Can X(5568) be a tetraquark state? *

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Abstract: Very recently, the D0 collaboration has reported the observation of a narrow structure, X(5568), in the decay process X(5568)$\rightarrow \mathrm{B}_s^0\pi^\pm$ using the 10.4fb^{-1} data of $\mathrm{p\bar{p}}$ collision at $\sqrt{s}=1.96$ TeV. This structure is of great interest since it is the first hadronic state with four different valence quark flavors, b, s, u, d. In this work, we investigate tetraquarks with four different quark flavors. Based on the diquark-antidiquark scheme, we study the spectroscopy of the tetraquarks with one heavy bottom/charm quark and three light quarks. We find that the lowest-lying S-wave state, a tetraquark with the flavor [su][$\mathrm{\bar{b}\bar{d}}$] and the spin-parity $J^P=0^+$, is about 150 MeV higher than the X(5568). Further detailed experimental and theoretical studies of the spectrum, production and decays of tetraquark states with four different flavors are vital to gain a better understanding of the nature and classification of hadron exotic states.

Keywords: exotic state, diquark, tetraquark

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Since the proposal of the concept of quarks by Gell-Mann [1], there have been great endeavors to test the quark model and search for exotic structures beyond this scheme. To date hundreds of hadrons have been discovered, and most of them can be accommodated in the naive quark model, in which mesons and baryons are composed of a quark–antiquark pair and three quarks, respectively. No firm evidence for the existence of exotic states beyond the quark model was established experimentally until the discovery of the X(3872) in 2003 [2–5]. The peculiar properties of the X(3872), the closeness of its mass to the $\mathrm{D\bar{D}^*}$ threshold, its tiny width and the large isospin violation in its production and decay, has sparked a renaissance of hadron spectroscopy studies. Since then one key topic in hadron physics has been the identification of the exotic hadrons. Many new interesting structures were discovered in the mass region of heavy quarkonium, named the XYZ states (for a review of these particles, see Refs. [6–9]). In particular, the charged structures with a hidden pair of heavy quark and antiquark such as the $\mathrm{Z_c^\pm}(4430)$ [10, 11], $\mathrm{Z_b^\pm}(10610,10650)$ [12], $\mathrm{Z_c^\pm}(3900)$ [13, 14], and $\mathrm{Z_c^\pm}(4020)$ [15] are undoubtedly exotic resonances. Moreover, candidates for exotic hadrons were extended to the pentaquark sector by the LHCb observations of two structures in the $\mathrm{J/\psi\, p}$ invariant mass distribution with masses (widths) $(4380\pm8\pm29)$ MeV ($(205\pm18\pm86)$ MeV) and $(4449.8\pm1.7\pm2.5)$ MeV ($(39\pm5\pm19)$ MeV), respectively [16]. These discoveries have opened up a new era of multi-quark spectroscopy and strengthened our belief that the hadron spectrum would be much richer than the quark model.

Most of the observed exotic X, Y, Z structures so far share a common feature, i.e. they consist of a hidden heavy quark-antiquark pair, $\mathrm{\bar{b}b}$ or $\mathrm{\bar{c}c}$. Various theoretical models aim to explain these exotics, many of them making use of heavy quark symmetry and chiral symmetry. Inspired by these symmetries, various combinations of heavy and light mesons have been examined and can be searched for. Of particular interest are those composed of a heavy meson and a light meson. Very recently, the D0 collaboration reported the first observation of such a structure in the final state $\mathrm{B}_s^0\pi^\pm$ [17]. Since the $\mathrm{B}_s^0\pi^\pm$ final state is made of four different flavors, b, s, u, d, the

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new structure is definitely exotic. Its mass and width have been determined as [17]

$$M_{\mathrm{X}}=(5567.8\pm2.9)\mathrm{MeV},\ \ \Gamma_{\mathrm{X}}=(21.9\pm6.4)\mathrm{MeV}. \quad (1)$$

The above results are obtained through a fit based on a Breit-Wigner parametrization of the S-wave decay of $X(5568)\to\mathrm{B}_{\mathrm{s}}^0\pi^{\pm}$. The statistical significance, including the look-elsewhere effect and systematic errors, is about 5.1σ [17].

