac{e}{c} Z_{N}^{1 j}\right) U_{N}^{1 i} Z_{\tilde{v}_{4}}^{1 k *}\right) P_{\mathrm{L}} \\
& \left.+Y_{v_{4}}^{*} Z_{\tilde{v}_{4}}^{1 k *} Z_{N}^{4 j *} W_{N}^{2 i} P_{R}\right] N_{i+3}^{\prime} \tilde{N}_{4}^{k *} \\
& -\bar{L}^{\prime}{ }_{i+3}\left[\left(Y_{e_{5}} Z_{N}^{4 j} W_{\mathrm{L}}^{1 i *} Z_{\tilde{e}_{5}}^{1 *}\right.\right. \\
& \left.-\frac{1}{\sqrt{2}}\left(\frac{e}{s} Z_{N}^{2 j}+\frac{e}{c} Z_{N}^{1 j}\right) W_{\mathrm{L}}^{1 i *} Z_{\tilde{e}_{5}}^{2 k}\right) P_{L} \\
& +\left(Y_{e_{5}}^{*} Z_{\tilde{e}_{5}}^{1 k} Z_{N}^{4 j *} U_{\mathrm{L}}^{2 i *}\right. \\
& \left.\left.+\sqrt{2} \frac{e}{c} Z_{N}^{1 j *} U_{\mathrm{L}}^{2 i *} Z_{\tilde{e}_{5}}^{1 k}\right) P_{R}\right] \chi_{j}^{0} \tilde{E}_{5}^{k}
\end{aligned}
$$

$$
\begin{align*}
& +\bar{N}^{\prime}{ }_{i+3}\left[\left(Y_{v_{5}} W_{N}^{1 i *} Z_{N}^{3 j} Z_{\tilde{v}_{5}}^{1 k}\right.\right. \\
& \left.+\frac{1}{\sqrt{2}}\left(\frac{e}{s} Z_{N}^{2 j}-\frac{e}{c} Z_{N}^{1 j}\right) W_{N}^{1 i *} Z_{\tilde{v}_{5}}^{2 k}\right) P_{\mathrm{L}} \\
& \left.\left.+Y_{v_{5}}^{*} Z_{\tilde{v}_{5}}^{2 k} Z_{N}^{3 j *} U_{N}^{2 i *} P_{R}\right] \chi_{j}^{0} \tilde{N}_{5}^{k}\right\}+ \text { h.c. } \tag{11}
\end{align*}
$$

The matrices $U_{\mathrm{L}}$ and $W_{\mathrm{L}}$ are used to diagonalize the exotic charged lepton mixing matrix [24-26], and $L_{4,5}^{\prime}$ are the mass eigenstates of the exotic charged leptons. The exotic slepton mass eigenstates are denoted by $\tilde{N}_{4,5}^{\prime}$ and $\tilde{E}_{4,5}^{\prime}$ with the rotation matrices $Z_{\tilde{v}_{4,5}}$ and $Z_{\tilde{e}_{4,5}}$. In the MSSM, there are couplings for MSSM neutralino-neutrino-sneutrino which should be transformed into BLMSSM with the rotations of the neutrinos and sneutrinos in Eqs. $(7,8)$.

In $\mathcal{W}_{\mathrm{L}}$ there is a new term $Y_{v} \hat{L} \hat{H}_{u} \hat{N}^{c}$ that can give corrections to the couplings of MSSM neutralino-neutrino-sneutrino. These new couplings are suppressed by the tiny neutrino Yukawa $Y_{v}$,

$$
\begin{align*}
& \mathcal{L}^{n}\left(\chi_{N} \chi^{0} \tilde{N}^{*}\right) \\
&=-\sum_{I, J=1}^{3} \sum_{i=1}^{4} \sum_{j=1}^{6} \bar{\chi}_{N_{\alpha}}\left(Y_{v}^{I J} Z_{N_{v}}^{I \alpha} Z_{N}^{4 i} Z_{\tilde{v}}^{(J+3) j *} P_{\mathrm{L}}\right. \\
&\left.+Y_{v}^{I J *} Z_{\tilde{v}}^{I j *} Z_{N}^{4 i *} Z_{N_{v}}^{(J+3) \alpha *} P_{R}\right) \chi_{i}^{0} \tilde{N}^{j *}+\text { h.c. } \tag{12}
\end{align*}
$$