After this first discovery, more experimental efforts to determine the properties of the X(5568) are needed. Meanwhile, theoretical interpretations of its nature are also required. The X(5568) is too far from the the $\mathrm{B}_{\mathrm{d}}^0\mathrm{K}^{\pm}$ threshold (5774 MeV) to be interpreted as a hadronic molecule of $\mathrm{B}_{\mathrm{d}}^0\mathrm{K}^{\pm}$. In addition, the interaction of $\mathrm{B}_s^0\pi^{\pm}$ is very weak and unable to form a bounded structure [18]. In this work, we will study tetraquark states using colored components, diquark and antidiquark, and bound by the long-range color forces. Tetraquark states in the large N_{c} limit of QCD has been explored in Refs. [19–21], which indicates that a compact tetraquark meson may have narrow decay widths which scale as $1/N_{\mathrm{c}}$. This gives reasonable candidates for the additional spectroscopic hadron series apart from the quark model. We want to see whether the X(5568) is a tetraquark state. In the past decades, tetraquark states, in particular those with hidden bottom and charm, have been explored in Refs. [22–34] and many references therein.

The QCD confining potential for the multiquark system can be generally written as [35]

$$V(\vec{r}_i)=L(\vec{r}_1,\vec{r}_2,\ldots)+\sum_{i>j}I\,\alpha_s S_{ij}\ , \quad (2)$$

where the $L(\vec{r}_1,\vec{r}_2,\ldots)$ stands for the universal binding interaction of quarks. The S_{ij} is two-body Coulomb and chromomagnetic interactions, with the $I=-4/3$ and $-2/3$ as the single gluon interaction strength in the quark-antiquark and quark-quark cases, respectively.

In the following, we will consider the tetraquark states with quark content $[\mathrm{qq}'][\bar{\mathrm{q}}\bar{\mathrm{Q}}]$, where q and q′ denote the light quarks, and Q denotes a heavy quark, bottom or charm. The effective Hamiltonian is composed of three kinds of interactions: spin-spin interactions of quarks in the diquark and antidiquark, and between them; the spin-orbital interaction; and purely orbital interactions. An explicit model that incorporates these interactions has been established in Ref. [24]:

$$H = m_\delta+m_{\delta'}+H_{SS}^{\delta}+H_{SS}^{\bar{\delta}'}+H_{SS}^{\delta\bar{\delta}'}+H_{SL}+H_{LL}, \quad (3)$$

with the functions

$$\begin{aligned}
H_{SS}^{\delta}&=2(\kappa_{\mathrm{qq}'})_{\bar{3}}(\mathbf{S}_{\mathrm{q}}\cdot\mathbf{S}_{\mathrm{q}'}),\\
H_{SS}^{\bar{\delta}'}&=2(\kappa_{\mathrm{Qq}})_{\bar{3}}(\mathbf{S}_{\bar{\mathrm{Q}}}\cdot\mathbf{S}_{\bar{\mathrm{q}}}),\\
H_{SS}^{\delta\bar{\delta}'}&=2\kappa_{\mathrm{q}\bar{\mathrm{q}}}(\mathbf{S}_{\mathrm{q}}\cdot\mathbf{S}_{\bar{\mathrm{q}}})+2\kappa_{\mathrm{q}'\bar{\mathrm{q}}}(\mathbf{S}_{\mathrm{q}'}\cdot\mathbf{S}_{\bar{\mathrm{q}}})\\
&\quad+2\kappa_{\mathrm{q}\bar{\mathrm{Q}}}(\mathbf{S}_{\mathrm{q}}\cdot\mathbf{S}_{\bar{\mathrm{Q}}})+2\kappa_{\mathrm{q}'\bar{\mathrm{Q}}}(\mathbf{S}_{\mathrm{q}'}\cdot\mathbf{S}_{\bar{\mathrm{Q}}}),\\
H_{SL}&=2A_\delta(\mathbf{S}_\delta\cdot\mathbf{L})+2A_{\bar{\delta}'}(\mathbf{S}_{\bar{\delta}'}\cdot\mathbf{L}),\\
H_{LL}&=B_{\delta\bar{\delta}'}\frac{L(L+1)}{2}\ .
\end{aligned} \quad (4)$$

In the above, m_δ and $m_{\delta'}$ are the constituent mass of the diquark $[\mathrm{qq}']$ and the antidiquark $[\bar{\mathrm{q}}\bar{\mathrm{Q}}]$, respectively. The spin-spin interaction inside the diquark and antidiquark is denoted as H_{SS}^{δ} and $H_{SS}^{\bar{\delta}'}$, respectively. $H_{SS}^{\delta\bar{\delta}'}$ reflects the spin-spin interaction of quarks between diquark and antidiquark. H_{SL} and H_{LL} are the spin-orbital and purely orbital terms respectively. The $\mathbf{S}_\delta$ and $\mathbf{S}_{\bar{\delta}'}$ corresponds to the spin operator of diquark and antidiquark, respectively. The spin operator of light quarks and heavy antiquark is given by $\mathbf{S}_{\mathrm{q}^{(\prime)}}$ and $\mathbf{S}_{\bar{\mathrm{Q}}}$, respectively. The symbol $\mathbf{L}$ denotes the orbital angular momentum operator. The coefficients $\kappa_{\mathrm{q}_1\bar{\mathrm{q}}_2}$ and $(\kappa_{\mathrm{q}_1\mathrm{q}_2'})_{\bar{3}}$ are the spin-spin couplings for a quark-antiquark pair and diquark in color antitriplet, respectively; $A_{\delta(\bar{\delta}')}$ and $B_{\delta\bar{\delta}'}$ denote respectively spin-orbit and orbit-orbit couplings.

For the lowest-lying tetraquark states with the quark content $[\mathrm{qq}'][\bar{\mathrm{q}}\bar{\mathrm{Q}}]$, their orbital angular momenta are vanishing, i.e. $L=0$. Among them, there are two possible tetraquark configurations with the spin-parity $J^P=0^+$, i.e.,

$$\begin{aligned}
|0_J\rangle_1&=\frac{1}{2}\big[(\uparrow)_{\mathrm{q}}(\downarrow)_{\mathrm{q}'}-(\downarrow)_{\mathrm{q}}(\uparrow)_{\mathrm{q}'}\big]\big[(\uparrow)_{\bar{\mathrm{q}}}(\downarrow)_{\bar{\mathrm{Q}}}-(\downarrow)_{\bar{\mathrm{q}}}(\uparrow)_{\bar{\mathrm{Q}}}\big],\\
|0_J\rangle_2&=\frac{1}{\sqrt{3}}\big\{(\uparrow)_{\mathrm{q}}(\uparrow)_{\mathrm{q}'}(\downarrow)_{\bar{\mathrm{q}}}(\downarrow)_{\bar{\mathrm{Q}}}+(\downarrow)_{\mathrm{q}}(\downarrow)_{\mathrm{q}'}(\uparrow)_{\bar{\mathrm{q}}}(\uparrow)_{\bar{\mathrm{Q}}}\\
&\quad-\frac{1}{2}\big[(\uparrow)_{\mathrm{q}}(\downarrow)_{\mathrm{q}'}+(\downarrow)_{\mathrm{q}}(\uparrow)_{\mathrm{q}'}\big](\uparrow)_{\bar{\mathrm{q}}}(\downarrow)_{\bar{\mathrm{Q}}}\\
&\quad-\frac{1}{2}\big[(\uparrow)_{\mathrm{q}}(\downarrow)_{\mathrm{q}'}+(\downarrow)_{\mathrm{q}}(\uparrow)_{\mathrm{q}'}\big](\downarrow)_{\bar{\mathrm{q}}}(\uparrow)_{\bar{\mathrm{Q}}}\big\}.
\end{aligned} \quad (5)$$