In the same way, the couplings of MSSM neutralinoexotic quark-exotic squark are obtained:

$$
\begin{aligned}
& \mathcal{L}\left(\chi^{0} q^{\prime} \tilde{q}^{\prime}\right)=\sum_{j=1}^{2} \sum_{i, k=1}^{4}\left\{\overline { \chi } _ { i } ^ { 0 } \left[\left(Y_{U_{5}} U_{3 k}^{*} Z_{N}^{3 i} U_{2 j}^{t^{\prime}}\right.\right.\right. \\
& \quad-\frac{e}{c} \frac{2 \sqrt{2}}{3} Z_{N}^{1 i} U_{2 j}^{t^{\prime}} U_{4 k}^{*}-\frac{1}{\sqrt{2}}\left(\frac{e}{s} Z_{N}^{2 i}\right. \\
& \left.\left.\quad+\frac{1}{3} \frac{e}{c} Z_{N}^{1 i}\right) U_{1 j}^{t^{\prime}} U_{1 k}^{*}-Y_{U_{4}} U_{1 j}^{t^{\prime}} Z_{N}^{4 i} U_{2 k}^{*}\right) P_{\mathrm{L}} \\
& \quad+\left(Y_{U_{5}}^{*} W_{1 j}^{t^{\prime}} Z_{N}^{3 i *} U_{4 k}^{*}+\frac{1}{\sqrt{2}}\left(\frac{e}{s} Z_{N}^{2 i *}\right.\right.
\end{aligned}
$$

$$
\begin{align*}
& \left.+\frac{1}{3} \frac{e}{c} Z_{N}^{1 i *}\right) W_{1 j}^{t^{\prime}} U_{3 k}^{*}-Y_{U_{4}}^{*} U_{1 k}^{*} Z_{N}^{4 i *} W_{2 j}^{t^{\prime}} \\
& \left.\left.+\frac{e}{c} \frac{2 \sqrt{2}}{3} Z_{N}^{1 i *} W_{2 j}^{t^{\prime}} U_{2 k}^{*}\right) P_{R}\right] t_{j+3}^{\prime} \tilde{\mathcal{U}}_{k}^{*} \\
& +\bar{\chi}_{i}^{0}\left[\left(Y_{d_{4}} U_{1 j}^{b^{\prime}} Z_{N}^{3 i} D_{2 k}^{*}\right.\right. \\
& +\frac{1}{\sqrt{2}}\left(\frac{e}{s} Z_{N}^{2 i}-\frac{1}{3} \frac{e}{c} Z_{N}^{1 i}\right) U_{1 j}^{b^{\prime}} D_{1 k}^{*} \\
& \left.-Y_{d_{5}} D_{3 j}^{*} Z_{N}^{4 i} U_{2 k}^{b^{\prime}}+\frac{e}{c} \frac{\sqrt{2}}{3} Z_{N}^{1 i} U_{2 j}^{b^{\prime}} D_{4 k}^{*}\right) P_{\mathrm{L}} \\
& +\left(Y_{d_{4}}^{*} D_{1 k}^{*} Z_{N}^{3 i *} W_{2 j}^{b^{\prime}}-\frac{1}{\sqrt{2}}\left(\frac{e}{s} Z_{N}^{2 i *}\right.\right. \\
& \left.-\frac{1}{3} \frac{e}{c} Z_{N}^{1 i *}\right) W_{1 j}^{b^{\prime}} D_{3 k}^{*}-Y_{d_{5}}^{*} W_{1 j}^{b^{\prime}} Z_{N}^{4 i *} D_{4 k}^{*} \\
& \left.\left.\left.-\frac{e}{c} \frac{\sqrt{2}}{3} Z_{N}^{1 i *} W_{2 j}^{b^{\prime}} D_{2 k}^{*}\right) P_{R}\right] b_{j+3}^{\prime} \tilde{\mathcal{D}}_{k}^{*}\right\}+ \text { h.c. } \tag{13}
\end{align*}
$$

In the mass basis the exotic quarks are $t^{\prime}$ and $b^{\prime}$, and their rotation matrices are $W^{t^{\prime}}, U^{t^{\prime}}, W^{b^{\prime}}$ and $U^{b^{\prime}} . \tilde{\mathcal{U}}$ and $\tilde{\mathcal{D}}$ are the exotic scalar quarks with their diagonalizing matrices $U$ and $D$.