In the above, $|0_J\rangle_1=|0_\delta,0_{\bar{\delta}'},0_J\rangle$, $|0_J\rangle_2=|1_\delta,1_{\bar{\delta}'},0_J\rangle$, and $|S_\delta,S_{\bar{\delta}'},S_J\rangle$ stands for the tetraquark; S_δ and $S_{\bar{\delta}'}$ stand for the spin of diquark $[\mathrm{qq}']$ and antidiquark $[\bar{\mathrm{q}}\bar{\mathrm{Q}}]$, respectively, while S_J denotes the total angular momentum of the tetraquark. In this paper, we only focus on the scalar and vector diquarks, i.e. $S_{\delta^{(\prime)}}=0,1$.

Using the basis defined in Eq. (5), one can derive the mass matrix for the $J^P=0^+$ tetraquarks

$$M=m_\delta+m_{\delta'}+\begin{pmatrix} -\frac{3}{2}((\kappa_{\mathrm{qq}'})_{\bar{3}}+(\kappa_{\mathrm{Qq}})_{\bar{3}}) & \frac{\sqrt{3}}{2}(\kappa_{\mathrm{q}'\bar{\mathrm{Q}}}+\kappa_{\mathrm{q}\bar{\mathrm{q}}}-\kappa_{\mathrm{q}'\bar{\mathrm{q}}}-\kappa_{\mathrm{q}\bar{\mathrm{Q}}}) \\ \frac{\sqrt{3}}{2}(\kappa_{\mathrm{q}'\bar{\mathrm{Q}}}+\kappa_{\mathrm{q}\bar{\mathrm{q}}}-\kappa_{\mathrm{q}'\bar{\mathrm{q}}}-\kappa_{\mathrm{q}\bar{\mathrm{Q}}}) & \frac{1}{2}((\kappa_{\mathrm{qq}'})_{\bar{3}}+(\kappa_{\mathrm{Qq}})_{\bar{3}}-2\kappa_{\mathrm{q}'\bar{\mathrm{q}}}-2\kappa_{\mathrm{q}'\bar{\mathrm{Q}}}-2\kappa_{\mathrm{q}\bar{\mathrm{q}}}-2\kappa_{\mathrm{q}\bar{\mathrm{Q}}}) \end{pmatrix}. \quad (6)$$

For the $J^P=2^+$, there exits only one tetraquark configuration:

$$|1_\delta,1_{\bar{\delta}'},2_J\rangle=(\uparrow)_{\rm q}(\uparrow)_{\rm q'}(\uparrow)_{\bar{\rm q}}(\uparrow)_{\bar{\rm Q}}, \tag{7}$$

with the mass

$$M(2^+)=m_\delta+m_{\delta'}+\frac{1}{2}\left((\kappa_{\rm qq'})_{\bar{3}}+(\kappa_{\rm Qq})_{\bar{3}}\right)+\frac{1}{2}\left(\kappa_{\rm q\bar{q}}+\kappa_{\rm q\bar{Q}}+\kappa_{\rm q'\bar{q}}+\kappa_{\rm q'\bar{Q}}\right). \tag{8}$$