### 4.2 Lepton neutralino couplings

At tree level, lepton neutralinos not only have relations with leptons and sleptons, but also act with neutrinos and sneutrinos:

$$
\begin{align*}
& \mathcal{L}\left(\chi_{\mathrm{L}}^{0} l \tilde{l}\right)=\sum_{\alpha, j=1}^{6} \sum_{I, J, j=1}^{3} \bar{\chi}_{N_{\alpha}}\left(\left[-\left(\lambda_{N_{\mathrm{c}}}^{I J}\right.\right.\right. \\
& \left.\quad+\lambda_{N_{\mathrm{c}}}^{J I}\right) Z_{N_{v}}^{(I+3) \alpha} Z_{N_{\mathrm{L}}}^{3 i} Z_{\tilde{\mathrm{v}}}^{(J+3) j *} \\
& \left.\quad+\sqrt{2} g_{\mathrm{L}} Z_{N_{\mathrm{L}}}^{1 i} Z_{N_{v}}^{I \alpha} Z_{\tilde{v}}^{J j *} \delta_{I J}\right] P_{\mathrm{L}} \\
& \left.\quad-\sqrt{2} g_{\mathrm{L}} Z_{N_{\mathrm{L}}}^{1 i *} Z_{N_{v}}^{(I+3) \alpha *} Z_{\tilde{\mathrm{v}}}^{(J+3) j *} \delta_{I J} P_{R}\right) \chi_{\mathrm{L}_{i}^{0}} \tilde{N}^{j *} \\
& \quad+\sum_{i, I=1}^{3} \sum_{j=1}^{6} \sqrt{2} g_{\mathrm{L}} \bar{\chi}_{\mathrm{L}_{i}^{0}}\left(Z_{N_{\mathrm{L}}}^{1 i} Z_{\tilde{\mathrm{L}}}^{I j} P_{\mathrm{L}}\right. \\
& \left.\quad-Z_{N_{\mathrm{L}}}^{1 i *} Z_{\tilde{\mathrm{L}}}^{(I+3) j *} P_{R}\right) e^{I} \tilde{L}_{j}^{-*}+\text { h.c. } \tag{14}
\end{align*}
$$

The couplings for lepton neutralino-exotic lepton-exotic slepton and lepton neutralino-exotic neutrino-exotic sneutrino read as

$$
\begin{aligned}
& \mathcal{L}\left(\chi_{\mathrm{L}}^{0} l^{\prime} \tilde{l}^{\prime}\right)=\sum_{i=1}^{3} \sum_{j, k=1}^{2}\left\{L _ { 4 } \sqrt { 2 } g _ { \mathrm { L } } \overline { \chi } _ { L _ { i } ^ { 0 } } \left(Z_{N_{\mathrm{L}}}^{1 i} U_{\mathrm{L}}^{1 j} Z_{\tilde{e}_{4}}^{1 k *} P_{\mathrm{L}}\right.\right. \\
& \left.\quad-Z_{N_{\mathrm{L}}}^{1 i *} Z_{\tilde{e}_{4}}^{2 k *} W_{\mathrm{L}}^{2 j} P_{R}\right) L_{j+3}^{\prime} \tilde{E}_{4}^{k *} \\
& \quad+L_{4} \sqrt{2} g_{\mathrm{L}} \bar{\chi}_{\mathrm{L}_{i}^{0}}\left(Z_{N_{\mathrm{L}}}^{1 i} U_{N}^{1 j} Z_{\tilde{v}_{4}}^{1 k *} P_{\mathrm{L}}\right.
\end{aligned}
$$

$$
\begin{align*}
& \left.-Z_{N_{\mathrm{L}}}^{1 i *} Z_{\tilde{v}_{4}}^{2 k} W_{N}^{2 j} P_{R}\right) N_{j+3}^{\prime} \tilde{N}_{4}^{k *} \\
& +\sqrt{2}\left(3+L_{4}\right) g_{\mathrm{L}} \bar{\chi}_{\mathrm{L}_{i}}^{0}\left(Z_{N_{\mathrm{L}}}^{1 i} Z_{\tilde{e}_{5}}^{1 j_{\mathrm{L}}} U_{\mathrm{L}}^{2 k} P_{\mathrm{L}}\right. \\
& \left.-Z_{N_{\mathrm{L}}}^{1 i *} W_{\mathrm{L}}^{1 k} Z_{\tilde{e}_{5}}^{2 j^{*}} P_{R}\right) L_{k+3}^{\prime} \tilde{E}_{5}^{j *} \\
& +\sqrt{2}\left(3+\mathrm{L}_{4}\right) g_{\mathrm{L}} \bar{\chi}_{\mathrm{L}_{i}}^{0}\left(Z_{N_{\mathrm{L}}}^{1 i} Z_{\tilde{v}_{5}}^{1 j *} U_{N}^{2 k} P_{\mathrm{L}}\right. \\
& \left.\left.-Z_{N_{\mathrm{L}}}^{1 i *} W_{N}^{1 k} Z_{\tilde{v}_{5}}^{2 j *} P_{R}\right) N_{k+3}^{\prime} \tilde{N}_{5}^{j *}\right\}+ \text { h.c. } \tag{15}
\end{align*}
$$

From the interactions of gauge and matter multiplets, we write down the couplings of lepton neutralino-lepton neutralino-lepton Higgs

$$
\begin{align*}
& \mathcal{L}\left(\chi_{\mathrm{L}}^{0} \chi_{\mathrm{L}}^{0} H_{\mathrm{L}}^{0 *}\right)=2 \sqrt{2} g_{\mathrm{L}} \sum_{i, j=1}^{3} \sum_{k=1}^{2} Z_{N_{\mathrm{L}}}^{1 i}\left(Z_{N_{\mathrm{L}}}^{3 j} Z_{\phi_{\mathrm{L}}}^{2 k *}\right. \\
& \left.\quad-Z_{N_{\mathrm{L}}}^{2 j} Z_{\phi_{\mathrm{L}}}^{1 k *}\right) \bar{\chi}_{\mathrm{L}_{i}}^{0} P_{\mathrm{L}} \chi_{\mathrm{L}_{j}}^{0} H_{\mathrm{L}_{k}}^{0 *}+\text { h.c. } \tag{16}
\end{align*}
$$

### 4.3 Baryon neutralino couplings

Baryon neutralinos interact with quarks and squarks, and their couplings are in the following form:

$$
\begin{align*}
& \mathcal{L}\left(\chi_{\mathrm{B}}^{0} q \tilde{q}\right)=\sum_{I, i=1}^{3} \sum_{j=1}^{6}\left\{\frac { \sqrt { 2 } } { 3 } g _ { \mathrm { B } } \overline { \chi } _ { \mathrm { B } _ { i } ^ { 0 } } \left(Z_{N_{\mathrm{B}}}^{1 i} Z_{\tilde{U}}^{I j *} P_{\mathrm{L}}\right.\right. \\
& \left.\quad-Z_{N_{\mathrm{B}}}^{1 i *} Z_{\tilde{U}}^{(I+3) j *} P_{R}\right) u^{I} \tilde{U}_{j}^{*}+\frac{\sqrt{2}}{3} g_{\mathrm{B}} \bar{\chi}_{\mathrm{B}_{i}^{0}}\left(Z_{N_{\mathrm{B}}}^{1 i} Z_{\tilde{D}}^{I j} P_{\mathrm{L}}\right. \\
& \left.\left.\quad-Z_{N_{\mathrm{B}}}^{1 i *} Z_{\tilde{D}}^{(I+3) j *} P_{R}\right) d^{I} \tilde{D}_{j}^{*}\right\}+ \text { h.c. } \tag{17}
\end{align*}
$$