In the case $J^P=1^+$, there are three possible tetraquark states, i.e.,

$$\begin{aligned}|0_\delta,1_{\bar{\delta}'},1_J\rangle&=\frac{1}{\sqrt{2}}\left[(\uparrow)_{\rm q}(\downarrow)_{\rm q'}-(\downarrow)_{\rm q}(\uparrow)_{\rm q'}\right](\uparrow)_{\bar{\rm q}}(\uparrow)_{\bar{\rm Q}},\\ |1_\delta,0_{\bar{\delta}'},1_J\rangle&=\frac{1}{\sqrt{2}}(\uparrow)_{\rm q}(\uparrow)_{\rm q'}\left[(\uparrow)_{\bar{\rm q}}(\downarrow)_{\bar{\rm Q}}-(\downarrow)_{\bar{\rm q}}(\uparrow)_{\bar{\rm Q}}\right],\\ |1_\delta,1_{\bar{\delta}'},1_J\rangle&=\frac{1}{2}\left\{(\uparrow)_{\rm q}(\uparrow)_{\rm q'}\left[(\uparrow)_{\bar{\rm q}}(\downarrow)_{\bar{\rm Q}}+(\downarrow)_{\bar{\rm q}}(\uparrow)_{\bar{\rm Q}}\right]\right.\\ &\quad\left.-\left[(\uparrow)_{\rm q}(\downarrow)_{\rm q'}+(\downarrow)_{\rm q}(\uparrow)_{\rm q'}\right](\uparrow)_{\bar{\rm q}}(\uparrow)_{\bar{\rm Q}}\right\}.\end{aligned} \tag{9}$$

Unlike the heavy quarkonium-like states, the tetraquarks with the quark content $[\rm qq'][\bar{q}\bar{Q}]$ do not have any definite charge parity and thus the above three 1^+ states can mix with each other.

Using the basis defined in Eq. (9), one can obtain the mass splitting matrix ΔM for $J^P=1^+$

$$\Delta M=\begin{pmatrix}\frac{1}{2}((\kappa_{\rm Qq})_{\bar{3}}-3(\kappa_{\rm qq'})_{\bar{3}}) & \frac{1}{2}(\kappa_{\rm q'\bar{Q}}-\kappa_{\rm q'\bar{q}}-\kappa_{\rm q\bar{Q}}+\kappa_{\rm q\bar{q}}) & \frac{\sqrt{2}}{2}(\kappa_{\rm q\bar{Q}}-\kappa_{\rm q'\bar{Q}}-\kappa_{\rm q'\bar{q}}+\kappa_{\rm q\bar{q}})\\ \frac{1}{2}(\kappa_{\rm q'\bar{Q}}-\kappa_{\rm q'\bar{q}}-\kappa_{\rm q\bar{Q}}+\kappa_{\rm q\bar{q}}) & \frac{1}{2}((\kappa_{\rm qq'})_{\bar{3}}-3(\kappa_{\rm Qq})_{\bar{3}}) & \frac{\sqrt{2}}{2}(\kappa_{\rm q'\bar{Q}}-\kappa_{\rm q'\bar{q}}+\kappa_{\rm q\bar{Q}}-\kappa_{\rm q\bar{q}})\\ \frac{\sqrt{2}}{2}(\kappa_{\rm q\bar{Q}}-\kappa_{\rm q'\bar{Q}}-\kappa_{\rm q'\bar{q}}+\kappa_{\rm q\bar{q}}) & \frac{\sqrt{2}}{2}(\kappa_{\rm q'\bar{Q}}-\kappa_{\rm q'\bar{q}}+\kappa_{\rm q\bar{Q}}-\kappa_{\rm q\bar{q}}) & \frac{1}{2}((\kappa_{\rm qq'})_{\bar{3}}+(\kappa_{\rm Qq})_{\bar{3}}-\kappa_{\rm q'\bar{Q}}-\kappa_{\rm q'\bar{q}}-\kappa_{\rm q\bar{Q}}-\kappa_{\rm q\bar{q}})\end{pmatrix}, \tag{10}$$