Similarly the couplings of baryon neutralino-exotic quark-exotic squark are deduced here:

$$
\begin{aligned}
& \mathcal{L}\left(\chi_{\mathrm{B}}^{0} q^{\prime} \tilde{q}^{\prime}\right)=\sum_{i=1}^{3} \sum_{j=1}^{6} \sum_{k=1}^{2}\left\{\sqrt{2} g_{\mathrm{B}} \bar{t}_{k+3}[-((1\right. \\
& \left.\quad+B_{4}\right) W_{1 k}^{t^{\prime} *} U_{3 j} Z_{N_{\mathrm{B}}}^{1 i}+\lambda_{Q} U_{1 j} W_{1 k}^{t^{\prime} *} Z_{N_{\mathrm{B}}}^{2 i} \\
& \left.\quad+\lambda_{U} W_{2 k}^{t^{\prime} *} Z_{N_{\mathrm{B}}}^{3 i} U_{4 j}+B_{4} U_{2 j} W_{2 k}^{t^{\prime} *} Z_{N_{\mathrm{B}}}^{1 i}\right) P_{\mathrm{L}} \\
& \quad+\left(B_{4} U_{1 k}^{t^{\prime} *} U_{1 j} Z_{N_{\mathrm{B}}}^{1 i *}+\left(1+B_{4}\right) U_{4 j} U_{2 k}^{t^{\prime *} *} Z_{N_{\mathrm{B}}}^{1 i *}\right. \\
& \left.\left.\quad-\lambda_{\mathrm{Q}}^{*} U_{1 k}^{t^{\prime} *} U_{3 j} Z_{N_{\mathrm{B}}}^{2 i *}-\lambda_{U}^{*} U_{2 j} U_{2 k}^{t^{\prime} *} Z_{N_{\mathrm{B}}}^{3 i *}\right) P_{R}\right] \chi_{\mathrm{B}_{i}}^{0} \tilde{\mathcal{U}}_{j} \\
& \quad+\sqrt{2} g_{\mathrm{B}}{\overline{b^{\prime}}}_{k+3}\left[\left(\lambda_{\mathrm{Q}} D_{1 j} W_{1 k}^{b^{\prime} *} Z_{N_{\mathrm{B}}}^{2 i}\right.\right.
\end{aligned}
$$

$$
\begin{align*}
& -B_{4} D_{2 j} W_{2 k}^{b^{\prime} *} Z_{N_{\mathrm{B}}}^{1 i}-\left(1+B_{4}\right) W_{1 k}^{b^{\prime} *} D_{3 j} Z_{N_{\mathrm{B}}}^{1 i} \\
& \left.-\lambda_{\mathrm{D}} W_{2 k}^{b^{\prime} *} Z_{N_{\mathrm{B}}}^{3 i} D_{4 j}\right) P_{\mathrm{L}}+\left(\lambda_{\mathrm{Q}}^{*} U_{1 k}^{b^{\prime} *} D_{3 j} Z_{N_{\mathrm{B}}}^{2 i *}\right. \\
& -\lambda_{\mathrm{D}}^{*} U_{2 k}^{b^{\prime} *} D_{2 j} Z_{N_{\mathrm{B}}}^{3 i *}+\left(1+B_{4}\right) D_{4 j} U_{2 k}^{b^{\prime} *} Z_{N_{\mathrm{B}}}^{1 i *} \\
& \left.\left.\left.+U_{1 k}^{b^{\prime} *} D_{1 j} Z_{N_{\mathrm{B}}}^{1 i *}\right) P_{R}\right] \chi_{\mathrm{B}_{i}}^{0} \tilde{\mathcal{D}}_{j}\right\}+ \text { h.c. } \tag{18}
\end{align*}
$$

Besides the baryon neutralino-baryon neutralino-baryon Higgs couplings, there are also interactions among baryon neutralinos and $X$ fields:

$$
\begin{align*}
& \mathcal{L}\left(\chi_{\mathrm{B}}^{0} \chi_{\mathrm{B}}^{0} H_{\mathrm{B}}\right)= \sqrt{2} g_{\mathrm{B}} \sum_{i, j=1}^{3} \sum_{k=1}^{2}\left(Z_{N_{\mathrm{B}}}^{2 j} Z_{\phi_{\mathrm{B}}}^{1 k *}\right. \\
&\left.-Z_{N_{\mathrm{B}}}^{3 j} Z_{\phi_{\mathrm{B}}}^{2 k *}\right) Z_{N_{\mathrm{B}}}^{1 i} \bar{\chi}_{\mathrm{B}_{j}}^{0} P_{\mathrm{L}} \chi_{\mathrm{B}_{i}}^{0} H_{\mathrm{B}_{k}}^{0 *}, \\
& \mathcal{L}\left(\chi_{\mathrm{B}}^{0} \psi_{X} \tilde{X}\right)=\left(\frac{2}{3}+B_{4}\right) \sqrt{2} g_{\mathrm{B}} \sum_{i=1}^{3} \sum_{j=1}^{2} Z_{N_{\mathrm{B}}}^{1 i} \bar{\chi}_{\mathrm{B}_{i}}^{0}\left[Z_{X}^{1 j *} P_{\mathrm{L}}\right. \\
&\left.-Z_{X}^{2 j *} P_{R}\right] \psi_{X} \tilde{X}_{j}^{*}+\text { h.c. } \tag{19}
\end{align*}
$$

## 5 Couplings of neutrinos beyond the MSSM

Because of the non-zero masses and mixing angles of light neutrinos, physicists are interested in neutrino physics which implies lepton number violation in the Universe. In the MSSM, the neutrino couplings are obtained, so we deduce the neutrino couplings beyond the MSSM in this work. From the supperpotential $\mathcal{W}_{\mathrm{L}}$ and the interactions of gauge and matter multiplets, we obtain $\mathcal{L}^{1}(v)$ and $\mathcal{L}^{2}(v) . \mathcal{L}^{1}(v)$ includes the neutrino couplings with Higgs: 1. neutrino-neutrino-neutral $C P$-odd Higgs; 2. neutrino-neutrino-neutral $C P$-even Higgs; 3. neutrino-lepton-charged Higgs; 4. neutrino-neutrino-lepton Higgs

$$
\begin{align*}
& \mathcal{L}^{1}(v)=\sum_{I, J=1}^{3}\left\{\sum_{\alpha=1}^{6} \sum_{j=1}^{2} Y_{v}^{I J} Z_{N_{v}}^{(J+3) \alpha} Z_{H}^{2 j} \bar{\chi}_{N_{\alpha}}^{0} P_{\mathrm{L}} e^{I} H_{j}^{+}\right. \\
& \quad-\sum_{\alpha, \beta=1}^{6} \sum_{j=1}^{2} \frac{i}{2} Y_{v}^{I J} Z_{N_{\nu}}^{I \alpha} Z_{N_{\nu}}^{(3+J) \beta} Z_{H}^{2 j} \bar{\chi}_{N_{\beta}}^{0} P_{\mathrm{L}} \chi_{N_{\alpha}}^{0} A_{j}^{0} \\
& -\sum_{\alpha, \beta=1}^{6} \sum_{j=1}^{2} \frac{1}{2} Y_{v}^{I J} Z_{N_{\nu}}^{I \alpha} Z_{N_{\nu}}^{(3+J) \beta} Z_{R}^{2 j} \bar{\chi}_{N_{\beta}}^{0} P_{\mathrm{L}} \chi_{N_{\alpha}}^{0} H_{j}^{0} \\
& \left.-\sum_{\alpha, \beta=1}^{6} \sum_{i=1}^{3} \lambda_{N^{c}}^{I J} Z_{N_{v}}^{(I+3) \alpha} Z_{N_{v}}^{(J+3) \beta} Z_{\phi_{\mathrm{L}}}^{2 i} \bar{\chi}_{N_{\alpha}}^{0} P_{\mathrm{L}} \chi_{N_{\beta}}^{0} H_{\mathrm{L}_{i}}^{0}\right\} \\
& + \text { h.c. } \tag{20}
\end{align*}
$$