and the mass matrix is given as

$$M=m_\delta+m_{\delta'}+\Delta M. \tag{11}$$

The spin-spin couplings have been extensively explored in previous analyses of mesons, baryons and the XYZ spectra in the quark model and diquark model. We quote the results from Refs. [24, 26, 28, 29] and summarize them in Table 1. The masses of diquarks [cq] and [bq] are determined through analysis of the X(3872) with $J^{PC}=1^{++}$ and $\rm Y_b(10890)$ with $J^{PC}=1^{--}$ in the diquark model, respectively. We quote $m_{\rm [cq]}=1.932$ GeV and $m_{\rm [bq]}=5.249$ GeV [26, 28, 36]. Using these results for the spin-spin, spin-orbit and orbit-orbit couplings and diquark masses, we can obtain the tetraquark spectrum. The tetraquark spectra with quantum numbers $J^P=0^+$, 1^+, and 2^+ are depicted in Fig. 1, in which (a) and (b) correspond to the tetraquark with a bottom and a charm quark respectively. The masses given in the figure are in units of GeV. The thresholds of the $\rm B_s\pi, B_s^*\pi, B_{s2}\pi$ and their charm analogues are shown in dashed lines. The masses of the X(5568) and the $\rm D_{s0}^*(2317)$ are also given in the figure.

Table 1. The spin-spin couplings (in units of MeV) for color-singlet quark-antiquark and color-antitriplet quark-quark pairs. The relation $\kappa_{ij}=(\kappa_{ij})_0/4$ for quark-antiquark coupling is obtained in the one gluon exchange model. Here q denotes the light u and d quarks, where we have adopted the isospin symmetry.

quark-antiquark	$\rm q\bar{q}$	$\rm s\bar{s}$	$\rm s\bar{q}$	$\rm c\bar{q}$	$\rm c\bar{s}$	$\rm c\bar{c}$	$\rm b\bar{q}$	$\rm b\bar{s}$	diquark	qq	ss	sq	cq	cs	bq
couplings $(\kappa_{ij})_0$	315	121	195	70	72	59	23	23	couplings $(\kappa_{ij})_{\bar{3}}$	103	72	64	22	25	6.6

Our results for tetraquarks with a charm quark in Fig. 1 are consistent with Ref. [24]. We find that the lowest tetraquark state is about 60 MeV higher than the discovered $\rm D_{s0}^*(2317)$. Switching to the bottom sector, we find this difference gets bigger: our prediction for the mass of the lower S-wave 0^+ tetraquark is about 150 MeV larger than the experimental result by D0 for the mass of the X(5568) in Eq. (1). The deviation in mass of the X(5568) still exists when reducing the diquark masses in a reasonable region. In the charm sector, the $\rm D_{s0}^*(2317)$ is about 70 MeV higher than the $\rm D_s^*\pi$ threshold, while in the bottom sector, the observed $X(5568)$ by D0 is only about 10 MeV higher than the $\rm B_s^*\pi$ threshold. In the heavy quark limit, the mass difference between the lowest tetraquark state and the $\rm D_s^*/B_s^*\pi$ presumably arises from the excitation of the light system, and might be at the same magnitude in the bottom sector and charm sector. Thus data may indicate that the X(5568) is too light to be the partner of the $\rm D_{s0}^*(2317)$. In order to simultaneously describe tetraquarks with four different flavors, a more comprehensive analysis is called for in future. In addition, the open bottom (charm) tetraquark states with strangeness number $S=-1$ constitute an isospin triplet and a singlet. Therein the tetraquark states with the quark content $[\mathrm{su}][\bar{\mathrm{b}}\bar{\mathrm{d}}]$, $[\mathrm{sd}][\bar{\mathrm{b}}\bar{\mathrm{u}}]$ and $1/\sqrt{2}([\mathrm{su}][\bar{\mathrm{b}}\bar{\mathrm{u}}]-[\mathrm{sd}][\bar{\mathrm{b}}\bar{\mathrm{d}}])$ constitute an isospin triplet, while the tetraquark with $1/\sqrt{2}([\mathrm{su}][\bar{\mathrm{b}}\bar{\mathrm{u}}]+[\mathrm{sd}][\bar{\mathrm{b}}\bar{\mathrm{d}}])$ is an isospin singlet. Due to the isospin symmetry, the masses of these tetraquark partners are identical to the values given in Fig. 1.