$\mathcal{L}^{2}(v)$ is composed of the couplings: 1. neutrino-sneutrino-MSSM neutralino; 2 . neutrino-sneutrino-
lepton neutralino; 3. neutrino-slepton-chargino:

$$
\begin{align*}
& \mathcal{L}^{2}(v)=\sum_{I, J=1}^{3} \sum_{\alpha, j=1}^{6}\left\{-\sum_{i=1}^{4} \bar{\chi}_{i}^{0}\left(Y_{v}^{I J} Z_{N}^{4 i} Z_{N_{v}}^{I \alpha} Z_{\tilde{v}}^{(J+3) j *} P_{\mathrm{L}}\right.\right. \\
& \left.\quad+Y_{v}^{I J *} Z_{N}^{4 i *} Z_{N_{v}}^{(J+3) \alpha *} Z_{\tilde{v}}^{I j *} P_{R}\right) \chi_{N_{\alpha}}^{0} \tilde{N}^{j *} \\
& \quad+\sum_{i=1}^{3}\left[\sqrt{2} g_{\mathrm{L}} Z_{N_{\mathrm{L}}}^{1 i} Z_{N_{v}}^{I \alpha} Z_{\tilde{v}}^{J j *} \delta_{I J}-\left(\left(\lambda_{N^{c}}^{I J}+\lambda_{N^{c}}^{J I}\right) Z_{N_{\mathrm{L}}}^{3 i}\right.\right. \\
& \left.\left.\quad+\sqrt{2} g_{\mathrm{L}} Z_{N_{\mathrm{L}}}^{1 i} \delta^{I J}\right) Z_{N_{v}}^{(I+3) \alpha} Z_{\tilde{\mathrm{v}}}^{(J+3) j *}\right] \bar{\chi}_{\mathrm{L} i}^{0} P_{\mathrm{L}} \chi_{N_{\alpha}}^{0} \tilde{N}^{j *} \\
& \left.\quad-\sum_{i=1}^{2} Y_{v}^{I J} Z_{+}^{2 i} Z_{N_{v}}^{(J+3) \alpha} Z_{\tilde{\mathrm{L}}}^{I j *} \bar{\chi}_{N_{\alpha}}^{0} P_{\mathrm{L}} \chi_{i}^{+} \tilde{L}_{j}\right\}+ \text { h.c. } \tag{21}
\end{align*}
$$

These neutrino couplings are favourable for the study of neutrinos in the BLMSSM.

## 6 Conclusion

In this work, we have briefly introduced the main content of the BLMSSM, which is an extension of the MSSM with local gauged B and L. In this model, there are new neutralinos and right handed neutrinos compared with the MSSM. We deduced the Feynman rules for neutrinos and neutralinos, and these can be used to further study neutrino masses and neutralinos in the BLMSSM. We also showed the mass matrices of particles such as lepton neutralinos and baryon neutralinos. Diagonalizing the corresponding mass mixing matrices one can obtain 3 lepton neutralino and 3 baryon neutralino masses.

We deduced the new couplings of the MSSM neutralinos, including MSSM neutralino-exotic leptonexotic slepton, MSSM neutralino-neutrino-sneutrino and MSSM neutralino-exotic quark-exotic squark. The couplings of lepton neutralinos were obtained from the superpotential and the interactions of gauge and matter multiplets: lepton neutralino-lepton-slepton, lepton neutralino-exotic lepton-exotic slepton and lepton neutralino-lepton neutralino-lepton Higgs. The baryon neutralino couplings were deduced in the same way: baryon neutralino-quark-squark, baryon neutralinoexotic quark-exotic squark, baryon neutralino-baryon neutralino-baryon Higgs, and baryon neutralinos- $\tilde{X}-\psi_{X}$. Finally, we obtained the couplings of neutrinos beyond the MSSM, which can be divided as: neutrino-neutrinoneutral $C P$-odd Higgs; neutrino-neutrino-neutral $C P$ even Higgs; neutrino-lepton-charged Higgs; neutrino-neutrino-lepton Higgs; neutrino-sneutrino-MSSM neutralino; neutrino-sneutrino-lepton neutralino; and neutrino-slepton-chargino. The obtained couplings are favourable for the study of neutrinos and processes relating to neutralinos in the BLMSSM.