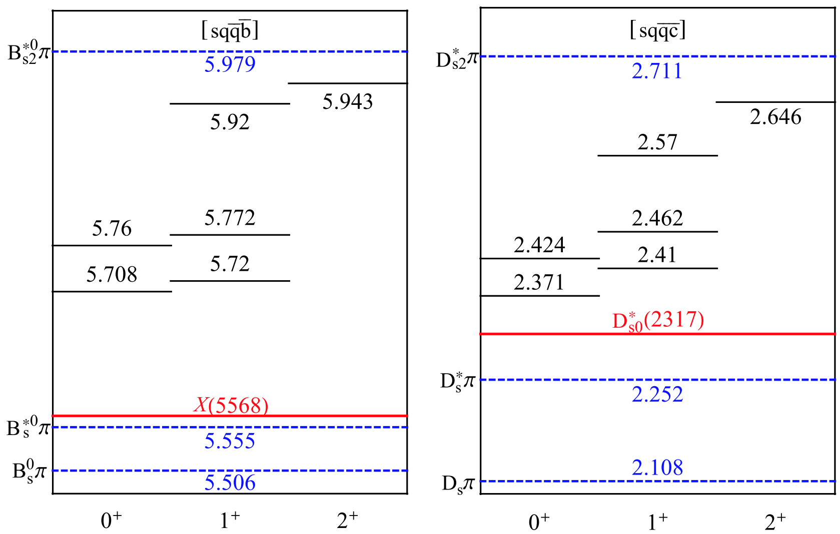

Fig. 1. The open bottom (a) and charm (b) tetraquark spectra in the diquark-antidiquark scheme. The masses are in units of GeV. The thresholds of the $\mathrm{B_s}\pi$, $\mathrm{B_s^*}\pi$, $\mathrm{B_{s2}}\pi$ and their charm analogues are shown in dashed lines. The masses of the X(5568) and the $\mathrm{D^*_{s0}}(2317)$ are also given.

In the past decades, the spectroscopy study of hadron exotics has played an important role in uncovering the hadron inner structure, and examining various models for hadrons with fundamental freedom. Many of the recently observed structures defy an ordinary interpretation as a $\bar{\mathrm{q}}\mathrm{q}$ meson or a qqq baryon. In this work, we have explored the tetraquarks with four different quark flavors. Based on the diquark-antidiquark scheme, we have calculated the spectroscopy of the tetraquarks with one heavy bottom/charm quark and three light quarks. We find that the lowest-lying S-wave state, a $J^P=0^+$ tetraquark with the flavor $[\mathrm{su}][\bar{\mathrm{b}}\bar{\mathrm{d}}]$, lies at around 5.7 GeV and is about 150 MeV higher than the X(5568). The identification of the X(5568) as a tetraquark with spin-parity $J^P=0^+$ is a challenge to the tetraquark model. The LHCb Collaboration did not see a signal for the X(5568) in an invariant mass scan of $\mathrm{B_s^0}\pi^{\pm}$ between 5.5 GeV and 5.7 GeV [37]. So it is worth searching for tetraquark states with four different flavors in the invariant mass of $\mathrm{B_s^0}\pi^{\pm}$ beyond 5.7 GeV. Further detailed experimental and theoretical studies of the spectrum, production and decays of tetraquark states with four different flavors in the future are called for towards a better understanding of the nature and the classification of hadron exotics.

Note added: After this work was finished, a series of papers also investigated the structure of the X(5568), with a diquark-antidiquark interpretation employed in Refs. [38–43]; the threshold rescattering effect and hadronic molecule structure are studied in Ref. [44] and Ref. [45] respectively; and decay relations in flavor $SU(3)$ symmetry are studied in Ref. [46].

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