## References

1 K. Abe et al (T2K Collab), Phys. Rev. Lett., 107: 041801 (2011)

2 P. Adamson et al (MINOS Collab), Phys. Rev. Lett., 107: 181802 (2011)
3 Y. Abe et al (DOUBLE-CHOOZ Collab), Phys. Rev. Lett.,
108: 131801 (2012)
4 F. An et al (DAYA-BAY Collab), Phys. Rev. Lett., 108:
14 S.T. Iqbal and Z. Lei, J. Phys. G, 42: 095003 (2015)
15 Y.-L. Tang, Nucl. Phys. B, 890: 263 (2014)
16 H.P. Nilles, Phys. Rept., 110: 1 (1984)
17 P. F. Perez and M. B. Wise, JHEP, 1108: 068 (2011)
18 P. F. Perez and M. B. Wise, Phys. Rev. D, 82: 011901 (2010)
19 P. F. Perez, Phys. Lett. B, 711: 353 (2012)

5 171803 (2012) $\quad$ R. Allahverdi, S. Bornhauser, B. Dutta et al Phys. Rev. D, 80:
20 J.M. Arnold, P. F. Perez, B. Fornal, and S. Spinner, Phys. Rev. D, 85: 115024 (2012)
$6 \quad 055026$ (2009) $\quad$ Allahverdi, S. Campbell, and B Dutta, Phys, Rev D, $\mathbf{8 5}$.
K. Abe et al, Phys. Rev. Lett., 107: 041801 (2011)

22 J. Ahn et al, Phys. Rev. Lett., 108: 191802 (2012) 035004 (2012)
7 R. Allahverdi, S. S. Campbell, B. Dutta, and Y. Gao, Phys. Rev. D, 90: 073002 (2014)
8 J. Rosiek, Phys. Rev. D, 41: 3464 (1990)
9 T.-F. Feng and X.-Y. Yang, Nucl. Phys. B, 814: 101 (2009)
10 H.P. Nilles, Phys. Rept., 110: 1 (1984)
11 H.E. Haber and G.L. Kane, Phys. Rept., 117: 75 (1985)
12 D.G. Cerdeno, M. Peiro, and S. Robles, Phys. Rev. D, 91: 123530 (2015)
13 D. G. Cerdeno and O. Seto, JCAP, 0908: 032 (2009)
24 S. M. Zhao, T. F. Feng, B. Yan et al, JHEP, 10: 020 (2013)
25 S. M. Zhao, T. F. Feng, H. B. Zhang et al, JHEP, 11: 119 (2014)

26 F. Sun, T. F. Feng, S. M. Zhao et al, Nucl. Phys. B, 888: 30-51 (2014)

27 P. Fileviez Perez and M.B. Wise, Phys. Rev. D, 84: 055015 (2011)

28 T. F. Feng, S. M. Zhao, H. B. Zhang et al, Nucl. Phys. B, 871: 223 (2013)
29 S. M. Zhao, T. F. Feng, X. J. Zhan et al, JHEP, 07: 124 (2015)
30 S. M. Zhao, T. F. Feng, H. B. Zhang et al, Phys. Rev. D, 92: 115016 (2015)


[^0]:    Received 19 February 2016，Revised 16 May 2016
    ＊Supported by Major Project of NNSFC（11535002）and NNSFC（11275036），Natural Science Foundation of Hebei Province （A2016201010），and Foundation of Hebei Province（BR2－201），and the Natural Science Fund of Hebei University（2011JQ05，2012－242）， Hebei Key Lab of Optic－Electronic Information and Materials，Midwest Universities Comprehensive Strength Promotion Project

    1）E－mail：dxx－0304＠163．com
    2）E－mail：zhaosm＠hbu．edu．cn
    

    Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence．Any further distribution of this work must maintain attribution to the author（s）and the title of the work，journal citation and DOI．Article funded by SCOAP ${ }^{3}$ and published under licence by Chinese Physical Society and the Institute of High Energy Physics of the Chinese Academy of Sciences and the Institute of Modern Physics of the Chinese Academy of Sciences and IOP Publishing Ltd

